PHYSICS 106
LABORATORY MANUAL

PHYSICS FOR THE LIFE SCIENCES II

Spring Semester, 2016

Contents

Introduction

> When you can measure what you are speaking about, and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science. *Lord Kelvin*

Purpose

Why does this course include a laboratory? In class you will be studying physical laws, many in the form of equations, and you will probably become proficient at manipulating the symbols and speaking the vocabulary of physics. Yet it is possible to do this without a clear understanding of what those symbols and formulas represent. For example, in the lecture part of this course you will study the equation

$$V = IR,$$

where V represents electrical potential, I is electric current, and R is resistance. These words may become familiar, and the symbols easy to use in solving problems from the text, but this does not assure that you understand potential, current, resistance, or the relationship among them. Physics deals with quantities that are *measurable*, and one of the best ways to find meaning in a physical law is to perform measurements of the quantities it relates. When you have learned how to measure potential, you may have a better idea of what potential is. So one purpose of the lab is to reinforce some concepts through the experience of making measurements. Another main purpose of the lab is to teach techniques in taking and analyzing data, and in reporting results. There will also be some emphasis on evaluating the amount of *uncertainty* that goes with measurements and results. Measurements are always imperfect and so the results are necessarily a bit uncertain, and it is clearly important to know the limits of their accuracy.

Schedule

The experiments planned for the semester are synchronized as well as possible with the material covered in the Physics 106 course. A multiple choice Assessment Exam will be assigned during the last lab cycle; it will count as a lab experiment (even though it does not test your lab skills). *This schedule may change during the semester. Your instructor will keep you updated.*

Dates	Experiment
1/11–12	Resonance
1/18–19	Electronic Instruments
1/25–26	Sound Waves
2/01–02	Equipotentials
2/08–09	DC Circuits
2/15–16	No Lab
2/22–23	Magnetic Fields
2/29	No Lab
3/07–08	AC Circuits
3/14–15	Thin Lenses
3/21–22	Break – No Lab
3/28–29	Break – No Lab
4/04–05	Diffraction
4/11–12	No Lab
4/18–19	Photoelectric Effect
4/25–26	Nuclear Radiation

You should be enrolled in a Physics Lab section along with your lecture. It is your responsibility to attend your scheduled lab section. Switching lab sections without authorization will result in loss of credit. If circumstances force a lab section switch, contact your lab instructor well in advance of the switch with a plan on how to complete the lab. Generally the equipment for each experiment will be available only during the week that the lab is scheduled.

Lab Partners

You will typically perform the experiment with *one partner*, but each of you will be responsible for writing your own lab report. Identical lab reports will be considered evidence of plagiarism and will be handled accordingly.

Laboratory Materials

You will need to purchase *two* laboratory notebooks; while one is being graded, the other is available for your use in the next lab. We recommend composition books with graph-paper pages. These are available in the campus bookstores. Your lab notebook need not be new; you may use your own partly-filled notebooks from last semester, or find notebooks abandoned from a past academic year and tear out the used pages. You should come prepared to do a complete lab during the 2-hour session. In addition to this laboratory manual and a notebook, you will need writing materials (ink preferred), a scientific calculator and your textbook (as a reference book).

Laboratory Apparatus

A list of apparatus is given with each experiment. Much of the apparatus is delicate and must be handled carefully. Please try to observe safe practice with the lab equipment. In particular,

- Do not attempt to adjust or work with the apparatus until you have been instructed in its use.

- Check each item of equipment you receive, making sure that you have all the items required and that all are in working order.

- Report any problems to your instructor at once.

- Do not, under any circumstances, make electrical connections to batteries or electrical outlets until your circuit has been checked and approved by the laboratory instructor.

Disregarding these simple precautions is not only dangerous, it can cost you time. Moreover, electrical instruments, in particular, are generally very sensitive. Many can be damaged in a fraction of a second if they are connected incorrectly.

Laboratory Procedure

Since the laboratory is required, attendance will be taken. The time that you will have to perform each experiment is limited, so read the experiment **before** the beginning of the laboratory period. Many of the experiments have a pre-lab question associated with them. These pre-lab exercises must be turned in at the beginning of the lab and then approved by your lab instructor before you begin the experiment.

Laboratory Reports

The notebooks are to be handed in at the end of each lab period. Please make sure that you hand in the notebook to your TA before you leave lab. *NOTE: Your TA cannot accept late lab reports.* If circumstances are extreme please see the Physics Lab Manager for possible extensions. It is likely that your score will be penalized under such circumstances.

A lab report should be written in such a way that someone who *did not* do the experiment can read it, together with the instructions in the lab manual, and understand exactly what happened. It certainly doesn't need to be long or wordy to be good. Ordinarily, it should include the following items.

NAMES. The title of the experiment, your name, and your partner's name.

DATE. The date the experiment was performed and report was handed in.

PURPOSE. A brief statement (in your own words) of the objective or purpose of the experiment.

THEORY. A brief explanation of the main ideas and formulas relevant to the experiment. Be sure to define the quantities that appear in your formulas.

DIAGRAMS. To anyone reading the lab report, a sketch of the apparatus is very helpful. A simple block diagram can often describe the experiment better than a great deal of written explanation.

METHODS AND MATERIALS. Similar to that given in the lab manual. You may just outline the procedure you used. However, be sure to note any modifications suggested by your lab instructor, or any changes in procedure that you devised.

DATA. All the important numbers you encounter, including *units* and *uncertainties*, plus any other relevant observations should be recorded as the experiment progresses. If you find it difficult to be neat and organized while the experiment is in progress, you might try using the left-hand pages of your notebook for doodles, raw data, rough calculations, etc., and later transfer the important items to the right-hand pages. Since eventually you will have to understand what the experiment is about, it's best to understand it in advance, and know what data are to be taken before you start. Often you can prepare a spreadsheet for recording data before lab. It's often helpful to graph the data as you acquire them in the lab; this allows you to see where more data are needed and whether some measurements are suspicious and should be repeated.

ANALYSIS/CALCULATIONS. A sample calculation should be included to show how results are obtained from the data, and how the uncertainties in the *results* are related to the uncertainties in the *data* (see Appendix A). By far the easiest way to do this is with a self-documented spreadsheet hardcopy taped into your lab notebook (see Lab 1).

RESULTS/CONCLUSIONS. One objective of laboratory work is to clarify theory and indicate when and how well it applies to practical situations. Give comparisons between your results and the theoretical or accepted values. Does your experimental range of uncertainty overlap the accepted value? Based on the Results, what does the experiment tell you? Connect the Conclusions to the Purpose you stated at the start of the lab.

LAB CRITIQUE. As a service to us and future students we would appreciate it if you would also include a short critique of the lab in your notebook. Please comment on such things as the clarity of this Lab Manual, performance of equipment, relevance of experiment, and if there is anything you particularly liked or disliked about the lab. This is a good place to blow off a little steam.

Don't worry; you won't be penalized, and we use constructive criticisms to help improve these experiments.

QUICK REPORT. As you leave lab, each group should turn in a 3×5 "quick report" card. You will be told in lab what information belongs on your card, but generally it includes the numerical results (properly recorded: significant figures, units, uncertainty) from your Conclusion. Always include: names (you and your partner), table number (find the number on the computer base), and date. These cards go directly to the Lab Manager who will use them to identify problems.

Grading

At the end of each experiment in this book there is a checklist which can serve as a guide for you so you won't forget some important part of the experiment. The TA will return your lab to you graded on a 100-point scale, and it may contain some helpful criticism, so please go over your labs carefully when they are returned. The TAs are expected to grade fairly and impartially, using a grading system something like the following. Please talk to your lab instructor if any problems arise.

Lab is an essential component of every experimental science so your complete participation in every physics lab is required. If circumstances require you to miss your scheduled lab section, it is your responsibility to find an alternative lab time. Contact, well in advance of your absence, the instructor of the lab you want to join and your lab instructor. The lab manager (Lynn Schultz, lschultz@csbsju.edu) may also be able to set up the lab equipment at an alternative time. **Warning:** failure to complete all labs may result in substantial grade reduction or failure of the course

Component	Quality			
	Inadequate	**Minimal**	**Adequate**	**Superior**
Purpose 0 – 5 pts	Purpose omitted, or not related to lab 0 pts	Purpose stated but not clearly related to the measured variables and calculations 1 pt	States an purpose clearly related to some, but not all of the important variables 2 – 3 pts	Correctly states an purpose related to all of the important variables 4 – 5 pts
Theory 0 – 10 pts	Theory omitted or unrelated to lab 0 – 1 pt	Unclear, or formulas not included 2 – 4 pts	Complete but with a few mistakes 5 – 7 pts	Clearly stated with all relevant formulas listed 8 – 10 pts
Methods/ Diagrams 0 – 10 pts	Omitted, or incoherent 0 – 1 pt	Unclear, steps out of order, or diagrams not included 2 – 4 pts	Complete but not well stated or diagrammed 5 – 7 pts	Clear, concise and complete 8 – 10 pts
Data 0 – 10 pts*	Data missing or incomplete 0 – 1 pt*	Data listed but not labeled correctly or without units 2 – 4 pts*	Data listed with some units and labels 5 – 7 pts*	Data well organized with clear labels and all units 8 – 10 pts*
Analysis/ Calculations 0 – 30 pts*	Calculations and graphs missing or not relevant 0 – 3 pts*	Calculations and graphs incomplete 4 – 12 pts*	Calculations and graphs nearly complete and correct 13 – 21 pts*	Complete with clear example calculations 22 – 30 pts*
Results/ Conclusion 0 – 25 pts	Missing or does not relate to purpose 0 – 2 pts	Conclusion not supported by data or analysis 3 – 9 pts	Written paragraph with supporting evidence but incomplete 10 – 16 pts	Written paragraph, logical arguments, included uncertainties 17 – 25 pts
Pre-Lab Question 0 – 10 pts	0 – 10 pts			
* The point breakdown between Data and Analysis/Calculations may vary, depending on the experiment, but will total 40 points				

1. Resonance

Pre-Lab Exercise:

1. When a 100-gram mass is hung on a spring, the spring stretches 1.5 cm. What would be the resonance frequency for this spring with a 50-gram mass attached?

2. Find the linear density of a string that oscillates as shown below when driven at a frequency of 35 Hz, if the mass is $m = 50$ grams and the string length is $L = 1.6$ m.
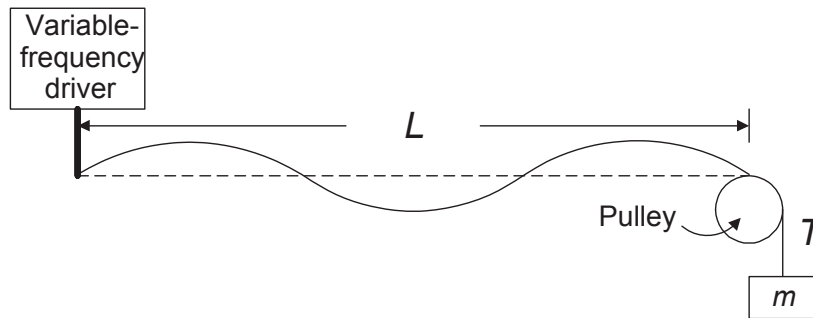
Introduction

Resonance occurs when an oscillating system is driven at a frequency that matches its natural frequency. The amplitude of the oscillator is the largest at resonance, and the maximum energy is transferred from the driving mechanism to the oscillator.

Resonance shows up in many sorts of oscillating system, including systems that cause sound. In music, resonance plays a role in both wind and string instruments.

In this experiment you will be working with two different oscillating systems: a mass on a spring, and a stretched string. In each case, you will be able to adjust the frequency at which the system is driven, and see resonance effects.

Theory

A. Mass on a spring

A spring that obeys Hooke's law, subjected to a force F will extend a distance x, which defines the spring constant $k = F/x$. A mass m hanging on this spring naturally oscillates at a frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{1.1}$$

It is possible to force the mass-on-a-spring to oscillate at other frequencies, however. A system for doing this is illustrated in Fig. 2.1(a). The variable-frequency driver can be some sort of a piston that vibrates vertically at a selected frequency f. The mass-on-a-spring will then also oscillate at frequency f, but with an amplitude A that varies with f something like the graph in Fig. 2.1(b); the amplitude is maximum when $f = f_0$. The energy of the mass-spring system is proportional to the square of the amplitude:

$$E = \frac{1}{2}kA^2. \qquad (1.2)$$

Hence, when driven at the natural frequency f_0, the oscillator has its maximum energy. It gets this energy from the driving mechanism, little by little. In any real oscillating system there is a limit to the amplitude; energy losses will grow, for instance, with drag forces that depend on speed, finally reaching the point where the energy lost in each cycle equals the energy added by the driver.
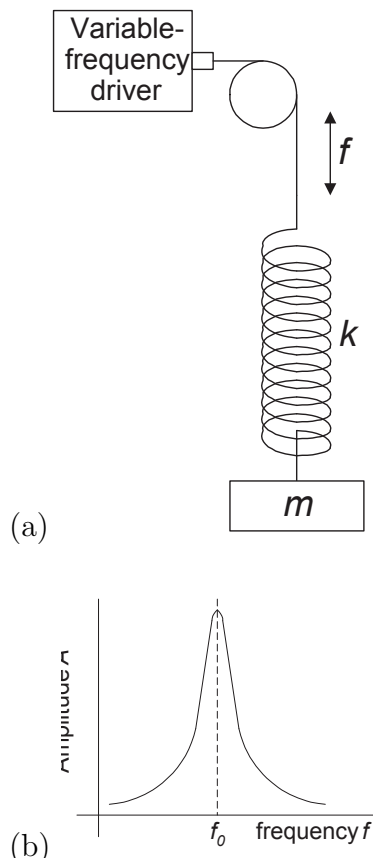

(a)

(b)

Fig. 2.1: (a) Mass-spring oscillator. (b)Amplitude vs. frequency.

B. Standing waves on a string

Standing waves can be produced when two waves of the same frequency and amplitude travel in opposite directions in the same medium. In this experiment waves are produced on a string by a variable-frequency driver, and they reflect at the other end of the string, which is held fixed by passing it over a pulley, as shown in Fig. 1.2. A
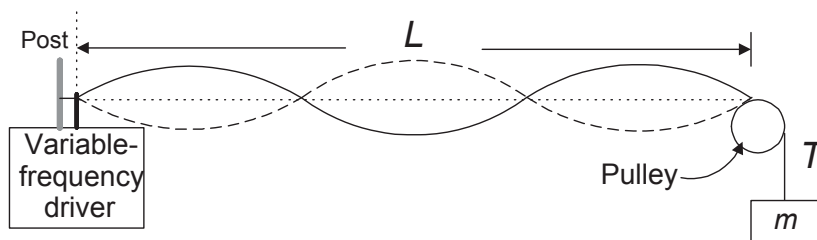

Figure 1.2: Standing waves on a string

mass m hangs on the string, giving a tension $T = mg$. It is found that the speed of

waves on such a string is given by

$$v = \sqrt{\frac{T}{\mu}} \tag{1.3}$$

where $\mu = \frac{\text{string mass}}{\text{string length}}$ is the *linear density* of the string. The wavelength λ of the waves on the string depends on the frequency f at which the driver shakes the string:

$$f\lambda = v. \tag{1.4}$$

Now, for these waves that travel both ways on the string to form a standing wave, it is necessary for the wavelength to fit an integer number of times along each round trip of the string; that is:

$$n\lambda_n = 2L, \text{ where } n = 1, 2, 3, \ldots \tag{1.5}$$

gives the wavelength λ_n of a standing wave. The integer n is the number of wavelengths in a round-trip on the string of length L. The points at which the string remains at rest are <u>nodes</u>, while the points with maximum displacement are <u>antinodes</u>. It is easy to determine n by counting the loops or the number of antinodes along the string. Thus, for example, in Fig. 1.2, $n = 3$. Because only certain wavelengths form standing waves, there are only certain frequencies that can be used to produce them. Combining Eqs. (1.4) and (1.5) gives

$$f_n = \frac{nv}{2L} \tag{1.6}$$

where v is determined by the tension and linear density of the string as in Eq. (1.3). These are the resonant frequencies of the string, at which the string receives the most energy from the driver. They are also referred to as *harmonics*, and the lowest of these, f_1 is called the *fundamental* frequency. From Eq. (1.6) it is expected that

$$f_n = n \cdot f_1 \tag{1.7}$$

Procedure

A. Mass on a spring

1. Measure the spring constant of the spring.

   To help you measure k for a spring, hang the spring on the stand, and attach a 100-gram mass to its lower end. The bottom of this mass can be used to as an initial reference point; sight along the bottom surface to the scale on the stand. Then hang another 100-gram mass beneath the first one, and find the distance x that the <u>upper</u> mass moves downward; this will be the change in the length of the spring. There is some uncertainty about x, of course, so estimate δx as well.
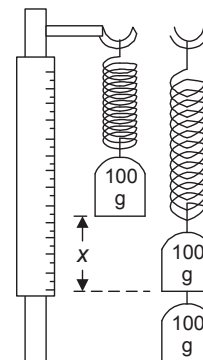
2. Calculate $k = F/x$ where $F = (0.1 \text{ kg})(9.8 \text{ m/s}^2) = 0.98 \text{ N}$, and its uncertainty:

$$\delta k = k \cdot \frac{\delta x}{x}. \tag{1.8}$$

3. Calculate the natural frequency of the spring with a 100-gram mass attached using Eq. (1.1), and its uncertainty:

$$\delta f_0 = \frac{1}{2} f_0 \frac{\delta k}{k} \tag{1.9}$$

4. Observe the mass-spring system driven at resonance.

   (a) Start with the driver on its side so it can pull on a string draped over a pulley. Hang the spring on the bottom end of the string. Attach one 100-gram mass to the bottom of the spring.

   (b) The driver is controlled by the function generator, and an amplifier increases the current that goes to the driver. The Amplitude knob of the function generator should be set no higher than about $\frac{1}{4}$ of maximum to keep the amplifier from overheating (it gets warm so be careful). Press the Frequency Range button for 20 Hz, so you can vary f between 0 and 20 Hz.

   (c) Set the function generator at a frequency somewhat lower than f_0; the driver piston should oscillate noticeably, but not more than a few millimeters.

   (d) Increase the frequency f gradually, and watch for the mass-spring oscillations to increase to a maximum. Find this resonant frequency and estimate a range of uncertainty.

   (e) Compare the observed resonant frequency with the natural frequency f_0 expected from your measurement of k. Do the values agree, considering their uncertainties?

B. Standing waves on a string

1. For this part, stand the driver so its piston moves vertically. A string should be attached by the hook to the post beside the driver shaft. You can place the other end over the pulley clamped at the opposite side of the table, as in Fig. 1.2. Hang a 200-gram mass on this end of the string to provide a known tension T.

2. Measure the length L from the driver shaft to the top of the pulley and estimate its uncertainty δL.

3. Press the function generator's Frequency Range button to allow frequencies up to 200 Hz. Starting near zero, gradually increase f until you see the string resonate at its fundamental frequency f_1.

4. Record f_1, with an estimate of its uncertainty δf_1 based on the range of frequencies that appear to give resonance.

5. Slowly increase f until you find f_2, f_3, etc., with estimates of their uncertainties δf_n. See how many harmonics you can find. Do you see a pattern? Do they follow Eq. (1.7)?

6. Calculate the linear density μ of the string using the tension T and one of your resonance frequencies f_n — pick the one that gives the most stable resonance. According to Eqs. (1.3)–(1.6),

$$\mu = \frac{n^2 T}{4L^2 f_n^2} \tag{1.10}$$

and its uncertainty is given by

$$\delta\mu = \mu \left(2\frac{\delta L}{L} + 2\frac{\delta f_n}{f_n} \right) \tag{1.11}$$

7. For a comparison value of μ, measure the mass of a known length of the string (something like 2 meters $\pm$ 5 mm), and see if $\frac{\text{string mass}}{\text{string length}}$ agrees with the value obtained using Eq. (1.10), within the range allowed by Eq. (1.11).

8. Quick Report: (A) natural frequency f_0 and observed resonant frequency (each with proper units, error, and sigfigs). Do they overlap? (B) linear mass density μ and measured mass/length (each with proper units, error, and sigfigs) Do they overlap?

Finish your lab report with a summary of your results and observations, conclusions, and any helpful critique of the experiment.

CHECKLIST

Pre-Lab Exercise	
Measurement of spring extension x with δx	
Calculation of spring constant k with δk	
Calculation of natural frequency f_0 with δf_0	
Observed resonant frequency compared with f_0	
Measurement of string length L and δL	
Observed fundamental frequency f_1 and δf_1	
Observed harmonics f_n	
Calculation of linear density μ and $\delta\mu$	
Measurement of μ from mass of 2-m length of string	
Summary, conclusions, critique	

2. Electronic Instruments

Pre-Lab:

Bring a musical instrument or some kind of tolerable noisemaker if you'd like to see a graph of its sound as measured with a microphone and displayed using the PC oscilloscope.

Purpose

This lab session is to introduce you to two important electronic instruments — the PC digital oscilloscope, and the function generator — which we will use repeatedly this semester.
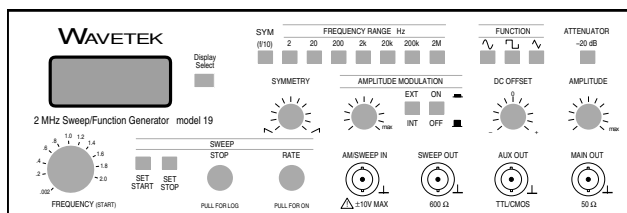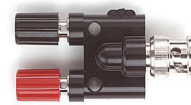
Apparatus

PC Digital Oscilloscope

Microphone

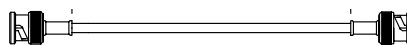

Function Generator

BNC/Banana plug adapter

Battery

Banana plug wires

BNC cables (2)

Introduction

Signals that vary in time—heart beat, the action potentials of nerves, vibrating vocal cords producing sound—are the hallmarks of life. This lab is an introduction to instruments used to measure such signals (oscilloscope) and to artificially produce such signals (function generator). The function generator produces time-varying electrical signals or *voltages* (which you will soon learn more about in class) with adjustable frequency and amplitude. The oscilloscope (a.k.a., 'scope') displays a graph of voltage vs. time. It is useful for seeing and measuring time-varying electrical signals. While an oscilloscope can only measure voltage, *transducers* are available to convert almost any imaginable physical quantity to an equivalent voltage. In this lab you will use a microphone to convert sound to voltage so it can be displayed on the oscilloscope.

Both instruments are designed to be versatile which means they have many controls—which can be confusing to the new users. Today you will be exploring only those features we'll need in this semester's labs.

Lab Report

This report will follow a format that is different from the one used in your previous physics labs. Simply keep a log and commentary on the steps given in the instructions. It might be helpful if you check off each step in the instructions when it has been completed. (Accidentally skipping a step may cause a problem during later steps.) Include sketches of observed waveforms, along with information about their units (the scale settings for horizontal and vertical axes). Also give answers to any questions asked in the instructions. There are a few calculations to be made but most the report will consist of your comments and observations.

Operating Basics

In many ways the PC digital oscilloscope is similar to the *Logger Pro* hardware/software which you used last semester. There the computer could monitor sensor outputs (location [smart pulley], temperature, pressure, . . .) and display the results as a graph—often a graph of measurement vs. time. Similarly the PC oscilloscope measures voltages and displays a graph of voltage versus time. Generally oscilloscopes are designed for periodic high-speed data collection (many samples per micro-second), but it has no problem displaying much slower changes. The PC oscilloscope you will be using is able to simultaneously measure and display two different inputted voltages (a.k.a., 'channels' abbreviated Ch).

The main oscilloscope program window is divided into functional areas. The display area in the center displays the waveform(s) being viewed along with waveform details and the instrument control settings. The most important controls are those that
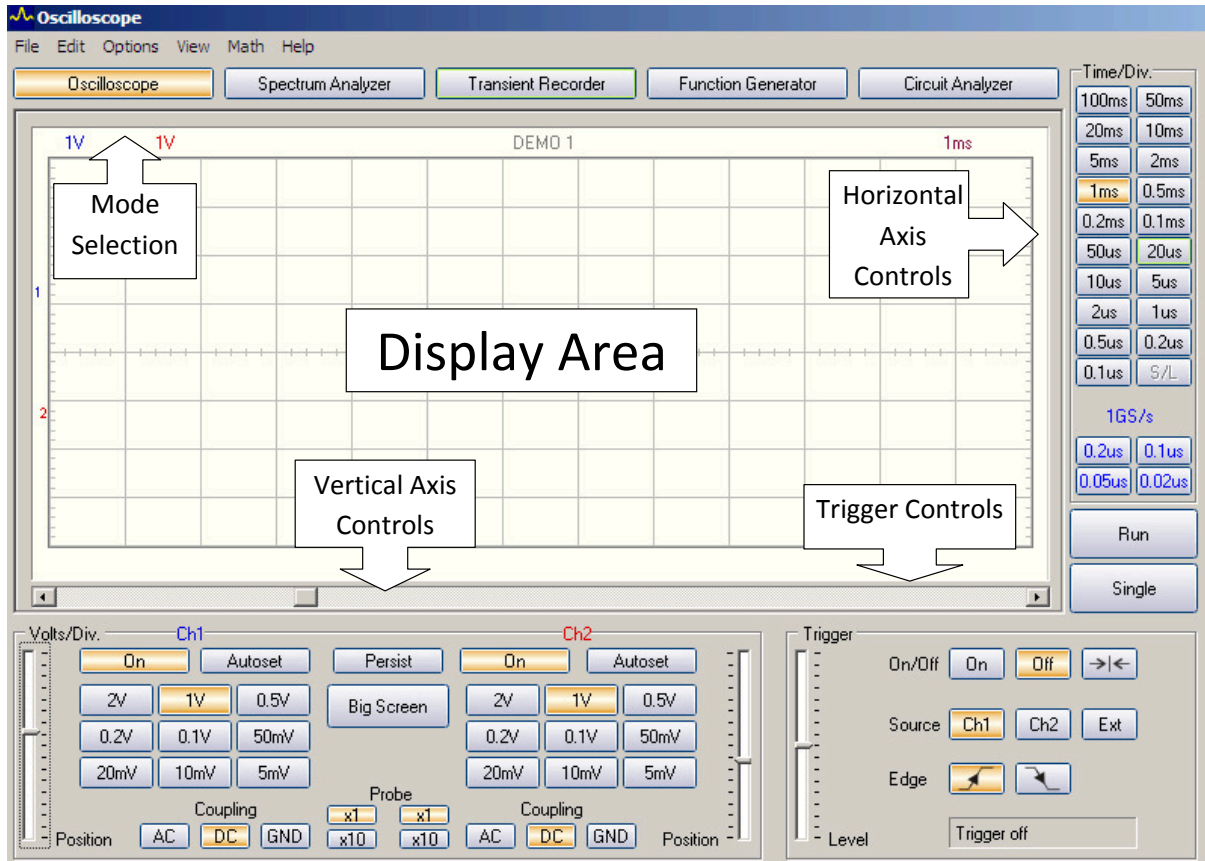
Figure 2.1: Oscilloscope front panel.

control the scales of the horizontal (time) and vertical (voltage) axes. The buttons on the right side of the window select the time scale used on the horizontal axis; the buttons located at the bottom of the window select the voltage scale for the vertical axis and how the channels will be displayed.

Procedure

A. Starting out

Start he PCSU1000 program. Check that the USB Device selected is PCSU1000 and then click OK. Note the basic control locations: Ch1 and Ch2 (settings for the voltage on the vertical axis) lower left, Time/Div (settings for the time scale on the horizontal axis) right hand edge above the large Run button, and the Trigger section lower right. From the File menu, select Recall Settings and open the <u>intro.set</u> file[1]. You will notice that channel 1 (Ch1) is on and channel 2 is off. Also, the vertical

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[1]You may have to browse to find the file intro.set. Try:
Windows (C:)►Program Files►Velleman►PCLab2000SE►data

scale is set to 1V (1 volt/division) and the horizontal scale to 1 ms/division. If you click on the **Run** button, located on the right, you will see a flickering horizontal line. This represent the data points (constant essentially zero voltage) being plotted vs. time. Since nothing is currently connected to the scope input, the voltage should be near zero. If the line is slightly above or below the center of the display (zero voltage) you can move the line to zero by dragging the Position slider (lower left edge) for channel 1 up or down. If you click on the **Run** button again (which stops the run), the display will hold steady with the last data set. The data points show some randomness because of low-voltage 'noise' which is always present.

Find the Volts/Div controls for the Ch1 input: at the bottom left side of the display you should see **1V** highlighted— each vertical unit on the display represents 1 volt. The trace (the dark line being graphed versus time) is at 0 volts. Next, connect a battery to the scope CH1 input, using wires and the BNC/Banana[2] plug adapter as shown here. You can obtain a reasonably good measurement of the battery voltage — something like 1.5 Volts. It is important to understand that an oscilloscope is nothing more than a sophisticated voltmeter. But it is designed especially for voltages that change quickly with time, as in the remainder of the experiment. Disconnect the battery before proceeding to the next part.
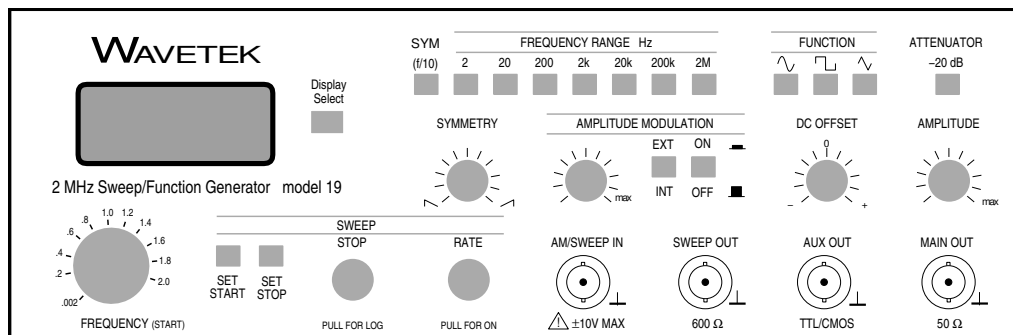
B. Applying a sine wave

Figure 2.2: Function generator front panel.

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[2]'BNC' (from bayonet Neill-Concelman) and 'banana' are names for common connectors. BNC connectors are used for high-frequency signals where avoiding noise is important; Banana connectors are used in less demanding situations. This adapter allows connection between the two types of connectors.

1. Connect a BNC cable from "MAIN OUT" of the function generator to "CH1" of the scope minitower.

2. Before turning the function generator on (switch is in the rear), push in the 2k FREQUENCY RANGE button and the sine wave (left FUNCTION button). All other buttons should be off (out). Adjust the FREQUENCY (big knob lower left), SYMMETRY and DC OFFSET knobs to their mid position. The AMPLITUDE knob (far right) should be set at approximately 25% (quarter turn clockwise).

3. Have your lab instructor check your settings, then turn the function generator on, and click on the scope **Run** button..

4. You should now see a recognizable sine wave on the oscilloscope display. If the wave is not steady (fixed), check that the Trigger (lower right) in On. Notice the information at the top of the display. The value of each scale division will be listed. Vertical units (top left) will be V or mV and horizontal units (top right) will be μs or ms.

C. Scale Adjustments

1. In the lower left of oscilloscope program window, find the collection of Ch1 buttons. Try adjusting the **Volts/Div** ("Div" is an abbreviation for "division"; each rectangle that makes up the grid in the scope's display is one division on a side). When you go from the 1V setting to the 2V setting does the apparent wave height increase or decrease? Notice that the vertical scale units at the top left of the display area change as you click different Volts/Div buttons. And the displayed wave shrinks or grows as you change the scale for Ch1.

2. In the above you changed the display but not the actual voltage. To change the voltage produced by the function generator, find its AMPLITUDE knob (far right) and twist it slightly. The displayed wave should change in accord with the actually changing voltage.

3. In the far right of the oscilloscope program window, find the horizontal **Time/Div** buttons. When you go from the 1ms setting to the 2ms setting does the apparent wave length increase or decrease? Notice that the horizontal scale units at the top right of the display area change as you click different Time/Div buttons. And the displayed wave expands or compresses as you change the scale on the horizontal axis

4. In the above you changed the display but not the actual voltage. To change the voltage produced by the function generator, find its FREQUENCY knob (far left) and twist it slightly. The displayed wave should change in accord with the actually changing voltage. Does the wave length of the displayed wave increase or decrease when you increase the function generator frequency?

D. View Menu

From the **View** menu, select **Waveform Parameters**.

1. Click on the boxes for the following items: DC Mean, Peak-to-Peak, Period, and Frequency. Click to deselect CH2. Record these numbers! The frequency should be very close to the value displayed by the function generator.

2. Sketch in your notebook the scope trace; be sure in include the scale settings. Be sure that your scope trace shows at least one full period of the sine wave, like Figure 2.3.

3. Record the Peak-to-Peak size in Div. Using the scale factor of Volts/Div calculate Peak-to-Peak size in volts. Record the Period size in Div. Using the scale factor of Time/Div calculate Period size in seconds. Compare the results of these calculations to the values displayed in the View Menu.

4. The DC Mean value in the View Menu should be close to zero since the sine wave voltage alternates between equal positive and negative peaks. The function generator's the DC OFFSET knob can be used to change the mean value — try it while observing the display. Sketch how the displayed trace changes.

5. Set the DC offset knob as close to zero as possible before proceeding. Kill the Waveform Parameters window.

E. Triggering

In this exercise, the function generator will be adjusted to produce a wave of changing amplitude.

1. In the AMPLITUDE MODULATION section near the center of the function generator, press the "ON" button and turn the AMPLITUDE MODULATION knob fully clockwise to its maximum position.

   The display should appear unstable, because the continuously updating display is sequentially displaying incongruent sections of the wave. In order to get a stable display the programs needs to select wave sections that all start at equivalent spots.

2. To see a single graph, toggle the **Run** button off. If you press the button repeatedly, you will notice that the waveform changes in appearance.

3. To improve the stability of a *running* waveform, the method the oscilloscope uses to commence graphing (triggering) can be adjusted. Check that the trigger button on the right side of the front panel is **On**. Drag the **Trigger Level** slider (just to the left of the Trigger On/Off button) up and notice the vertically rising

extra-dark tick mark on the left side of the display. As you move the Trigger Level slider up and down, this tick mark moves up and down, and the displayed wave section moves its 'initial' start point up and down. The definition of 'initial' is a bit tricky as the scope is designed to display both pre- and post-trigger values. Notice on the bottom of the graph a matching extra-dark tick mark; that is the location of the trigger event in time. The location of this 'initial' time can be moved my adjusting the large horizontal slider just below the plot.

If you move the Trigger Level slider to a level that is uniquely high, every wave section will have nearly the same start point and hence have nearly the same shape throughout the wave section. (If you move it too far the display will stop being refreshed—because no wave section reaches that height; you have effectively stopped the display just as you did by toggling the Run button.)

4. You may find it helpful to increase the time per division (horizontal scale), in order to see more clearly how the amplitude of the sine wave is changing and the effect of changing the triggering level. When the Trigger Level is near zero the display is much less steady than it is when you select a uniquely high or low Trigger Level.

   By adjusting the Trigger Level you are changing the voltage which starts or triggers the graphing process. Choosing a fairly high trigger level eliminates spurious signals ('noise') from triggering a graph. Noise is relatively significant when investigating weak signals. Finally note the Edge buttons which selects triggering by a rising or falling voltage at the specified level.

5. Turn the AMPLITUDE MODULATION of the function generator OFF before proceeding to the next section.

F. Sine Wave Measurements, Cursors

1. The AMPLITUDE of the function generator should be set anywhere from 25% to 50%. Make whatever adjustments of the scope that are necessary to obtain a full screen display of the sine wave of your choice.

2. Open the **View** menu and select **Markers (DSO)**. Click on the display area and notice the horizontal and vertical dashed lines called cursors.

3. Drag the horizontal lines (voltage cursors) up and down while noticing how the **dV** at the bottom left of the display change. (The voltages in parenthesis show the values of the each cursor separately; dV is the voltage difference between the cursors.) Do the same with the time (vertical) cursors while observing the **dt** and **1/dt** values below. (1/dt will be the frequency of an appropriately marked periodic wave.)

4. Adjust the two cursors to coincide with the positive and negative peaks of the sine wave. Record the peak-to-peak voltage.
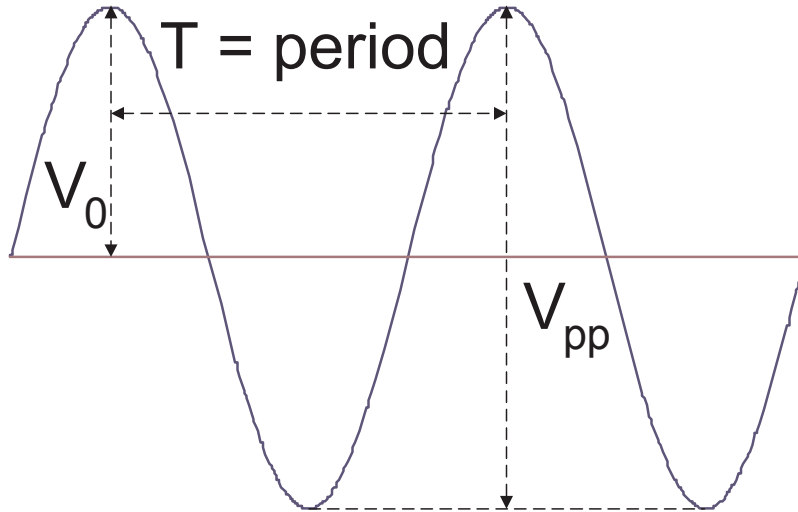
Figure 2.3: Sine-wave properties.

5. Drag the time cursors and record the period **dt** of one sine wave cycle and the corresponding frequency.

6. Without moving the cursors, select **Print...** from the **File** menu. Tape the hardcopy in your lab notebook.

7. Compare the frequency of the sine wave found with the time cursors to the value displayed on the function generator. The scope uncertainty is 3% plus any additional uncertainty in your ability to place the cursors, while the function generator uncertainty is 1.5%.

G. Separating AC and DC signals

1. Adjust the DC OFFSET knob on the function generator to move the waveform upward on the display. Record how the display changes when you rotate the DC OFFSET knob. You may also need to make the trigger more positive to obtain a stable waveform.

2. Find the **Coupling** button in the Ch1 display window. Change the highlighted selection from DC to AC. Notice how the waveform is again centered vertically in the display and the waveform is insensitive to the DC OFFSET knob. The voltage is actually changing when you rotate the DC OFFSET knob, but the scope is automatically subtracting the constant component of the waveform.

3. Change the Coupling selection back to "DC" before proceeding. Return the DC OFFSET to near zero.

H. Dual Channel Operation

1. Connect the "AUX OUT" output of the function generator to CH2 (channel 2) on the scope minitower while leaving the "MAIN OUT" output connected to CH1.

2. Set the amplitude of the function generator to approximately 25%. On the oscilloscope front panel, click Ch2 to **On**. Notice the appearance of a second waveform.

3. Press **Autoset**. The two overlapping waveforms may be separated vertically by dragging the two **Position** sliders. Position the waveforms so they are both clearly visible. The vertical scales for both waveforms may be changed independently but only one time scale can be selected. Try adjusting the two vertical scales.

4. Change the amplitude of the function generator and record how the two waveforms are affected differently.

5. With the trigger on, either waveform can be used as a trigger by selecting the appropriate source. Try both sources.

6. Sketch the resulting dual trace.

7. Turn Ch2 off before proceeding to the next section.

I. Audio Signal

1. Find the **Coupling** buttons under **Ch1** in the front panel. Change the highlighted selection from DC to AC.

2. Disconnect the function generator from the oscilloscope, and connect a microphone to the oscilloscope with the amplifier. The microphone may have switch a that must be on for it to work. Try to sing, hum or whistle a sustained tone into the microphone and simultaneously press the "Autoset" button. "Autoset" can be useful when attempting to locate a signal of unknown amplitude and/or frequency. Now make a continuous sound using your musical instrument (or voice if you didn't bring one). When you have a regular pattern, toggle the "Run" button to capture the signal.

3. Sketch (record scales!) and describe the waveform produced by your audio signal. Using the Markers (DSO) cursors, measure the frequency, or frequencies as best you can.

4. Change the coupling selection back to "DC" before proceeding.

J. Complex Waveform

In this last exercise, you will investigate a signal that is a composite of several different waveforms. The electrical wiring and instruments in the lab room produce the signal.

1. Disconnect the BNC cable from CH1 of the oscilloscope minitower and replace it with a BNC/banana plug adapter with the red post connected to the banana plug lead. The other end of the lead should extend away from the scope and remain unconnected — it will act as an antenna to pick up electrical signals. If you hold on to the free end of the cable, the antenna effect will be enhanced.

2. Use what you have learned about the oscilloscope settings to obtain a stable display. Using the cursor tool as you did in part **F**, determine at least two frequencies (one of which is below 100 Hz) that are part of this complex waveform. You will need to adjust the time scale to see different frequencies. Draw a sketch of the display and list any frequencies that you were able to measure.

Quick Report Question

What are the frequency and peak-to-peak voltage of the oscilloscope signal shown below if the Ch1-input button was on the .5 mV/div scale and the time/div button was on the .2 ms/div scale?
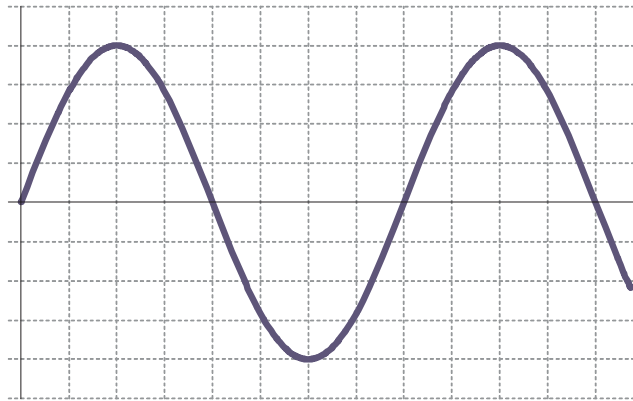
Lab Critique

Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

3. Sound Waves

Pre-Lab Questions:

1. What should be the distance between antinodes of a standing wave of sound with a frequency 9 kHz, if the room temperature is 24°C?

2. If signals at 310 Hz and 330 Hz are combined, what should be the beat frequency?

Purpose

In this lab you will measure several characteristics of waves: velocity, nodes in standing waves, and beats due to on superposition. These characteristics apply to most any wave; we will explore them using sound.

Apparatus

- Sound tube apparatus: oscilloscope, function generator, speaker, movable microphone, graduated tube

- Beats apparatus: dual sine wave generator, oscilloscope, speakers, microphone (one in each room)

Introduction

Generally when waves are simultaneously present, they combine in the simplest possible way: superposition or simple addition. But the results—constructive and destructive interference— can be surprising. For example, two musicians simultaneously playing nearly the same note with identical amplitude (volume) produce a varying volume at an intermediate pitch, a phenomenon know as 'beats'. Waves moving in opposite directions in a tube, add to produce a 'wave' that is fixed in place: a 'standing wave'. Standing waves are at the core of almost all musical instruments. Perhaps the most fundamental characteristic of a wave is its speed: the distance it travels per time. You will measure this directly with a sound wave and its sequence of echoes, and indirectly through the behavior of standing wave.

Theory

A. Standing Waves

Standing waves are produced whenever two waves of the same frequency and amplitude travel in opposite directions in the same medium. In this experiment the standing waves are produced by multiple reflecting the waves from a source, S in a closed tube.

When measuring standing waves, one observes regions of maximum disturbance (antinodes) and regions of minimum disturbance (nodes). Since these regions are fixed in space, the wave pattern is called a *standing wave*. Wave theory tells us that the separation between *two consecutive nodes* or between *two consecutive antinodes* is one-half the wavelength of the sound. Since the velocity of sound, v, is equal to the product of the frequency, f, and wavelength, λ,

$$v = f\lambda \tag{3.1}$$

the velocity of sound can be calculated from measurements of the frequency and wavelength of the standing waves. The resulting uncertainty is, of course

$$\delta v = \left(\frac{\delta f}{f} + \frac{\delta \lambda}{\lambda} \right) v \tag{3.2}$$

where δf and $\delta \lambda$, respectively, are the uncertainties in f and λ. This velocity can then be compared to the expected value from the following formula for dry air:

$$v = 331.3\sqrt{1 + \frac{T_C}{273}} \tag{3.3}$$

where T_C is the temperature in degrees centigrade.

B. Sound Pulses and Echoes

The function generator can be made to produce pulses, or clicks from the speaker, and these clicks reflect back and forth along the tube. Each arrival of a click echo at the microphone will appear as a voltage spike on the oscilloscope. By measuring the time between these echoes, the time t for each trip down the tube can be determined, and if the length L of the tube is also measured, the speed of sound can be calculated:

$$v = L/t \tag{3.4}$$

with uncertainty

$$\delta v = v \left(\frac{\delta L}{L} + \frac{\delta t}{t} \right). \tag{3.5}$$

C. Beats

Sound waves can be represented mathematically with either a sine or cosine function. The sum of two simultaneously-produced sound waves of the same amplitude A but different frequencies (f_1 and f_2)) can be represented as follows, through the use of a trigonometric identity.

$$A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t) = 2A\cos\left(2\pi\frac{f_1 - f_2}{2}t\right)\cos\left(2\pi\frac{f_1 + f_2}{2}t\right) \qquad (3.6)$$
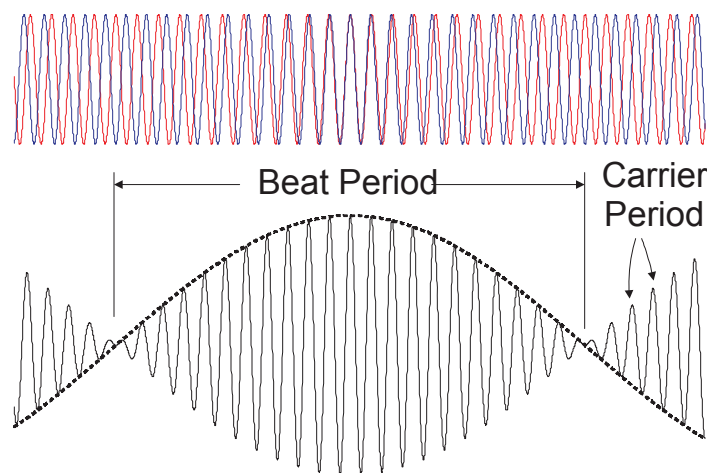

Figure 3.1: Two waves of different frequencies superposed to cause beats

The left side of the equation is displayed in the upper part of Fig. 3.1: the two waves have the same amplitude but different frequencies. If the two waves are added together, the result will appear as seen in the lower part of Fig. 3.1, which is described by the right side of Eq. (3.6). Notice that where the two waves are in phase (near the center) we have constructive interference (and a large amplitude) whereas where the waves are 180° out-of-phase we have destructive interference. The average of the two frequencies, $\frac{f_1 + f_2}{2}$ applies to the rapid oscillations of the resulting wave, which is known as the *carrier wave*. The frequency $\frac{f_1 - f_2}{2}$ applies to the slow variations of the carrier amplitude, shown as a dashed line in the lower figure.

What is it that a person actually hears when two sounds of equal amplitudes, but slightly different frequencies, are produced? Notice in the second diagram that the carrier wave reaches peak amplitude twice during each cycle represented by the dashed line. Therefore, the frequency of the varying amplitude (known as the beat frequency) will be

$$f_{beat} = f_1 - f_2, \qquad (3.7)$$

while the carrier wave oscillates at the rapid frequency given by

$$f_{carrier} = \frac{1}{2}\left(f_1 + f_2\right). \qquad (3.8)$$

Procedure

A. Standing Waves:

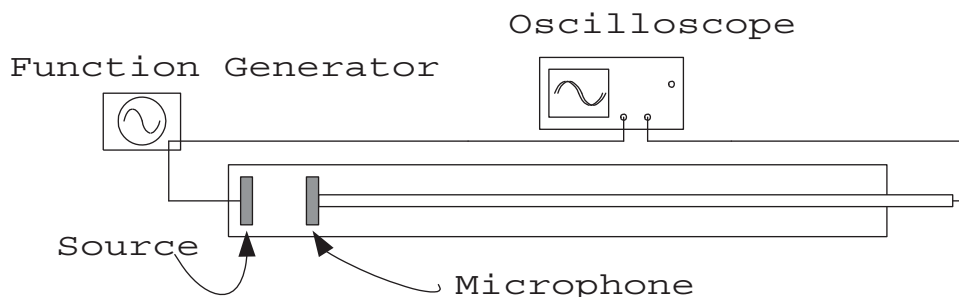1. Set up the apparatus as sketched in Fig.3.2. The function generator should be

Figure 3.2: Apparatus for measuring the speed of sound.

set to produce a sine wave. The output should have a "T" connector allowing connection to both CH1 of the oscilloscope minitower and the sound tube transducer ('speaker'). Open the PC oscilloscope program as described on page 19. From the File Menu, select Recall Settings and open the <u>sound.set</u> file[1]. Set the function generator's AMPLITUDE to the middle of its range, set FUNCTION to sine waves, FREQUENCY RANGE to 20k, and adjust its FREQUENCY knob to mid-range— 8 to 10 kHz. Turn on the function generator (switch in rear).

2. Connect the microphone to CH2 of the oscilloscope minitower.

   Now slide the microphone to a point near the center of the tube. Adjust the function generator frequency (not the amplitude) slightly to produce the strongest signal on Ch2. You will probably need to adjust the Volts/Div control for channel 2 and the position sliders for both channels to clearly see both waveforms on the display.

3. Slide the microphone to a point close to the transducer and gradually move the microphone away from the transducer. Record the position of the microphone each time an antinode (maximum) is observed. Do this for at least 10 antinodes.

4. Using an Excel spreadsheet, record the separations s between successive antinodes, the mean $\bar{s}$, and the standard deviation of the mean, $\sigma_{\bar{s}} = \delta\bar{s}$.

5. Remembering that the separation distance s between consecutive between consecutive antinodes is one-half the wavelength of sound, it should be clear that $\lambda \pm \delta\lambda = 2(\bar{s} \pm \delta\bar{s})$. Calculate the speed of sound v in the sound tube for the antinode measurements, using Eq.(3.1).

---

[1]You may have to browse to find the file intro.set. Try:
Windows (C:)▶Program Files▶Velleman▶PCLab2000SE▶data

6. Using your uncertainty in the wavelength and assuming the fractional uncertainty in frequency to be $\frac{\delta f}{f} = 0.015$, calculate the uncertainty in v using Eq. (3.2).

7. Compare your results with the speed predicted from Eq. (3.3) based on temperature. **Q1:** Does your calculated range for $v \pm \delta v$ overlap with the prediction based on temperature?

B. Pulses and Echoes

1. Move the microphone to the far end of the tube. Change the function generator output from sine waves to square waves. Change the coupling for channel 1 from AC to DC. Expand the y-scale to approximately 2 ms/div.

2. Starting at about 50 Hz, gradually reduce the frequency until you see the echoes clearly on the scope.

3. Adjust the vertical and horizontal scales of the oscilloscope to obtain a display similar to Fig. 3.3. The square wave represents a voltage pulse applied to the transducer — you may be able to hear the clicking sound it produces. The microphone output should show a decaying series of sharp spikes — a pulse followed by a series of echoes within the tube — along with other noise. The time between these spikes is the time needed to travel twice the length of the tube. To help you be sure you are seeing the correct spikes, try adjusting the square-wave frequency — the positions of these spikes won't change, because they depend on the speed of sound, not the frequency of the clicks.

4. When you see the echoes clearly, click on the Run button to freeze the display.

5. Drag the time cursors to determine the time delay between the sound pulse and a later echo; for example, t_4 as illustrated in Fig. 3.3, is the time of travel for four round trips of the sound made by the speaker.

6. Calculate the mean time for a pulse to travel the length of the tube; in Fig. 3.3 the mean time t is $t_4/8$ because the pulse traveled the tube length eight times (four round trips). The fractional uncertainty $(\frac{\delta t}{t})$ should be about .03 for this measurement.

7. Then calculate the speed of sound in the tube using Eq. (3.4), and its uncertainty from Eq. (3.5). The length uncertainty δL is probably not negligible so estimate this for yourself.

8. Compare with the speed predicted from Eq. (3.3) based on temperature. **Q2:** Does your calculated range for $v \pm \delta v$ overlap with the prediction based on temperature?

9. Make a final results table recording your velocity of sound measurements $(v \pm \delta v)$ from part A (standing waves) and part B (pulses and echoes) and the predicted
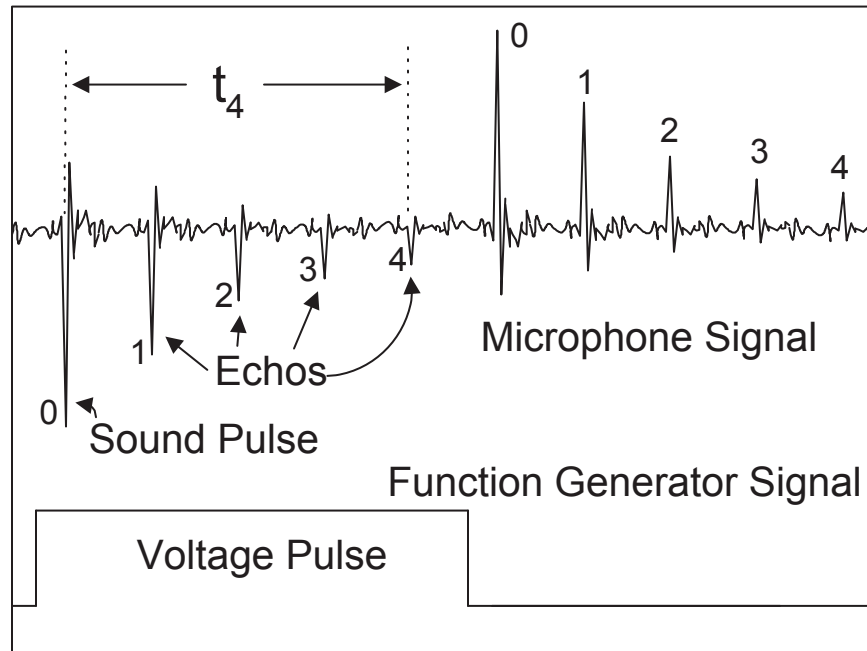
Figure 3.3: Sound tube signals on oscilloscope.

sound speed from Eq. (3.3). Your table should properly display units, significant figures, and errors.

C. Beats

The instruments used in this part of the experiment have been set up as a separate station. Your group will need to move to that station to obtain the necessary measurements.

Two function generators are connected to speakers and frequency counters. The sound mix coming from the two speakers can be detected with a microphone connected to an oscilloscope.

1. Adjust the frequencies so $|f_1 - f_2|$ is around 15 Hz. Turn on the speaker switch.

2. If the scope is not already running, click on the **Run** button.

3. It may be necessary to change the oscilloscope time base to obtain recognizable beats like in Fig. 3.1. When the beats are evident, click on **Run** button to freeze the display.

4. When you have obtained a good beat pattern, please turn off the speaker switch.

5. Read f_1 and f_2 directly from the frequency counters. Read to the nearest 1 Hz, but don't try for more digits — the counters may vary by about 1 Hz primarily from drift in the function generators producing f_1 and f_2.

6. Use the mouse to drag the cursors onto two consecutive <u>nodes</u>, and read the frequency (1/dt) of the <u>beats</u>. It should correspond to the difference between f_1 and f_2 according to Eq. (3.7). Print a copy of this display.

7. Then decrease the time per division so that you can measure the frequency of the carrier wave (the fast oscillations between beats) by moving the cursors to consecutive wave crests. This frequency should correspond to the average of f_1 and f_2, according to Eq. (3.8).

8. **Q3:** Did your measurement of the beat frequency match the prediction Eq. (3.7)? Did your measurement of the carrier frequency match the prediction Eq. (3.8)?

9. Adjust f_1 and f_2 so they differ by just a few Hz. **Q4:** Describe the sound you hear. Suppose you want to adjust the frequency of one musical instrument to match the frequency of another. **Q5:** How could you use audible beats to do this?

Final steps

1. Quick Report: Two measurements of the speed of sound: (A) by standing waves and (B) by echoes. Report these with proper units, errors, and significant figures. Did they overlap with the speed of sound calculated from room temperature?

2. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Pre-Lab Questions	
Sketch of the Sound Tube Apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
Room temperature recorded	
Table of antinode separations with mean and SDOM	
Measurement of λ, with uncertainty	
Calculation of sound speed and uncertainty from wavelength measurements	
Calculation of sound speed from pulse echoes	
Q1, Q2, Final results table for sound speed (units, significant figures, errors)	
Measurement of beat frequency f_{beat}, and carrier frequency f	
Q3, Comparison of f_{beat} and $f_{carrier}$ with Eqs. (3.7) and (3.8).	
Q4, Q5 description of audible beats, and tuning of musical instruments	

4. Equipotentials and Field Lines

A new concept appeared in physics, the most important invention since Newton's time: the field. It needed great scientific imagination to realize that it is not the charges nor the particles but the field in the space between the charges and the particles that is essential for the description of physical phenomena. The field concept proved successful when it led to the formulation of Maxwell's equations describing the structure of the electromagnetic field.

Einstein & Infield *The Evolution Of Physics* 1938

Pre-Lab Question

Assume Fig. 4.2 is a to-scale equipotential plot with the A equipotential at 1 V, the B equipotential at 2 V, and the C equipotential at 3 V. Find the direction and magnitude of the electric field at the spot labeled **P**. (This will require finding a ruler to measure Δx.)

Purpose

To explore the concepts of electrostatic potential and electric field and to investigate the relationships between these quantities. To understand how equipotential curves are defined and how they can display the behavior of both the potential and field.

Apparatus

- DC power supply

- digital multimeter (DMM)

- conductive paper with electrode configurations (2)

- graph paper

- cork board, push pins

Introduction

Every electric charge in the universe exerts a force on every other electric charge in the universe. Alternatively we can introduce the intermediate concept of an electric

field. We then say that the universe's charge distribution sets up an electric field $\vec{\mathbf{E}}$ at every point in space, and then that field produces the electric force on any charged particle that happens to be present.

The electric field $\vec{\mathbf{E}}$ is defined in terms of the force, $\vec{\mathbf{F}}$, on a "test charge" q brought to the spot where we want to measure $\vec{\mathbf{E}}$:

$$\vec{\mathbf{E}} = \vec{\mathbf{F}}/q \tag{4.1}$$

Note that the field exists independently of any charges that might be used to measure it. For example, fields may be present in a vacuum. At this point is is not obvious[1] that the field concept really explains anything. The utility of the electric field concept lies in the fact that the electric field exists a bit independently of the charges that produce it. For example, there is a time delay between changes in the source charge distribution and the force on distant particles due to propagation delay.

The electric force $\vec{\mathbf{F}}$ does work ($Fs\cos\theta$) on a charge, q, as it is moved from one point to another, say from A to B. This work comes from a change in the electrical potential energy $\Delta PE = PE_B - PE_A$ of the charge. Consider, for a moment, a line or surface along which the potential energy is constant. No work is done moving along such a line or surface, and therefore the force must be perpendicular to this surface. Electric forces (and hence the electric fields) must be **perpendicular** to surfaces of constant potential energy. This concept is central to our experiment.

Example: Gravitational potential energy. Consider a level surface parallel to the surface of the Earth. The potential energy mgh does not change along such a surface. And of course, both the gravitational force $m\vec{\mathbf{g}}$, and the gravitational field $\vec{\mathbf{g}}$ are perpendicular to this surface, just as we would expect from the above reasoning. Electrical fields and potential energies are less intuitive because less familiar, but they work in much the same way.

We can define a new quantity, the potential difference (a.k.a., voltage) $\Delta V = V_B - V_A$ between points A and B, in the following way:

$$\Delta V = \frac{\Delta PE}{q} \tag{4.2}$$

that is, just as the electric field is the force per charge, the voltage is the potential energy per charge. Although the potential energy difference $\Delta PE = PE_B - PE_A$

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[1]*Star Wars Episode IV*:
LUKE: You don't believe in the Force, do you?
HAN: Kid, I've flown from one side of this galaxy to the other. I've seen a lot of strange stuff, but I've never seen anything to make me believe there's one all-powerful force controlling everything. There's no mystical energy field that controls my destiny.
Of course, we should replace "force" in the above with "field".
Contrast the above with Han's view in *Episode VII*:
FINN: We'll figure it out. We'll use the Force.
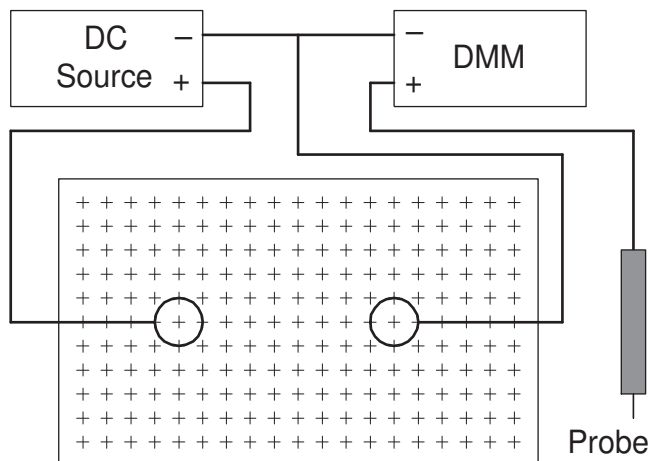HAN: That's not how the Force works!

Figure 4.1: Electric field mapping circuit.

does depend on q, the ratio $\Delta PE/q$ (and therefore ΔV) is independent of q. That is, in a manner analogous to the electric field, potential difference does not depend on the test charge q used in its definition, but only on the charge distribution producing it.

Finally, note that only the potential *difference* has been defined. Specifying the potential itself at any point depends on assigning a value to the potential at some convenient (and arbitrary) reference point. Often we choose the potential to be zero "at infinity", i.e., far away from the source charges, but this choice is arbitrary. Moreover, if we measure only potential differences, the choice of "ground" (zero volts) doesn't matter.

Theory

In this experiment you will use different configurations of 'electrodes' which have been painted on high resistance paper with conducting ink. When the electrodes are connected to a "battery" (actually a DC power supply: a battery eliminator) an electric field is set up that approximates the electrostatic conditions discussed in lecture. Under truly electrostatic conditions the charges would be fixed, and the battery could be disconnected and the electric field would continue undiminished. However, in this experiment the battery must continue to make up for the charge that leaks between the electrodes through the paper. These currents are required for the digital multimeter (DMM) to measure the potential difference. Nevertheless, the situation closely approximates electrostatic conditions and measurement of the potential differences will be similar to those obtained under strictly electrostatic conditions.

Consider Fig. 4.2 where the electrodes are a circle and a line. Suppose we fix one electrode of the digital multimeter (DMM) at some (arbitrarily chosen) point O and then, using the second electrode as a probe, find that set of points $A_1, A_2, A_3, \ldots$ such
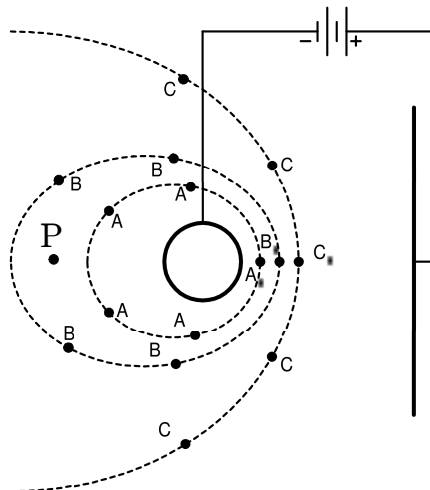
Figure 4.2: Some equipotential curves for the circle-line electrodes.

that the potential differences between any of these points and O are equal:

$$V_{A_1O} = V_{A_2O} = V_{A_3O} = \ldots. \tag{4.3}$$

Hence the potentials at points $A_1, A_2, A_3, \ldots$ are equal. In fact, we can imagine a continuum of points tracing out a continuous curve, such that the potential at *all* the points on this curve is the same. Such a curve is called an *equipotential*. We label this equipotential V_A. We could proceed to find a second set of points, $B_1, B_2, B_3, \ldots$ such that the potential difference V_{B_nO} is the same for $n = 1, 2, 3, \ldots$; i.e.,

$$V_{B_1O} = V_{B_2O} = V_{B_3O} = \ldots. \tag{4.4}$$

This second set of points defines a second equipotential curve, which we label V_B. The actual values of the potentials assigned to these curves, V_A and V_B, depend on our choice of the reference potential V_O. However, the potential *difference* between any two points on these two equipotentials does not depend on our choice of V_O; that is,

$$(V_A - V_O) - (V_B - V_O) = V_A - V_B \tag{4.5}$$

and so the potential difference $V_A - V_B$ can be determined from the measured values V_{AO} and V_{BO}.

A mapping of electrostatic equipotentials is analogous to a contour map of topographic elevations, since lines of equal elevation are gravitational equipotentials. The elevation contours are closed curves; that is, one could walk in such a way as to remain always at the same height above sea level and eventually return to the starting point. Likewise, electrostatic equipotentials are closed curves.

The electric field at a given point is related to the spatial rate of change of the potential at that point, i.e., the *gradient*, in the potential. Continuing the analogy with the contour map, we find the electric field is analogous to the slope of the landscape, with

(a) Two equipotential curves and the minimum distance, Δx, between them.

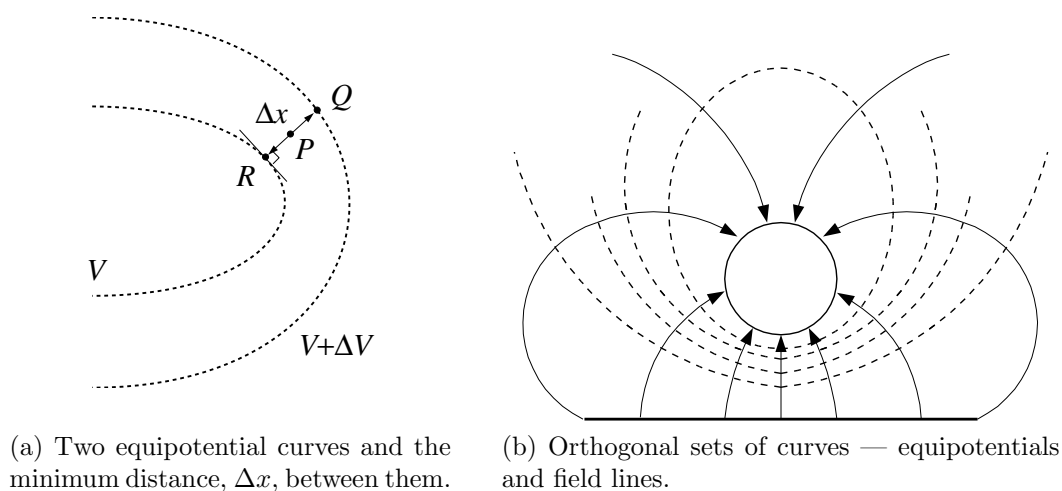(b) Orthogonal sets of curves — equipotentials and field lines.

Figure 4.3: Equipotential curves

the direction of the electric field corresponding to the direction of steepest downward slope at the point in question. Thus, as shown in Figure 4.3a, with one equipotential at potential V and a second, nearby one, at potential $V + \Delta V$, the magnitude of the electric field between points Q and R is approximated by

$$E \approx \left| -\frac{\Delta V}{\Delta x} \right|, \tag{4.6}$$

where Δx is the minimum distance between the two equipotentials. The direction of the electric field is perpendicular to the equipotential curve and pointing towards smaller voltages.

We can now envision two entire families of curves, as illustrated in Figure 4.3b: the first, a set of equipotentials, and the second, a set of curves whose tangent at a point indicates the direction of the electric field at that point. The members of the latter family, the electric field lines, are also called "lines of force" because the electric field is in the direction in which a positive charge q would move if it were placed at that point. As mentioned earlier, the equipotentials form closed curves, while the field lines start on positive charge and end on negative charge. The two families of curves are orthogonal (perpendicular), because at any given point, say A, the tangent to the equipotential through A is perpendicular to the field line through A. The direction of a field line is in the direction of the electric field.

The arrows on field lines indicate the direction of the electric field.

Conventionally, equipotential are selected with their potentials in a sequence of uniform steps (i.e., constant ΔV). The spacing (Δx) between the equipotentials then immediately gives the relative strength of the corresponding electric field at the point: small Δx corresponding to large E.

Conventionally, the spacing of the field lines is also designed to give a qualitative measure of the magnitude (strength) of the electric field; that is, the field is stronger where the field lines are more concentrated. (In 3 dimensional space we talk about the "density of field lines", meaning the number of lines per cross sectional area. In our plane figures, this corresponds to the separation between the field lines.) Once the equipotentials have been mapped, the field lines can be found by drawing the orthogonal family of curves.

Procedure

1. A pair of electrodes have been defined by a pattern painted with conducting silver ink on black high resistance paper. Record (copy or trace) the electrode patterns onto a piece of graph paper. Make copies for each lab partner.

2. Place one of the conductive sheets on the cork board. Connect each painted electrode to the power supply (battery eliminator) using a positive (+, red) or negative (−, black) wire and a pushpin. Do not break the conducting path of the painted electrode with a pushpin hole—instead the pushpin should be placed directly alongside the painted electrode. The pushpin will then press and hold the ring terminal of the wire to the painted electrode without scraping off the paint. You can hold the sheet in place with additional pins at corners. Connect the black (negative, common or "COM") terminal of the DMM to the negative terminal on the power supply. (Figure 4.4 briefly describes the operation of the multimeter.) You will use the (red) probe connected to the other voltage terminal of the DMM to measure voltage difference.

3. Have your lab instructor check your set-up. The DMM should be set to read DC Volts (FUNCTION ⎓V), with a RANGE of 20 Volts. Set the power supply to about 10 Volts DC.

4. With the free probe of the DMM, touch the positive electrode and record the potential difference V_{PO} between the two points probed by the DMM. Press firmly to make good electrical contact; however, do not puncture or otherwise damage the paper (or the painted electrode). Touch the DMM probe to various points on the positive electrode. (This may not be possible if the electrode itself is very small, i.e., a "point charge".) Each piece of the electrode should be at the same potential; if the potential seems to vary check that you have firm connections to the electrode and contact your instructor if you cannot correct the problem.

   Touch the free probe of the DMM to the negative electrode and record the (nearly zero) reading V_{NO}. (Again: this electrode should also be an equipotential. If you cannot achieve an equipotential electrode by reseating the ring terminal on the painted electrode, contact your instructor.) In both readings, O refers to a fixed location at the negative terminal of the power supply. V_{NO} and V_{PO} are the potential differences between the negative (N) and positive (A)
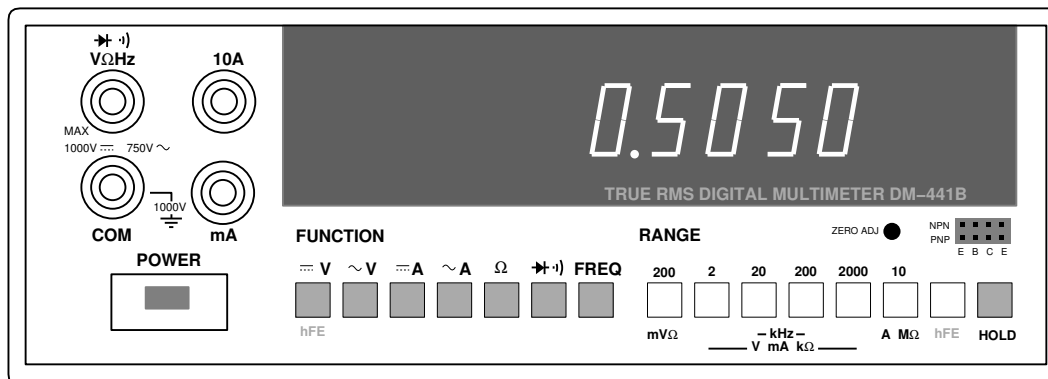
Figure 4.4: The DM-441B digital multimeter. When used as a volt/ohm meter, the leftmost inputs ("VΩHz" and "COM") are used. When used as an ammeter, the bottom inputs ("mA" and "COM") are used. "COM" refers to common or ground: the low terminal. The FUNCTION buttons determine the mode of the multimeter: V (voltmeter), A (ammeter), and Ω (ohmmeter). AC modes are denoted with $\sim$ and DC modes are denoted with $=$. Note that accurate measurement requires you use the appropriate RANGE: the smallest possible without producing an overscale (a flashing display).

electrodes, and the reference O. Compute V_{PN}, the potential difference between the positive and negative electrodes.

5. In the vicinity of the negative electrode, find a point where the DMM reads 2.0 Volts as the free probe is touched to the paper. Remember, do not puncture or otherwise damage the paper. Record this point as accurately as you can on the (white) graph paper.

   Now move the free probe of the DMM to another nearby point where the reading is again 2.0 V and again record this point on your graph paper. Obtain a series of such points, all corresponding to a DMM reading of 2.0 V, by gradually moving the probe from point to point. *Continue until the set of points closes on itself or, perhaps, goes off the edge of the paper.*

6. Repeat the previous step for some other DMM readings, say 3.0 V, 4.0 V, 5.0 V, 6.0 V, 7.0 V, and 8.0 V. You should obtain at least five sets of equipotential points.

7. Draw the equipotential curves on the graph paper by connecting the points at the given potential with a smooth curve. Each such curve is an equipotential and can be labeled by the appropriate DMM reading.

8. Repeat Steps 2 through 7 for the second electrode configuration.

Analysis

Show your equipotentials to the lab instructor. The instructor will assign a point —
call it A — where you should find the electric field.

Carry out the following steps:

1. Draw the segment of the equipotential that goes through A. (A may have
   been selected so that it does not lie on an equipotential you have measured.
   Nevertheless you should be able to estimate the orientation of that unmeasured
   equipotential by comparison with the neighboring, measured equipotentials.)

2. Draw the perpendicular to this equipotential through A.

3. Find the distance Δx, by measuring in cm using a ruler, between two adjacent
   equipotentials, which differ by ΔV. Find the magnitude and direction of the
   electric field $\vec{\mathbf{E}}$ at point A, i.e., $\vec{\mathbf{E}}_P$. Record the magnitude and indicate the
   direction by a vector placed at A on the graph paper. (Δx could also be defined
   as the shortest distance between the adjacent equipotentials measured along a
   line the goes through the point A.)

4. Sketch the field lines orthogonal to the equipotentials for this configuration of
   electrodes, and indicate their direction by small arrowheads. (See Figure 4.3b.)

5. Quick Report: You value of E with proper units, and appropriate significant
   figures (error not required).

6. Following the suggestions given in the Introduction to this Lab Manual, record
   in your notebook a brief critique of this lab.

<div align="center">CHECKLIST</div>

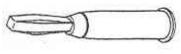|	
Pre-Lab Questions	
Sketch of the Apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
2 pages of equipotential curves	
One set of equipotentials with sketched field lines	
Calculation: E magnitude and direction	

5. DC Circuits

Pre-Lab Questions:

1. Use Eqs. (5.6) and (5.13) to calculate the equivalent resistance R_{par} and its uncertainty δR_{par} when resistors of 4.300 kΩ and 2.600 kΩ, with uncertainties of 0.5%, are connected in parallel.

2. Calculate the equivalent resistance R_{ser} and its uncertainty δR_{ser} if the same two resistors are connected in series.

Purpose

In this experiment you will become familiar with three of the most commonly used electrical measurements (current, voltage and resistance), two of the most commonly used pieces of equipment in the electronics lab (power supply and digital multimeter), and some characteristics of two common electrical components — the resistor and capacitor.

Apparatus

1. power supply,

2. 2 digital multimeters (DMM), (DC δV: 0.5%, DC δI: 0.5%, δR: 0.5%)

3. 2 resistors of different resistance,

4. plug-in board, with 6 circuit templates,

5. an assortment of banana plug-in leads.

6. *RC* box, LabQuest interface, *Logger Pro*

Theory

Current, Voltage and Resistance

In many materials electrical charges may move freely. Positive and negative ions are often free to move in liquid solutions and in gases; In metals valence electrons move freely. Current is a measure of this charge movement or flow. The SI unit of current is the **Ampere** (or 'amp', A) which is defined as a coulomb of charge per second. Since either positive or negative charges may be flowing (depending upon the material), a

simple convention is adopted in which the direction of current flow is defined as the direction of flow of equivalent positive charge. If negative charges are flowing, the current will be in the opposite direction as the velocity.

The flow of charge, like the flow of water in a river, cannot appear out of nowhere or suddenly disappear. The current flowing into a point along an electrical circuit must be equal to the current flowing away from the point, even if the circuit contains many branches or pathways.

Voltage (potential difference) is a measure of the energy transferred to or from charges. The SI unit of potential difference is the **Volt** (V), which is defined as a joule of energy per coulomb of charge. In electrical circuits the electric field produced by a battery or power supply is generally the source of this energy. Electrical currents are affected by voltage differences. A simple analogy would be water current in a river. If the water started out with a higher energy (possibly a higher elevation) the rate of flow of water would be greater at a given point in the river.

Current and voltage are related. In ohmic materials, there is a linear relationship between current through the material and voltage across it. The resulting proportionality constant is known as **resistance**. The type of material an object is made from and the geometry of the object determines resistance. Ohmic materials obey Ohm's law:

$$V = IR \tag{5.1}$$

where V represents voltage difference, I the current, and R the resistance, with R measured in **Ohms** (Ω). An ohm is equivalent to a volt per ampere.

In a complex circuit with one current input and one current output but with many branches and materials, we define the equivalent resistance to be the total voltage drop (input to output) divided by the current flowing in (which is the same as the current flowing out).

Circuits

Symbols are used to represent the parts of a circuit through which current flows. Some examples of symbols for circuit components and equipment are shown in Fig. 5.1.
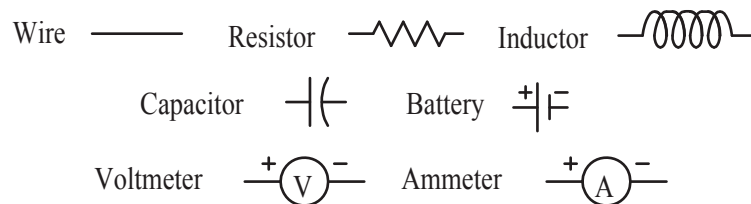

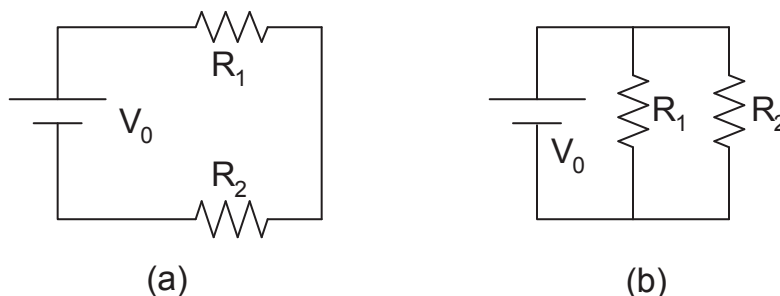Figure 5.1: Some circuit component symbols.

Figure 5.2: Two resistors connected (a) in series and (b) in parallel with a power supply.

In Figure 5.2(a) two resistances of different value are connected in **series**, or end-to-end, in a circuit with a power supply voltage of V_0. Since the circuit has no branches, the current will be the same at all points, so the current through resistances R_1 and R_2 is $I_1 = I_2 = I_0$. The voltage drop across the first resistor added to the voltage drop across the second resistor must equal the total voltage drop across the pair. Thus:

$$V_0 = V_1 + V_2 = I_1 R_1 + I_2 R_2 = I_0(R_1 + R_2). \tag{5.2}$$

Thus, the equivalent resistance for the series circuit, R_{ser} is:

$$R_{ser} = \frac{V_0}{I_0} = \frac{V_1 + V_2}{I_0} = R_1 + R_2. \tag{5.3}$$

In Figure 5.2(b), the two different resistors are connected in **parallel** so each branch experiences the same voltage: $V_0 = V_1 = V_2$. The current I_0 from the battery divides into two branch currents: $I_0 = I_1 + I_2$. From Ohm's Law, the total current is

$$I_0 = I_1 + I_2 = V_1/R_1 + V_2/R_2 = V_0(1/R_1 + 1/R_2) \tag{5.4}$$

so the equivalent resistance for parallel circuits is:

$$R_{par} = \frac{V_0}{I_0} = \frac{V_0}{V_0(1/R_1 + 1/R_2)} = \frac{1}{(1/R_1 + 1/R_2)} \tag{5.5}$$

or more simply:

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{5.6}$$

Capacitor Discharge

Another important circuit component is the capacitor, a charge-storing device which you will probably have learned about in class by now. If a resistor is connected across the terminals of a charged capacitor, current will begin to flow thus discharging the capacitor. As the capacitor discharges, the resulting potential difference (voltage)

across the plates will decrease exponentially. The exponential function describing the discharge is

$$V = V_0 e^{-t/RC} \tag{5.7}$$

where V_0 represents the initial voltage, R the resistance in ohms, C the capacitance in farads and t the elapsed time since the discharge began. You can easily test this relationship by monitoring the voltage as a function of time.

Procedure

A. Use of the power supply and multimeter

Do not turn on the power supply or attempt any multimeter measurements before your circuit has been checked by your lab instructor. The correct operation of the power supply, multimeter and plug-in board will be explained at the start of the lab session. Here are a few important points:

1. The voltage control on the power supply should be set to its minimum value before it is turned on, and the current limit should be set to its maximum. Turn it off when not in use.

2. Leave the multimeter on between measurements. Generally high accuracy measurements required a 'warmed up' instrument.

3. When used to measure current, the multimeter breaks the wire through which you are measuring the current. (Current measurement is always *through* a wire/device.)

4. When used to measure voltage difference, the multimeter leads are attached to the two places you seek the voltage difference between. (Voltage difference is always *across* a device.)

5. When the multimeter is used to measure resistance, the component being measured must be disconnected from the circuit.

6. When any measurement is made with a multimeter, start out in a range which is higher than the anticipated measurement then switch to a lower range, if necessary.

7. The meter accuracies are listed in the Apparatus section. These accuracies assume that at least four digits are being displayed by the meter. If three or fewer are displayed, switch to a lower range.

8. For most circuits tested in this experiment, a template has been prepared that diagrams the circuit using schematic symbols. This may be helpful to you in making the various connections correctly. Each template is designed to guide you through one set of measurements.

9. Record every digit displayed by the meter (this should be four or more digits). Also, calculate uncertainties (e.g., if a voltage is measured as 1.3842, $\delta V = 1.3842 \times .005$).
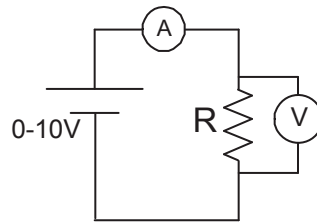
B. Single resistor

Figure 5.3: Current and voltage measurements for a single resistor.

In this part of the experiment you will measure current I through a resistor as the voltage V is varied.

1. First, use one of the multimeters to measure the resistance R of one of the resistors. Press the Ω button on the meter, and connect the two leads to the two ends of a resistor. Read R off the meter's display.

2. Then you will need both multimeters to make the other measurements. One should be set to measure DC current (the $\dddot{}$ A button) and the other set to measure DC voltage (the $\dddot{}$ V button).

3. Make sure the power supply voltage is turned down to minimum.

4. Connecting wires: Normally the wires going to the power supply and multimeters are color-coded red (R) and black (B) to correspond with the color coded terminals on the instruments. Jumper leads, which complete the circuit by connecting the various parts together, are not necessarily color-coded. Selecting a variety of colors for the jumpers may be helpful in keeping track of your wiring. Make all of the required connections by plugging components and leads into the designated places.

5. The circuit to be connected is diagrammed in Figure 5.3. Connect the red terminal of the power supply to the red terminal of the ammeter (marked mA). From the black ammeter terminal (COM), connect to one end of the resistor. From the other end of the resistor run a wire to the black terminal of the power supply. Thus, current from the power supply must pass through both the ammeter and the resistor to complete its circuit back to the power supply. Try increasing the power supply voltage slightly, then back down to zero; you should see a corresponding variation in the current.

6. Now connect the voltmeter's two terminals (marked COM and V Ω Hz) to the two ends of the resistor. If all is correct, when you increase the voltage slightly, you should see both the current and the voltage increase. If it works, you're ready to take data. If it goes in reverse, you can switch the voltmeter wires. If it doesn't work right, get help.

7. Enough data must be collected to construct graphs and do a curve fit. Vary the voltage of the power supply **(never greater than 10V)** and measure the corresponding currents and voltages until you have a sufficient number of data points — say 10 or so. Record the data in a spreadsheet for easy transfer to *WAPP+*.

8. Use *WAPP+* to plot V (on the y axis) versus I (on the x axis) and test for a linear fit. You can enter your x and y errors either as Excel columns or as a formula (you must select which option on the opening *WAPP+* page). For example for $V = y$, the error formula would be `.005*y`. According to Ohm's law, Eq. (5.1), the slope of the line should be the resistance. Compare the slope (*WAPP+* "B" parameter with its uncertainty) to your multimeter measurement of the resistance R. **Q1:** do they agree? Tape the plot and fit report into your notebook.

9. Make a final results table properly displaying (units, errors, significant figures) the value of R from *WAPP+* and the value of R from the direct multimeter measurement.

C. Two resistors in series and in parallel

In this part, two resistors will be used. To assist you in getting all the necessary measurements, there is a sequence of paper templates showing how everything should be plugged into a circuit board. In the first circuit the two resistors will be connected in series. In the second, they will be connected in parallel. The circuits are shown in Fig. (5.4). However, instead of having <u>four</u> different meters connected to the circuits, you will be using your <u>two</u> meters to measure the necessary voltages and currents. To help you accomplish this, three separate templates are provided for each of the two circuits, showing the different ways in which the multimeters are connected. The circuit remains the same — the templates are just to help you get the meter readings, and you needn't use them if you have understood how to connect ammeters and voltmeters.

1. <u>Before</u> you wire either of the circuits, use one of the multimeters to measure the resistances R_1 and R_2 of the two resistors and record the values in your notes.

2. Starting with a series circuit [see Fig. 5.4(a)], identify the appropriate template and wire the circuit for the first set of measurements. When your circuit has been checked, turn on the power supply and adjust it to approximately 10 volts.
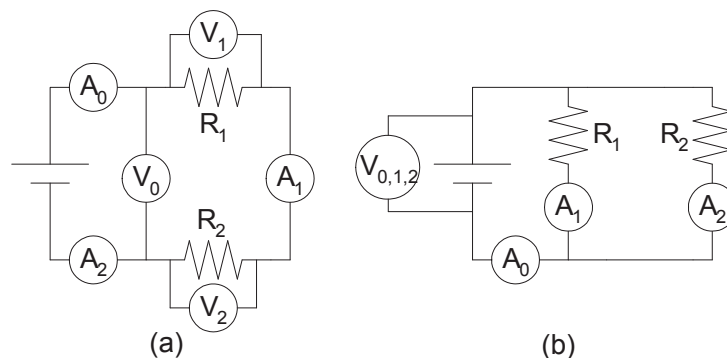
Figure 5.4: Current and voltage measurements for (a) a series circuit, and (b) a parallel circuit.

This <u>constant</u> source voltage will be used for each of the circuits in this part. (Note: if the power supply voltage drifts during your measurements, consult your lab instructor.)

3. Make the measurements indicated on the first template and record them in your lab notes. Then repeat the process for the second and third templates. You will end up with three current measurements (all should be nearly the same) and three voltages (all should be different). Make the following comparisons:

   (a) Currents I_1, I_2 and I_0 should be the same.

   (b) $V_1 + V_2$ should equal V_0.

   (c) The equivalent resistance of the series resistances is defined as

   $$R_{equ} = V_0/I_0. \tag{5.8}$$

   According to Eq. (5.3) it should equal the sum of the individual resistances, $R_{ser} = R_1 + R_2$.

   Note: The resistances you measured directly should be accurate to within 0.5%, so the uncertainty for the sum is also 0.5%:

   $$\delta R_{ser} = \delta R_1 + \delta R_2 = .005 \cdot R_1 + .005 \cdot R_2 = .005 \cdot R_{ser}. \tag{5.9}$$

   The resistance R_{equ} measured from voltages and currents has a fractional uncertainty

   $$\delta R_{equ} = R_{equ} \left(\frac{\delta V_0}{V_0} + \frac{\delta I_0}{I_0} \right) = R_{equ}(.005 + .005) = .01 \cdot R_{equ}. \tag{5.10}$$

   Q2: Does the measured equivalent resistance R_{equ} agree with the theoretical R_{ser} (Eq. (5.3)), within the uncertainties?

   (d) Make a final results table properly displaying (units, error, significant figures) R_{equ} and R_{ser}.

4. Follow the same approach with the parallel circuit [see Fig. 5.4(b)]. You will notice a different pattern of currents and voltages from what you saw with the series circuit. Here

   (a) Voltages V_1, V_2 and V_0 should be the same.
   (b) $I_1 + I_2$ should equal I_0.
   (c) The equivalent resistance of the parallel resistances is defined as

$$R_{equ} = V_0/I_0. \tag{5.11}$$

   According to Eq.(5.6) it should equal

$$R_{par} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}. \tag{5.12}$$

   Note: for the uncertainty in the calculated equivalent resistances (parallel case), applying the methods of Appendix A produces the result:

$$\delta R_{par} = .005 \cdot R_{par}. \tag{5.13}$$

   Q3: Do R_{equ} and R_{par} agree within the appropriate uncertainties?

   (d) Make a final results table properly displaying (units, error, significant figures) R_{equ} and R_{par}.

D. Capacitor discharge

The capacitor and resistor are in a small container with a switch to control charging and discharging of the capacitor, and with appropriate wiring to monitor the voltage across the capacitor terminals. By monitoring $V(t)$ you can test Eq. (5.7).

1. Plug the RC box into analog channel 1 of the LabQuest interface. Make sure the switch on the RC box is set to C (charge).

2. Start the Logger Pro program by clicking on the icon on the desktop. If you see the message **"No Device Connected"** in the upper left corner of the screen, the interface must be unplugged at the computer end of the cable and then, after pausing a few seconds, re-plugged into the computer. You will then see the message disappear.

   Under the file menu, open the file named **rc**. For certain custom analog sensors (like the one used in this experiment) a confirmation window will open that asks you to connect the sensor. Click the **"Connect"** button. A toolbar, a data table, and a voltage vs. time graph will appear. The most useful buttons on the toolbar will be **"Autoscale"** and **"Zoom"**.

   In the lower left corner of the screen, note the box labeled **"Potential"**. This reading shows the charge on the capacitor measured in volts. Do not proceed to step 3 until the potential is somewhere between 2.0 and 2.5 volts. The potential box is active even when data is not being collected.

3. When you are ready to begin, change the RC box switch to D (discharge) and immediately click the Collect button. Data will be collected for four minutes and stop automatically.

4. The resistor and capacitor are visible on the surface of the RC box. The resistor is rated as 1000 Ω and the capacitor as 0.1 F. The product of R multiplied by C is referred to as the *time constant* τ and has the units of seconds. For these components, the time constant is therefore expected to be approximately 100 seconds.

5. When the computer has stopped collecting data, select a portion of the run to be copied to *WAPP$^+$*(or Excel) by left clicking and dragging the mouse pointer over the desired portion of the graph. A light grey shaded area will parallel the entire vertical axis, while a darker grey area will follow the dragging of the mouse. Zooming applies only to the dark grey area.

   When selection a portion of the data for copying and pasting, the selection may be done from the graph, but be sure that you click on the data table slider (right edge of the table) before you make a copy. If you do not, you will copy the image from the graph, and not the numerical data.

6. In *WAPP$^+$*:

   (a) Select a formula-based Y error of 1% and no (i.e., negligible) X error

   (b) Paste the data into block data form.

   (c) Select an exponential fitting function $y = A\exp(Bx) = Ae^{Bx}$.

   (d) According to the exponential function, the A parameter represents the voltage $t = 0$ and the B parameter represents $-1/RC$ which should be about -0.01 sec^{-1}. **Q4:** How close do the values come? Compare the *WAPP$^+$* results to these predictions.

   (e) Include the *WAPP$^+$* fit report and plot in your lab report.

Quick Report: for series: R_{ser} (Eq. (5.3)) and R_{equ} (Eq. (5.8)); for parallel: R_{par} (Eq. (5.6)) and R_{equ} (Eq. (5.11)) all with proper units, error, and significant figures.

Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Pre-Lab Questions	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
V vs. I fit report and plot for single resistor	
Q1: Comparison of slope with measured resistance of single resistor	
Final results table: slope & measured resistance (units, errors, significant figures)	
Measured resistances R_1, R_2 for two-resistor circuits	
Comparison of expected I and V relationships for series circuit	
Q2: Compare R_{equ} and $R_1 + R_2$ (final results table)	
Comparison of expected I and V relationships for parallel circuit	
Q3: Compare R_{equ} and $(R_1 \cdot R_2)/(R_1 + R_2)$ (final results table)	
$WAPP^+$ fit report and plot of capacitor discharge	
Q4: Comparison of $WAPP^+$ parameter B with $-1/RC$	

6. Magnetic Fields

Pre-Lab Questions

1. Find the units that result from the items on the right side of Eq. (6.2), i.e., $\mu_0 n I / B_e$.

2. If B_e is 1.5×10^{-5} T, and the solenoid has 1000 turns/m, how much current in solenoid would be needed to deflect the compass needle by 45°?

Purpose

To explore some basic properties of magnetic fields: (A) that the source of magnetic fields is electric current, (B) that simple superposition applies when magnetic fields from different sources are present, and (C) that, in contrast to electric field lines, magnetic field lines form loops.

Apparatus

1. power supply

2. digital multimeter (DC $\delta I = 0.5\%$)

3. solenoid assembly with compass and resistor mounted inside

4. banana plug leads

5. coil with banana plugs and compass array

Theory

The solenoid

A solenoid is a length of wire wound into a helix. If the turns are tightly wound, the solenoid may be considered a series of closely spaced current loops, each with a magnetic field looping through and around the turn. It can be shown that the individual looping fields will add together to form a field remarkably like the field of a bar magnet. More importantly, the field inside the solenoid is fairly strong and uniform (except near the ends) and is directed along the axis of the solenoid, as illustrated in Figure 6.1. The magnetic field at the center of the solenoid you will be using today is given by
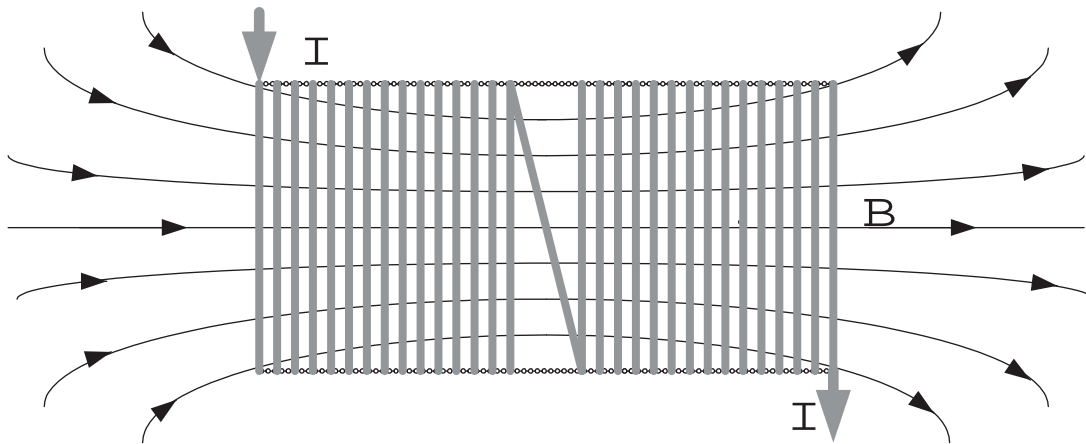
$$B_s = .67 \mu_0 n I, \tag{6.1}$$

Figure 6.1: Magnetic field lines through a finite solenoid of coils carrying current I. Note an application of the right hand rule: if you point your thumb in the direction of the current in this figure, your fingers—when inside the coil— should point to the right just as magnetic field shown.

where B_s is the magnetic field in Teslas (T), $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, $n = N/\ell$ is the number N of turns of wire in a distance of ℓ meters, and I is the current in Amperes. The correction factor of .67 takes into account the geometry of our solenoid, which is finite in length and has a gap in the loops of wire. (If this were an infinite solenoid the correction factor would be 1.)

Compass

The needle of a compass points in the direction of the magnetic field it experiences. We say that a compass needle shows north, because that is (generally speaking) the direction of the Earth's magnetic field. The needle is a magnetic dipole made of a small permanent magnet. Just like a current loop, it feels a torque aligning it to the external magnetic field, so it twists until it is aligned. We will use a compass to show us the direction of the magnetic field that results from the superposition of the Earth's field with that of the solenoid.

The Earth's magnetic field is like that of a coil

The source of the Earth's magnetic field is a huge electric current continuously flowing in the Earth's liquid core; really just a huge version of the coil you will be using in lab to map magnetic field lines. (Confusingly the Earth's core is largely iron and you will use iron to augment the magnetic field in lab. However, in the Earth's core the iron is so hot that the iron there lacks this ability to augment magnetic fields; technically the Earth's core is well above the Curie temperature of iron. Again: the iron in the
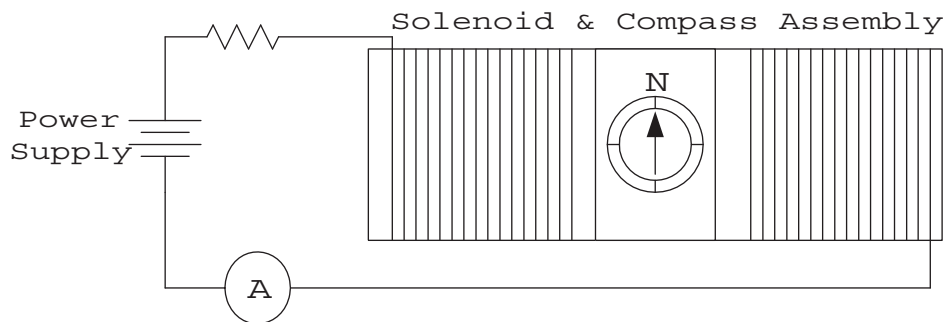
Figure 6.2: Solenoid circuit.

Earth's core is no more magnetic than copper: it is electric current that produces the Earth's magnetic field.)

When you map the magnetic field of a coil, you should find that the field enters one end of the coil (where the lines become concentrated, and hence the field is strongest), comes out the other and loops around on the outside of the coil. The magnetic field lines are loops that go through the coil's center. Similarly the Earth's magnetic field actually points directly down at the North Pole, and out at the South Pole, and only near the Equator is it fully horizontal pointing north. In Collegeville the inclination of the Earth's magnetic field is quite large—around 20 degrees from vertical—so the horizontal component is relatively small. One of the goals in this experiment is to determine the horizontal component of the Earth's magnetic field at our location.

Measuring the Earth's magnetic field

As shown in Figure 6.2, the compass is placed in the center of the solenoid, and the apparatus is oriented so that with no current flowing the compass needle points perpendicular to the coil-axis in the direction "N", i.e., $\theta = 0$. When a current is sent through the coil, the needle will be deflected to point in the direction of the resulting magnetic field. The current will be supplied by an adjustable power supply, and accurately measured with a digital multimeter. If we denote the magnetic field of the Earth by B_e and the magnetic field of the solenoid by B_s, the resulting total magnetic field should be the vector sum of the two. As shown in Figure 6.3, we predict

$$\tan \theta = \frac{B_s}{B_e} = \frac{0.67 \mu_0 n}{B_e} \, I \qquad (6.2)$$

Thus a graph of $\tan \theta$ vs. I should be a straight line.

Note: since the compass needle is only free to rotate in the horizontal plane, only the horizontal components of the magnetic field are detected. Thus B_e is actually just the horizontal component of the Earth's magnetic field. (Of course, the solenoid is oriented so that its field is fully horizontal.) At our latitude it turns out that the vertical component of the Earth's field is more than twice as large as the horizontal
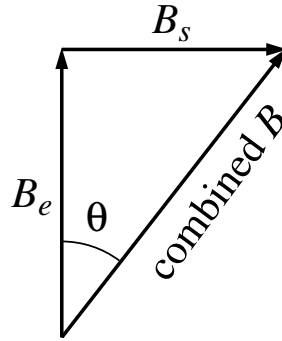
Figure 6.3: Adding two magnetic fields: B_e from the Earth and B_s from the solenoid. The angle of the resulting magnetic field is called θ.

component.

Procedure

Mapping the magnetic field of a coil

1. Connect the coil to the power supply. Insert the iron bar in the coil.

2. On the power supply, twist the CURRENT knob fully counterclockwise to MIN. Turn the power supply on and gradually twist the CURRENT knob until the meter shows a current of about 1 ampere (it need not be exact).

3. Place on the compass array near the side of the coil and tap the array until a consistent pattern of compass needles is displayed.

4. Sketch the magnetic field lines around the coil. Be sure to put small arrow heads on your magnetic field lines showing the direction of the field lines.

5. One end of your coil should show magnetic field lines entering and the other, magnetic field lines leaving. **Q1:** On your sketch mark 'N' to show which end of the coil is the north-seeking end. (Careful!)

Measuring the Earth's magnetic field

1. Position the solenoid/compass assembly so that the compass needle is aligned with north (N) on the compass housing.

2. Without moving the assembly, connect the power supply in series with the assembly and ammeter (initially set to read current up to 200 mA DC) as shown in Fig. 6.2. If you connected the ammeter in a reasonable way, the current values

will be positive, and the compass needle will be deflected clockwise towards small values of the angles.

3. Take a series of readings from the compass and multimeter while increasing the voltage. Readings can be taken in approximately ten-degree or smaller increments from zero to forty degrees. Use the multimeter on an appropriate scale: at least four digits should be displayed and recorded. The uncertainty in current (δI) can then be approximated as 0.5%. [Note: Avoid using (0,0) as a data point: a 0.5% error for that current would claim $\delta I = 0$ which is impossible.] The uncertainty in the compass reading $\delta\theta$ must be estimated based on your ability to read the compass scale. Typically for an analog scale, the uncertainty is estimated to be $\pm$ half of the smallest scale division, but you may wish to assign a larger uncertainty if you believe that the compass is more difficult to read.

4. Reverse the leads on the power supply (only) so the current will flow the other direction through the solenoid, and take additional ten-degree (or smaller) increment readings to forty degrees. For this last set of measurements the currents will be negative. (It may also seem logical to record an angle like $350°$ as $-10°$, but this is optional.)

5. When the measurements have been completed, turn off the power supply and check whether the compass is still aligned with North. If the solenoid/compass assembly has moved, the measurements should be redone.

6. An Excel spreadsheet will be useful for calculating and transferring the data to $WAPP^+$. You may want to set up a table similar to:

θ (degrees)	$\delta\theta$ (degrees)	$\tan(\theta)$	$\delta\tan(\theta)$	I (A)	δI (A)
10					
$\vdots$					
350					
$\vdots$					

Note: Excel requires angles in radians, so your formula for calculating $\tan\theta$ might look like `=tan(radians(A2))` or `=tan(A2*PI()/180)` depending on if you want to directly use the conversion factor to radians of $\pi/180$ or use Excel's built in function `radians(...)` that does the same thing. Similarly the error in $\tan\theta$ could be calculated from $\delta\tan(\theta) = |\tan(\theta+\delta\theta) - \tan(\theta)|$ or, applying a bit of calculus, $\delta\tan(\theta) = (1 + \tan(\theta)^2) \cdot \delta\theta$. In either case, all those angles would need to be converted to radians. Remember to self-document any formulas you use! It may actually prove easiest to just enter the errors as formulas in $WAPP^+$, for example the Y error formula would just be `(1+y^2)*pi/180` where I've assumed $\delta\theta = 1° = \pi/180$ radians and of course $y = \tan\theta$.

7. Do list the currents I in amps! If non-zero, they can be assumed to have an uncertainty of 0.5% (again either as a $WAPP^+$ formula or a column in Excel).

8. Tell $WAPP^+$ your plan for entering errors, and then transfer the appropriate spreadsheet data to $WAPP^+$, letting Y correspond to $\tan\theta$, and X to I.

9. Plot $\tan\theta$ versus I, and fit it to a linear function. The slope of the line will be used to find B_e in the next steps.

10. Determine the value of $n = N/\ell$ (turns/length) for the solenoid assembly by counting turns of wire over a given distance. Do enough turns (50 or more) to be sure that $\delta n/n$ is no more than a few percent. Of course:

$$\frac{\delta n}{n} = \frac{\delta N}{N} + \frac{\delta \ell}{\ell} \tag{6.3}$$

Assume a counting error δN of one; estimate $\delta \ell$ yourself.

11. The slope (B) of the $\tan\theta$ vs. I line should be equal to $.67\mu_0 n/B_e$ according to Eq.(6.2). Use your $WAPP^+$ slope to determine B_e. For an uncertainty, use the formula

$$\delta B_e = B_e \left(\frac{\delta n}{n} + \frac{\delta\text{slope}}{\text{slope}} \right). \tag{6.4}$$

12. Compare your calculated value of B_e with the very approximate value for northern U.S.A. (1.5×10^{-5} T). Magnetic materials in the building will affect the magnitude and direction of B_e from table to table in the lab.

13. Quick Report: Your value of B_e with proper units, error, significant figures.

14. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Pre-Lab Questions	
Sketch of the Apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
Sketch of coil magnetic field showing field lines and **Q1** N pole	
Tabulated values for $\theta, \delta\theta, \tan\theta$ and I	
Uncertainties in $\tan\theta$ and I in $WAPP^+$ or Excel (self-documented!)	
$WAPP^+$ fit report & plot of $\tan\theta$ vs. I	
Calculation of B_e with δB_e	
Comparison with approximate value of B_e	

7. AC Circuits

Pre-Question

Consider a series RLC circuit with the following characteristics: $R = 2\ \Omega$, $L = 1$ mH, $C = 1\ \mu$F, carrying a current $I = .5$ A at a frequency of $f = 5$ kHz. Calculate the voltage across the resistor, V_R; the voltage across the inductor, V_L; and the voltage across the capacitor, V_C. Draw a phasor diagram of these three voltage and calculate the net voltage across the RLC combination.

Purpose

Circuits with time dependent signals ("AC circuits") are more useful and more complicated than circuits where the voltages and currents are constant ("DC circuits"). You have met the capacitor (C), the inductor (L) and the resistor (R) in previous labs; today you will investigate how these devices act in AC circuits. You will particularly examine the series RLC circuit, and measure how the amplitudes and phase angles vary with frequency. And you will examine how these devices can be used in speaker design.

Apparatus

Function generator	Resistor	Set of plug-in leads
PC Oscilloscope	Capacitor	Plug-in board
Inductor (solenoid)	3 templates	Solenoid core
Pair of mounted speakers	Adaptor box & Amplifier	Switch box
Light bulb with leads		

Theory

A. The RLC circuit

You have studied each of these components previously in DC circuits. Three important properties should stand out.

1. Resistors have a linear, instantaneous relationship between the current flowing through them and the voltage across them.

2. Capacitors store charge (equal/opposite amounts: $+Q$ & $-Q$ on separated plates). An incoming current will increase the charge on the corresponding

plate, resulting in an increasing voltage. Note that since the net charge stored by a capacitor is zero (i.e, $+Q$ & $-Q$ add to zero), the current exiting a capacitor must equal the current coming in. Thus we can still talk about the current flowing 'through' a capacitor even though no electron goes from one plate to the other.

3. Inductors produce a magnetic field, which is proportional to the current flowing through the inductor. A change in that current induces an opposing voltage according to Lenz's Law.

B. Phase relationships

In alternating currents the voltage varies as a sinusoidal function of time. If an alternating current is flowing through a resistor, the voltage instantaneously adjusts to the current. Voltage and current remain exactly in step; the current and voltage are said to be 'in phase'. If the same alternating current is applied to a capacitor or inductor, the phase relationships are different.

When current begins to flow into an uncharged capacitor, there is virtually no voltage drop across the capacitor. As long as there is an inward current, charge accumulates, and the voltage will increase. Only when the current switches from incoming to outgoing (i.e., is zero) will the voltage voltage start to decrease. Thus the voltage peaks after the current peaks, in fact when the current is zero. The voltage and current are 90° out of phase, with the voltage lagging the current. Graphically, the phase relationships appear as in Fig. 7.1(b).
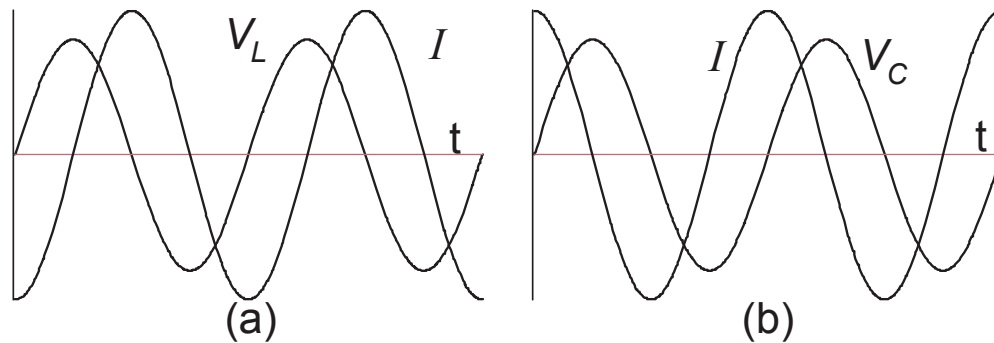
Figure 7.1: Current-voltage graphs for (a) an inductor and (b) a capacitor. In the inductor the voltage peaks before the current peaks. In the capacitor the voltage peaks after the current peaks.

In the case of the inductor, only if the current is changing does Lenz's law convert the changing the magnetic field to a voltage (emf) opposing the current change. When the current is a maximum, it is neither increasing nor decreasing, and the induced voltage is zero. The maximum voltage actually occurs earlier—as the current switches from

negative to positive—as that is the moment of maximum changing current. Again the voltage is a maximum when the current is zero, but this time the voltage peaks occur before the current peaks. The voltage and current are 90° out of phase, with the voltage leading the current. Graphically, the phase relationships appear as in Fig. 7.1(a).

Phasors are a useful physical picture of these phase relationships. Recall that both voltage and current are sinusoidal functions with angular representations (eg $V = V_o \sin \omega t$). Picture the current as a counterclockwise rotating vector whose x component represents the actual current. (The y component is purely imaginary, but it helps to make the picture work.) The voltage across a capacitor would then be a rotating vector that is lagging by 90°; the voltage across an inductor would then be a rotating vector that is leading by 90°; and the voltage across a resistor would be exactly in phase with the current. In a series circuit, every device experiences the same current, and the total voltage (V_S) is just the sum (vector sum for phasors) of the voltage across each device. It is convenient to display the phasor at the moment when the current vector is aligned with the x axis (i.e., the physical current is a maximum). The result of vector voltage addition is then a nice right triangle. (At any other moment the voltage sum is that same triangle just rotated at a less convenient angle.) As shown in Fig. 7.2(b), we can then find the net 'phase angle' between the net voltage and the current ϕ is $\arccos(V_R/V_S)$ or $\arctan((V_L - V_C)/V_R)$, etc.
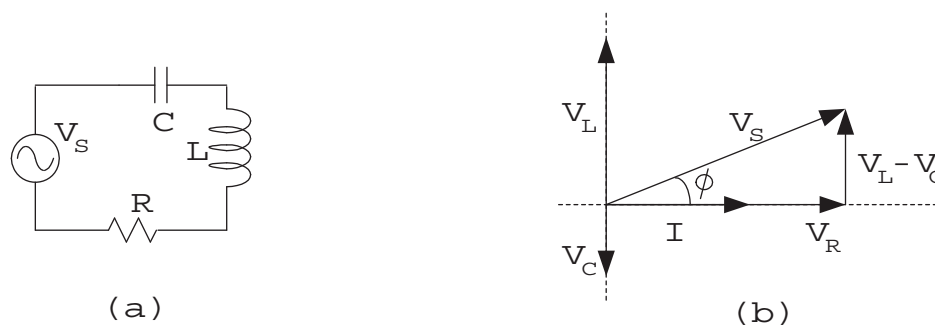
Figure 7.2: (a) *RLC* series circuit with an AC source. (b) Phasor diagram for the circuit.

C. Frequency dependence

In addition to differences in phase, inductors and capacitors show frequency dependent 'resistance', formally called reactance. The induced voltage in an inductor, which opposes current changes, varies with the *rate of change* of current. The greater the rate of change, or the higher the frequency f, the greater the opposition. The **inductive reactance** X_L, is proportional to the frequency f:

$$X_L = 2\pi f L. \tag{7.1}$$

When current flows to a capacitor and charge accumulates on the plates, the amount of charge accumulated will depend on charging time. The longer the period of the AC cycle (the lower the frequency), the greater the charge build up and the greater the voltage. **Capacitive reactance** X_C is a measure of this opposition:

$$X_C = \frac{1}{2\pi f C}. \tag{7.2}$$

In the phasor diagram, the three voltages $V_R, V_L,$ and V_C can be replaced with IR, IX_L, and IX_C. The aim here is to think about inductors and capacitors as devices with a 'resistance' that depends on the frequency. This new 'resistance', (properly called **reactance** or more generally **impedance**), represented by Z, can be defined as the phasor sum of resistance, inductive reactance, and capacitive reactance:

$$Z^2 = R^2 + (X_L - X_C)^2 \tag{7.3}$$

An interesting "what if?" presents itself. What if an AC current is applied to an RLC series circuit in which $X_L = X_C$? At this frequency, the total phasor sum of the two reactances $(X_L - X_C)$ is zero; the impedance Z in then minimized and so is V_S which now equals the remaining leg of the triangle: V_R. In order for this situation, known as 'resonance', to occur we must have:

$$X_L = 2\pi f L = X_C = \frac{1}{2\pi f C}. \tag{7.4}$$

Solving for this special frequency (which we call the resonance frequency f_0) gives

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \tag{7.5}$$

This type of circuit is the heart of electronic "tuning". It can be used to select frequencies or bypass frequencies.

Procedure

A. The RLC circuit

In this part, a function generator with attached amplifier and adapter box will act as the source of alternating current supplied to a series circuit containing an inductor, capacitor, and resistor. The capacitance of the capacitor is printed on the capacitor (it has an uncertainty of 2%). The inductance of the coil will be determined later.

1. Connect the entire series RLC circuit to the function generator. The circuit should be arranged as in Fig. 7.3 (corresponding to template #1). Note that for this step you should connect only the "first location" CH1 V_R; you will use

the "second location" in the following step. The switch box contains an 'active' switch that requires power from the LabQuest interface.

Make sure the iron core is inserted in the inductor.

The amplitude of the function generator should be set near the middle of its range and left there throughout this part of the experiment. Select the 20k frequency range button, and adjust the frequency knob to about the middle of its range. Open the PC oscilloscope program (see page 19) and load the file **ac.set**. Click on **Run** and then **Autoset**. Note that Ch1 is displaying V_R. It may be necessary to adjust the trigger level to obtain a stable display. You will notice some noise in the signal due to the unshielded wiring. While watching the oscilloscope display, gradually change the frequency of the function generator. When the greatest voltage amplitude (V_R) is seen on the oscilloscope, you will be very close to resonant frequency f_0. Now turn on Channel 2 which connected to the switch box.
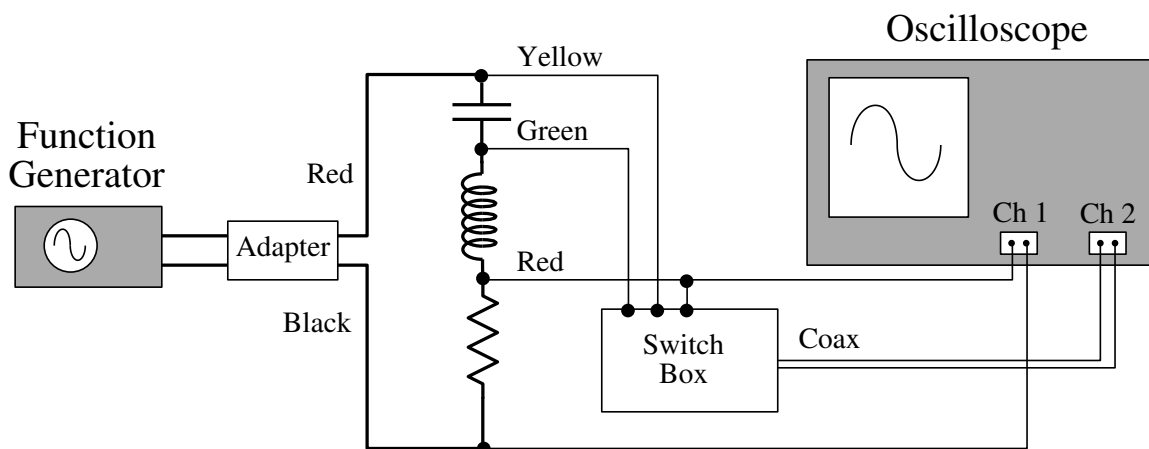
Figure 7.3: Connection of instruments to an RLC series circuit.

2. Turn the switch to $\mathbf{V_r}$, which is the combined voltage across L and C. To precisely determine the resonant frequency f_0 carefully adjust the frequency to give a minimum V_r voltage, as seen on Channel 2. Be certain you are seeing both a maximum on Channel 1 (V_R) and a minimum on Channel 2. You should also notice that the two waveforms are exactly in phase. In fact, you may find that making adjustments by observing phase may be easier than by observing amplitude. An accurate value for f_0 will be very important in later calculations. Record the frequency. Make use of the horizontal lines ('cursors') that can be dragged with the mouse to determine the peak-to-peak voltage of both V_R (Channel 1) and V_r (Channel 2). To obtain these voltages, use one channel at a time (turn the other channel off), adjust the cursors and read the peak-to-peak voltage in the lower left corner of the display. V_R is measured by Ch1 and V_r is measured by Ch2. V_r represents a voltage drop that is partially due to the resistance of the wire in the inductor. Even though the resistivity of copper wire

is small, the long length of wire in the coil results in some noticeable resistance. Additionally power dissipation due to eddy currents in the iron core contribute to V_r.

Without changing the frequency, use the scope to measure the peak-to-peak voltage across each of the other two components individually and the source voltage V_S (move the CH1 red lead from the V_R position to the V_S position). Turn the switch to V_C. When measuring V_S (Channel 1) and V_C (Channel 2), you will notice that V_S leads V_C by 90°. Now turn the switch to V_L. V_S (Channel 1) will lag V_L (Channel 2) by 90°. Also, the magnitudes of V_C and V_L will both be larger than V_S. The two measured values of V_S should be the same. If they vary slightly, use the average of the two to represent V_S in your calculations. You should end up with five measured voltages (V_r, V_R, V_C, V_L, and V_S) along with your measurement of the resonant frequency f_0. Here is a question to answer when you finish this experiment: Why don't V_r, V_R, V_C, and V_L add up to equal the source voltage V_S? That's what you might expect from your experience with DC circuits, or knowledge of Kirchoff's rules. To learn the answer, keep going.

3. In your notes, place all of these measurements in a table along with their estimated uncertainties. The f_0 uncertainty will be a combination of the 1.5% function generator uncertainty along with an estimate of your ability to pinpoint the resonant peak.

4. Calculate the inductance L of the coil from the resonant frequency; rearranging Eq. (7.5) gives

$$L = \frac{1}{4\pi^2 f_0^2 C} \tag{7.6}$$

and the rules for finding uncertainties (p. 99) give

$$\delta L = L \left(2 \frac{\delta f_0}{f_0} + \frac{\delta C}{C} \right). \tag{7.7}$$

If frequency f_0 is in Hz, and capacitance C is in Farads, then L will be in henries. The capacitance should be approximately $C = 1 \ \mu F$, with a fractional uncertainty $\frac{\delta C}{C} = 0.02$.

5. Your measurement of V_L is actually a phasor combination of the voltage drop across the resistance of the copper wire in the coil and the voltage drop resulting from induction ($V_{L'}$). As the phasor diagram in Fig. 7.4 shows, At resonance $V_{L'}$ and V_C are nearly equal in magnitude. Notice that V_L (the phasor sum of V_r and $V_{L'}$) is expected to be slightly larger in magnitude than V_C. Is V_L actually slightly larger than V_C according to your measurements? Discuss in your conclusion.

6. Now find f_0 with the iron core removed from the inductor. To find f_0, follow the instructions outlined for observing V_R and V_r. From Eq. (7.6), judge whether the inductance has increased or decreased without the core. Explain.
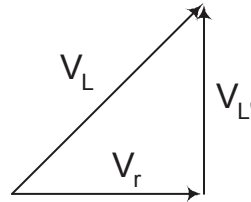
Figure 7.4: Pure Inductance Voltage Drop.
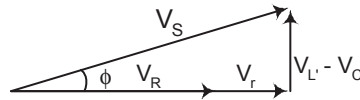
Figure 7.5: Phasor Diagram.

7. In the previous measurements, f was kept constant as you measured V_R, V_r, V_S, V_C & V_L. For this part, you will change f and will (unfortunately) notice changes in V_S. In contrast to textbook voltage sources, real sources will reduce their output voltage when connected to smaller impedances, which is what happens as your approach resonance.

   With the iron core removed, change to template #2, replace the resistor with the light bulb and reduce the function generator frequency (initial f_0) found in step 2 by approximately 1.0 kHz. The circuit will no longer be in resonance.
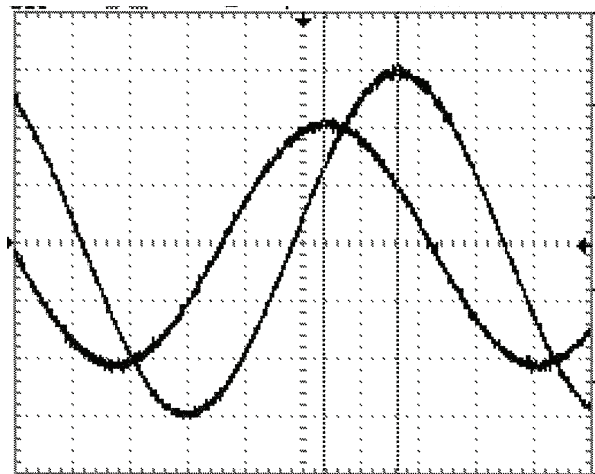
Figure 7.6: Measuring Phase Angle ϕ.

   Use channel 1 to trigger the display. The scope display should look like Figure 7.6.

   The cursors (vertical lines this time) will soon be used to determine the phase difference between V_S and V_R by measuring a time difference between the adjacent peaks from the two channels. The ratio of this Δt to the wave period

$T = 1/f$ will equal the ratio of ϕ to $360°$, that is:

$$\phi = 360° \cdot f \cdot \Delta t \tag{7.8}$$

Adjust the function generator amplitude to maximum and slowly increase the frequency back to f_0. You should notice that the light bulb lights up and becomes brightest at f_0; the maximum power transfer occurs at resonance. Recall that, in general, the power transferred in a circuit is given by

$$P = IV_R = IV_S \cos \phi = P_0 \cos \phi \tag{7.9}$$

where $\cos \phi$ is called the power factor. (Figure 7.5 may remind you of the proof of this result.)

Now reduce the frequency back to the point where the bulb just begins to glow. Use the cursor tool to find Δt between the two waveforms, and calculate ϕ. Use the phase angle (ϕ) to calculate the fractional power (P/P_0) now dissipated in the light bulb.

B. Speaker crossover frequency

You are probably familiar with the terms woofer and tweeter. They refer to speakers that are designed to reproduce low frequencies and high frequencies, respectively. If the speakers are both connected to the same signal source, how do the low frequencies get to the woofer and high frequencies to the tweeter? To make the frequency separation, a 'crossover network' is employed. Inductors offer high reactance to high frequencies and capacitors offer high reactance to low frequencies. The following circuit, Fig. 7.7 (corresponding to template #3) illustrates how to take advantage of the properties of inductors and capacitors to channel frequencies to the appropriate speaker. The oscilloscope is connected as shown to allow the voltages across the two
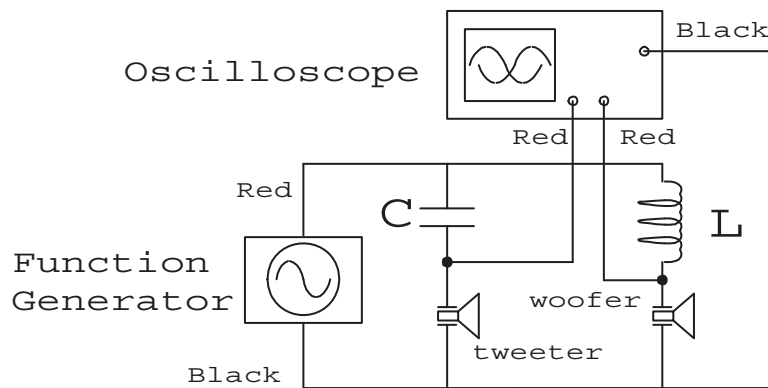

Figure 7.7: Connections to measure speaker crossover.

speakers to be compared.

Technically, the crossover frequency is the frequency at which the *sound outputs* of the tweeter and woofer have decreased by an equal number of decibels from their normal values. This would require use of calibrated microphones, so instead you will find a frequency where the *voltages* across both speakers are equal, which is a nearly equivalent way of defining "crossover". Insert the iron core into the inductor to increase its inductance. Using template #3, connect one channel of your oscilloscope across the tweeter and the other across the woofer. Make sure the black ground leads from the oscilloscope and function generator connect together as shown. Adjust the frequency of the function generator until the peak-to-peak voltages are the same for both channels. (Note: make sure the oscilloscopes voltage scales are the same for both channels.) Record the crossover frequency. The uncertainty in frequency must be estimated from your ability to pinpoint the occurrence of the crossover which may well be greater than the 1.5% function generator uncertainty.

Lab Report

Along with your data, your lab report should include the following:

1. For RLC resonance:

   (a) Determine the inductance of the coil with and without the iron core. Include calculated uncertainties.

   (b) Show your calculations and comparisons from procedure parts A.3, A.5, and A.7.

2. For speaker crossover: determine the crossover frequency (equal speaker voltages). Include the estimated uncertainty.

3. Quick Report: the inductance of the coil with and without the iron core each with proper units, error, and significant figures.

4. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

<div align="center">CHECKLIST</div>

Pre-Lab Question	
Theory, with relevant equations and quantities defined	
Resonant frequencies with uncertainties	
Table of AC voltages for the RLC circuit	
Calculated inductances with uncertainties	
Comparison of V_C and V_L	
Phase angle and power reduction for "off-resonance"	
Crossover frequency with uncertainty	
Brief explanation why $V_r + V_R + V_C + V_L$ can exceed V_S	

8. Diffraction

Pre-Lab Questions:

1. Suppose you shine laser light of wavelength 633 nm at a hair of thickness 75 μm. How wide should be the central bright zone of its diffraction pattern on a wall 2 m away?

2. If a grating has 13,400 lines per inch, what is the line spacing d in microns?

Purpose

To observe features of light showing its wave nature.

Apparatus

Laser box, single-slit slides, diffraction grating with magnet attached, blank slide frame with magnet attached, meter tape, darkened room.

Introduction

The light rays we used in the previous lab to explain the behavior of lenses suggest that light is a particle. And indeed that was the opinion Newton published in his *Opticks* (1704). However, particularly after the experiments of Thomas Young (c. 1801), wave theories of light seemed to offer simpler explanations phenomena like double-slit interference. In the 1860s, Maxwell's theory of electromagnetism seemed to close the book on the nature of light. However in 1900 Max Planck's work on blackbody radiation seemed to require particles of light (photons), and today we talk about a dual nature of all 'particles' as a result of the quantum revolution of the 1920s.

In this experiment we bring together effects in which the wave nature of light is visible. (Oddly enough is doing so we will make prominent use of lasers whose operation would require photons to explain.) When two or more waves occupy the same space, they '*interfere*'; that is, the result is the simple sum of the waves ('superposition'). In some places the terms added together will reinforce each other; in other places they cancel. Waves also *diffract*, or bend around the edges of an obstacle. By making careful measurements of interference and diffraction effects, you can determine details about light sources and small objects or openings.

In particular, in this experiment you will be measuring:

1. the wavelengths of two different lasers (one emitting red light, the other one green) using a diffraction grating.

2. the size of a small single slit, using laser light.

3. the thickness of a hair, using a laser of known wavelength.

For each measurement the apparatus is set up as illustrated in Fig. 8.1. Laser light of wavelength λ is directed toward some diffracting object — a slit or a hair or a grating — and makes a pattern of light and dark 'fringes' on a screen some distance L from the object. You will be measuring the positions y_i of features in the diffraction pattern. From y_i and L you calculate the angles θ_i of these features, and from these you determine the desired information about the object or laser light.
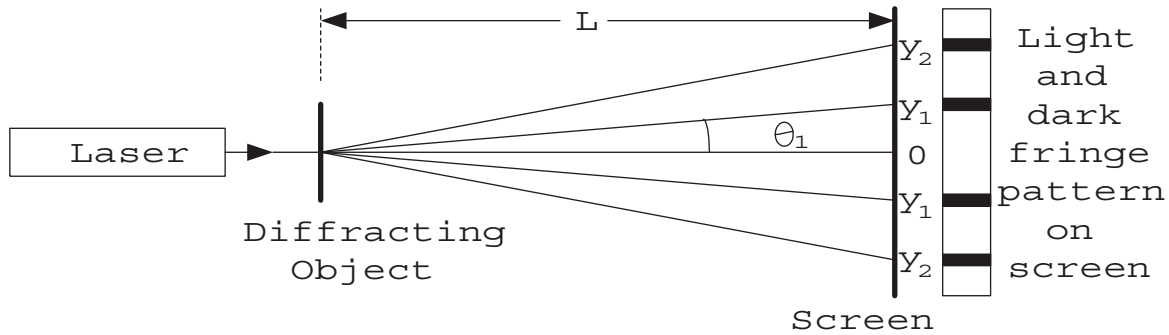
Figure 8.1: Diffraction measurements.

Theory

Diffraction grating

A diffraction grating is a large number of narrow slits separated by a distance d. For diffraction by a grating, a simple formula gives the angular locations of intensity <u>maxima</u>, or bright spots in the pattern (the result of 'constructive interference'). They occur at angles θ_m satisfying the formula

$$d \sin \theta_m = m\lambda, \quad m = 0, \pm 1, \pm 2, \ldots \tag{8.1}$$

So if d is known and several angles θ_m are measured, several estimates of the wavelength λ may be calculated: $\lambda = \frac{d}{m} \sin \theta_m$.

Single-slit diffraction

When light of wavelength λ passes through a slit of width a, there will be intensity minima, or dark fringes, at angles θ_m that satisfy

$$a \sin \theta_m = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots \tag{8.2}$$

At these angles there is 'destructive interference' of waves from all points in the slit. For a derivation of this formula, check your physics text. If λ is known and several angles θ_m are measured, then several estimates of the slit width a can be calculated: $a = m\lambda / \sin \theta_m$.

Diffraction by a thin obstacle

Babinet's Principle, which you might find in your physics text, says that the diffraction patterns of complementary objects are alike. This means that if, instead of an opening of width a, we use an obstacle of width a, the same diffraction pattern results. So in Eq. (8.2) the minima y_m can also be used to find several estimates of the width of a hair.

Procedure

Do not look into the lasers or point the beams at anyone.
Some of the measurements will require a dark room. To see what you are doing, it works well to have a spreadsheet on the computer monitor for doing your calculations and to provide light to see what you are doing. It is recommended that you prepare a spreadsheet right at the start, so that only the positions y_m of the maxima or minima need to be entered, and the calculations of λ and a and will be done automatically. Spreadsheet layouts are in the following sections.

Nature of white light: a color combination

Figure out which switches control the red and green lasers, and make sure the lasers are switched off so you won't be looking into them. Indeed flip the box on its side so those lasers point directly into the table top. The other three switches control an LED (now facing up) beneath them. Turn on all three of these LED switches and notice that the LED appears to be white. With the diffraction grating covering your eye (which will be a couple of feet above the LED on the tabletop), look—not at the LED—but to the side of the LED. See there separate blobs of red, green and blue light; the grating is separating those colors from the white LED light source. Record the apparent color of the LED with all the possible different switch combinations.

Measurement of green laser wavelength λ_g

1. Now turn off the LED, aim the lasers toward the wall beside your table and switch your green laser on. Put the diffraction grating over the opening so the magnetic strip holds it properly in place. According to the label on the grating, it has 13,400 lines/inch, or 528 lines/mm, so $d = (1/528)$ mm. Aim the laser beam through the grating to observe the diffraction pattern on a white meter tape fastened to the side bench at the end of your lab table. If necessary, elevate an edge of the laser box with sheets of paper or wooden shims. You may have to rotate the grating slightly to see the entire pattern on the meter tape. Do not mark on the meter tape!

2. Measure the distance L from the grating to the tape. Record the locations of the diffracted spots on the tape.

3. Make a table for m, y_m, θ_m, and λ_g:

m	y_m	$\theta_m = \arctan(y_m/L)$	$\lambda = \frac{d}{m} \sin \theta_m$
1			
2			
			average $\overline{\lambda_g}$
			SDOM $\sigma_{\overline{\lambda_g}}$

The distances y_m are from the center of the pattern, but it is more accurate to measure the distance between corresponding maxima on both sides of the pattern and divide by two; this will correct for the fact that the beam might not be perpendicular to the viewing screen and result in half as many table rows (just $m > 0$). Note that in Excel arctan is `ATAN`. Compute the resulting values of λ_g.

4. Get an estimate of λ_g and its uncertainty by taking the average and standard deviation of the mean of your computed values. Compare with the accepted value of $\lambda_g = 532$ nm.

Measurement of red laser wavelength λ_r

Use the same procedure as above, but with the red laser beam going through the diffraction grating. Mark the diffraction pattern maxima, and use them to estimate λ_r. It may vary a bit with temperature, but λ_r should be in the vicinity of 650 nm.

Make a final results table properly displaying (error, units, significant figures) your results for λ_g and λ_r (in nm). Also include in this table the accepted values of these wavelengths. Comment if your results are consistent with the accepted values.

Single-slit measurement

1. Shine the green laser beam through a single slit.

2. Get a nice symmetric pattern of fringes on the screen. Measure the distance L from the slit to the screen, and measure the positions of the diffraction minima centers. You should be able to measure many (say out to $m = \pm 6$) minima locations. An easy way to make these measurements is to hold a scrap sheet of paper on the pattern and mark the minima locations with pencil. You can then measure the pencil marks in good light on a table top. The distances y_m are from the center of the pattern, but it is more accurate to measure the distance between corresponding minima on both sides of the pattern and divide by two.

3. Make a table of your data and calculations:

m	y_m	$\theta_m = \arctan(y_m/L)$	$a = m\lambda/\sin\theta_m$
1			
2			
...	...	...	...
6			
			average a
			SDOM $\sigma_{\bar{a}}$

Fill out the calculations of a in your table assuming that $\lambda = 532$ nm.

4. From your results for a, compute the average and the standard deviation of the mean as an uncertainty in the average. Properly report (error, units, significant figures) your value for a in μm and compare it to the given width of the slit.

Measurement of hair thickness

1. Obtain a hair somehow from someone.

2. Using the slide frame, position the hair in the center of the green laser beam.

3. View the diffraction pattern on the meter tape or paper screen, and measure the locations of minima in the pattern. Also measure the distance L from the hair to the screen. Yep, marking the tape is still verboten.

4. Make a table for m, y_m, θ_m and a as shown above. Calculate estimates of a, again assuming $\lambda = 532$ nm.

5. Find the average value for a in μm and standard deviation of the mean. Is this a reasonable estimate of the thickness of a hair? Hair thicknesses are generally in the range 50–100 μm.

Quick Report: your values of λ_g, λ_r, and single-slit a all with proper units, error, significant figures.

Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

<div align="center">CHECKLIST</div>

Pre-Lab Questions	
Sketch of the Apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
Observations on colors of LED	
Measurements of grating diffraction maxima for green light	
Calculation of λ_g and $\delta\lambda_g$	
Measurements of grating diffraction maxima for red light	
Calculation of λ_r and $\delta\lambda_r$	
Final results & comparison λ_g, λ_r (units, error, significant figures)	
Measurements of single-slit diffraction pattern minima	
Calculations of slit width a with δa	
Final results & comparison a (units, error, significant figures)	
Measurements of hair diffraction pattern minima	
Calculations of hair diameter	

9. Thin Lenses

Pre-Lab Question

An object that is at 20 ± 1 cm from a lens has an image at 10 ± 1 cm. What is the focal length (including uncertainty; use high-low method) of the lens?

Purpose

To investigate the relationship between object, image, and focal length in thin lenses. To make and test a simple telescope from two lenses.

Apparatus

- two converging lenses with differing focal lengths

- graduated optical rail, clamps, lens holders

- LED object, screen

Introduction

The formation of images by lenses is fundamental to many optical instruments. A knowledge of image formation by lenses is basic to the understanding of all optical systems from the human eye to the most complex telephoto lens. Complicated systems can be analyzed by applying a simple image-object relationship to each lens in the system. In this experiment, we will investigate the relationship of the image, the object, and the focal length of thin lenses.

Theory

Two common types of lenses are illustrated in Figure 9.1. In the simplest lenses the surfaces are sections of spheres. In the illustrated example, the front and rear surfaces of the lenses have matching radius-of-curvature; in general, the two spherical surfaces can have quite different radii (for example, 'plano' lenses have one flat surface). The line joining the centers of curvature is called the optical axis of the lens. Lenses formed with these geometries will either converge or diverge parallel rays of light. Each lens is characterized by a focal length f which is determined by the radii of curvature and index of refraction of the glass used in the lens.
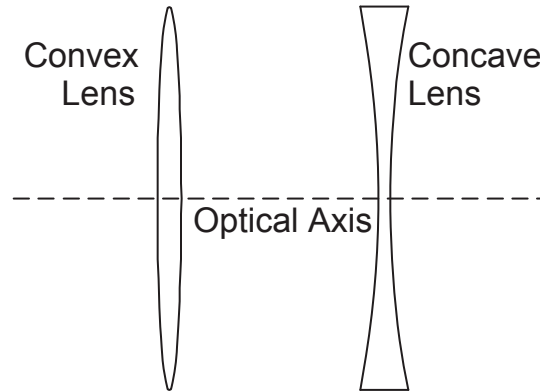
Figure 9.1: Shapes of convex and concave lenses.

The relationship of the image of an object formed by a lens and the focal length of the lens can be determined graphically using the technique of ray tracing and two or more of the convenient rays as shown in Figure 9.2. This relationship is given in your text:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{9.1}$$

where p is the object distance from the lens, and q is the image distance. The rules
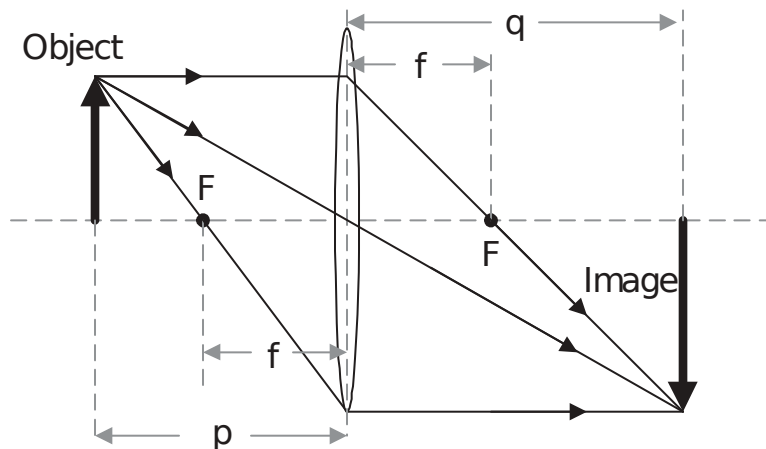
Figure 9.2: Principle rays showing image formation by a convex lens.

for geometrical location can be briefly summarized:

1. A ray parallel to the optical axis passes through the far focal point.

2. A ray passes through the center of the lens undeflected.

3. A ray passing through the near focal point emerges parallel to the optical axis.

Equation (9.1) relating p, q, and f can be derived using geometrical arguments based on similar triangles. It will be our goal to test this relationship experimentally.

Procedure

The measurements in this experiment will be made using an optical rail, which is a track carrying movable clamps for the lens holders, a view screen and the object of interest. Each movable clamp has a pointer which indicates its position on a calibrated centimeter scale which is attached to the track. Make sure this pointer is below the center of the lens or screen when you use it to make distance measurements. The basic layout of the pieces is shown in Figure 9.3.

Figure 9.3: Apparatus for measuring object and image relationships.

1. You will have two converging lenses for the experiment, one thick and one thin. Handle the lenses by their edges to avoid getting fingerprints on them. You will need to know the focal lengths of the two lenses.

   - One way to estimate the focal length of the lens is by locating the real image of a far distant object, since, according to Eq. (9.1), if $p \approx \infty$ then $q = f$. Ideally, the object should be very fa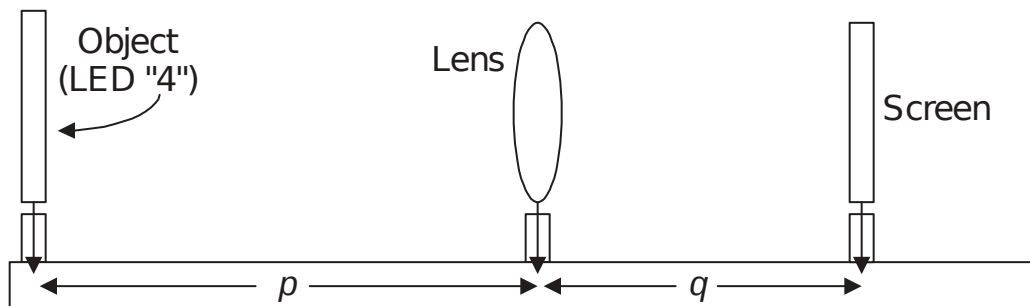r from the lens, but the overhead ceiling lights in the lab will be high enough to give a decent estimate. Use the lens to form an image of the ceiling lights on a piece of paper below the lens, and the distance from the lens to this focused image will be approximately f. (Note: the lens is making a real image of the ceiling lights on the paper; you should view the image without looking through the lens yourself.) The position of the sharpest image is somewhat uncertain due to the "depth of field" of the lens. Use that to determine δf.

   - The thinner lens should have the longer focal length, f_{Long}, and the thicker one has f_{Short}. Record the estimated focal lengths f_{Long} and f_{Short} for both lenses, and estimates of their uncertainties, δf.

2. Next, examine the object/image relationship in detail:

   - An LED numeral (4) 'object', a lens holder, and a white screen (to capture the image) should be mounted on an optical bench. Make sure that the actual LED/lens/screen location matches the location marker on the optical bench scale. (The holders can be mis-rotated so the object location is a bit in front or behind the scale marker.)

- Make sure the LED, the lens and the screen are parallel and lie along the axis of the lens.

- Position the lens slightly more (try about 10 mm) than its f from the object. The separation between LED and lens is p.

- A real image will be formed somewhere on the other side of the lens. Adjust the position of the screen to get the sharpest image possible. The separation between lens and screen is q.

- As noted above because of 'depth of field' the exact location of 'sharpest image' will be uncertain. Find the smallest and largest values of q that still look in focus and use half that range as δq. **Note:** for some values of p, δq will be large and for others it will be small. **You must estimate δq every time you change p.**

When you have everything under control,

(a) Record the object and image distances, p and q. The numbers marked on the edge of optical rail can be helpful, but remember that p and q are separations <u>from the lens</u>; they must be positive quantities! Use Excel to calculate p and q for you, something like:

image location	lens location	object location	p	q	δq
			=B2-C2	=A2-B2	

but the order of the subtraction (i.e., =B2-C2 or =C2-B2) will depend on how you've ordered the objects on your optical bench.

(b) You must estimate δq—the uncertainty in the image location—from the range of distances in which the image looks reasonably 'sharp' (i.e., in focus) every time to make a measurement. The uncertainties in p (just based on reading the scale) are probably negligible compared to the uncertainties in q, so you need not estimate δp.

(c) Note whether the image is upright or inverted, enlarged or reduced.

(d) Move the lens farther away (e.g., double the value of p), and repeat the procedure recording the distances p, q, and δq, and the image type.

(e) Do all this for at least 6 different object distances. About half the images should be enlarged and half reduced; the range of both p and q should span at least a factor of two. You should also see substantial variation in δq.

As you go along, it is helpful to sketch a graph of image distance q vs. object distance p in your notebook. You will find the curve helpful in deciding at what object distances to take more data. If your $WAPP^+$ plots (see below) do not show nicely spaced points, take additional points to fill in the gaps.

3. Repeat the 6 measurements of q vs. p for the other converging lens. On the Excel spreadsheet your table of data might look like:

Thick Lens: $f_{Short} =$

p (mm)	q (mm)	δq (mm)	image type?

Thin Lens: $f_{Long} =$

p (mm)	q (mm)	δq (mm)	image type?

4. Using your q vs. p data for each lens separately, test the lens equation in the following way:

   (a) Use $WAPP^+$ to fit to the lens equation with $Y = q$ and $X = p$. After selecting appropriate error choices (probably: Y errors individual and no X errors) paste your $p, q, \delta q$ data into $WAPP^+$.

   (b) According to Eq. (9.1), $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$, and $WAPP^+$ has an appropriate fitting function for this case: 'Inverse x & y'; that is, $\frac{1}{y} = A + B/x$.

      i. Check the graph and the reduced χ^2 to see if the function fits the data. The fitting parameter B evidently should be -1 for each set of data. Do yours come out close, within uncertainty δB?

      ii. Set $B = -1$ and fit the data again. Print out a fit report and plot for your lab notebook.

      iii. Recall that use of an unusually scaled axis can turn curves into straight lines. For the 'Inverse x & y' function Inverse scales for X and Y should produce a straight line. For one of your lenses, have $WAPP^+$ make a inverse-inverse plot of your data and function.

   (c) The A parameter corresponds to $1/f$. Calculate $f = 1/A$ and $\delta f = f \cdot \frac{\delta A}{A}$. Compare with the quick-estimate value you found for f

5. Construct a simple astronomical telescope using the two lenses you have just worked with by separating them by a distance of about $f_{Short} + f_{Long}$ and using the f_{Short} lens as the eyepiece.

   (a) Look through the telescope at the brick wall out in the hall, and note whether the image is inverted or upright, reduced or magnified.

   (b) Estimate the magnification of this telescope by observing the bricks through the telescope with one eye, while keeping the other eye open too. You should be able to compare the relative sizes of the bricks seen with and without the telescope, and the ratio gives the magnification. For example, if one brick seen with the telescope looks as tall as four bricks do without the telescope, then the magnification is 4. Clearly this will be a fairly crude estimate, but try your best.

(c) The magnification of a telescope is equal to the ratio of the focal lengths of the lenses used. Does the ratio f_{Long}/f_{Short} agree with the magnification you estimated visually? (Be sure to include error estimates in answering this question.)

6. Quick Report: your values of f_{Long} and f_{Short} from $WAPP^+$ with proper units, error, significant figures.

7. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Pre-Lab Question	
Sketch of the Apparatus	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
Quick-estimate values f of two lenses, including uncertainties	
Object and image data for each lens	
Comment: $WAPP^+$ results for B compared to -1.	
Graphs and fit reports of q vs. p for each lens for $B = -1$	
Graph of $1/q$ vs. $1/p$ for a lens	
Determination of f from $WAPP^+$ A, comparison with initial estimates of f	
Notes on the simple telescope and appearance of image	
Estimation of telescope magnification, comparison with f_{Long}/f_{Short}	

10. Photoelectric Effect

Pre-Lab Questions

1. Cesium has a work function of 1.81 eV[1]. What is the longest light wavelength expected to produce photoelectrons from a cesium surface?

2. What stopping potential should be measured for a cesium phototube illuminated with a wavelength 400 nm?

Purpose

This experiment is to introduce you to the photoelectric effect and a method for measuring Planck's constant h which relates the energy of a photon of light to its frequency.

Introduction

In the late 1800s light was recognized as an electromagnetic wave, described by Maxwell's theory of electromagnetism, and supported by much experimental work, notably that of Heinrich Hertz. Hertz used tuned circuits to produce electromagnetic waves, and he detected them at a distance by observing sparks in another tuned circuit. In such an experiment in 1887, Hertz discovered that the sparks between metal plates were facilitated by shining light on the plates. It was later learned that light may cause a metal to emit electrons — a process now called the *photoelectric effect.*

It was understood from Maxwell's theory that electromagnetic waves carry energy; by definition, the intensity of light is its energy/area/time. So it was expected that when a light source had delivered enough energy to the metal, an electron could be liberated. For low intensity light, it should take longer, but it would be a matter of sufficient intensity and sufficient time.

Instead, it was found that the emission of electrons occurred with no measurable time delay, regardless of the light's intensity, but only for light of sufficiently high frequency. For each metal there is a *cutoff frequency*: the lowest that would produce photoelectrons. Higher intensity served only to produce more photoelectrons per second, but if the frequency was too low no photoelectrons were emitted even at high intensities. The photoelectric effect was one of the most puzzling difficulties in physics at the time: Maxwell's theory explained many other phenomena very well, but not this one. The solution came with the quantum theory of light that Max Planck developed to explain 'Blackbody Radiation'.

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[1]eV is a unit of energy; to convert it to joules multiply by the conversion factor 1.60×10^{-19} J/eV. On the other hand if $\phi = 1.81$ eV, $\phi/e = 1.81$ V.
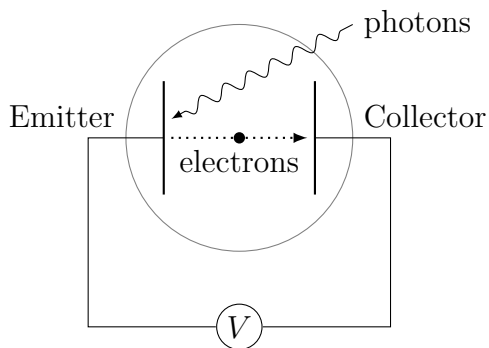
Figure 10.1: The photoelectric effect. Light causes emission of electrons from a metal. The emitter becomes more positive as electrons leave, and the collector becomes more negative, eventually preventing any more electrons from moving from emitter to collector. A voltmeter measures the resulting potential difference between the metal plates.

Blackbody radiation is the light emitted by a hot object, e.g., the red glow of an electric heater filament. The problem was to account for precise spectrum of colors emitted by a heated object: exactly how much red, yellow, orange, etc. light was emitted from the hot object. In 1900, Planck announced a blackbody radiation theory which matched well with experiments, but required the novel assumption that atoms emit light in discrete packets having a fixed energy E related to the frequency f of the light, where

$$E = hf \tag{10.1}$$

and the constant h, now known as Planck's constant, is

$$h = 6.63 \times 10^{-34} \text{J} \cdot \text{s}. \tag{10.2}$$

Soon after Planck's success, Einstein, in 1905, proposed an explanation for the photoelectric effect: that light behaved like a stream of individual particles, each carrying energy E according to Planck's formula (10.1). Thus each particle of light, or *photon*, as they came to be called, either carried sufficient energy to knock an electron loose, or else it didn't. Hence, the emission of photoelectrons could be understood to depend on the frequency of the light.

Theory

Figure 10.1 illustrates a way the photoelectric effect may be observed. Light is shone on a clean metal surface inside a vacuum chamber, known as the photocell. The illuminated metal is the electron emitter. There is another metal surface nearby to collect these 'photoelectrons', so it is known as the collector. In metals — unlike most other materials — some of the electrons are itinerant rather than bound to a particular atom: they are free to move around inside the bulk of the material. They cannot easily leave the metal as a Coulomb force would pull them back. That force is limited so given enough energy (ϕ — known as the *work function*) they can escape. When light strikes the metal, each photon carries energy $E = hf$. If $E < \phi$, no electrons are released from the metal. But if $E > \phi$, emission may occur, and the emitted electron carries any excess energy as kinetic energy KE. So the equation for

conservation of energy in the photoelectric effect is

$$hf = \phi + KE. \tag{10.3}$$

In this process, as illustrated in Fig. (10.1), the emitter acquires a positive electric potential, while the collector becomes more negative; essentially, a capacitor is being charged by the photoelectric effect. The process continues until a potential difference V_s, the *stopping potential*, is reached. V_s tells us the maximum potential energy of electrons leaving the emitter, namely eV_s, where e is the elementary charge:

$$e = 1.6 \times 10^{-19} \, \text{Coulomb}. \tag{10.4}$$

So we can rewrite Eq. (10.3) in the form

$$hf = \phi + eV_s. \tag{10.5}$$

It is easier for us to use the wavelength of the light $\lambda = c/f$ instead of the frequency. Then if V_s is measured for different wavelengths, Eq. (10.5) predicts

$$V_s = \frac{hc}{e\lambda} - \frac{\phi}{e}. \tag{10.6}$$

The accepted values of h, c and e combine conveniently into one constant:

$$\frac{hc}{e} = \frac{(6.626 \times 10^{-34} \, \text{J·s}) \, (2.998 \times 10^8 \, \text{m/s})}{1.602 \times 10^{-19} \, \text{C}} = 1240 \, \text{V·nm}. \tag{10.7}$$

So, a plot of V_s versus $1/\lambda$ is expected to fit a function of the form $y = A + B/x$, with $B \approx 1240$ V·nm. The parameter A tells the value of the emitter work function ϕ expressed in electron-volts, because $\phi = e \cdot A$. From Eq. (10.6) it is also apparent that $V_s = 0$ for the "cutoff wavelength" $\lambda_c = hc/\phi$, the wavelength for which the photons carry the minimum energy to remove an electron from the metal — longer wavelength photons produce no emission.

It may be helpful to think of of the photoelectric effect as a 'clean' version of photosynthesis: a photon is absorbed and — if the photon has a high enough frequency — an electron is emitted to a higher energy state. In the physics version the emitted electron is charging a capacitor; in the biological version the electron enters the electron transport chain that ends with ATP synthesis.

Experiment

Apparatus

Figure 10.2 shows the experimental apparatus in more detail.

- To prevent charges from flowing from the collector back to the emitter through a standard voltmeter, a high-impedance device known as an operational amplifier is used. So there is some additional circuitry to run this op-amp and convert its output into V_s.

- Lasers and an ultraviolet LED are used to provide photons with known wavelengths λ. | Do not look into the end of a laser, or point it at anybody. |

- The emitter is coated with a film of Antimony and Cesium (Sb_3Cs), and its work function probably varies from place to place, as well as changing with usage. To protect the emitter surface and improve your results, you should use a piece of diffusing glass with the lasers. This will help average the photoelectron emission over the emitter surface, and reduce effects of high intensity.
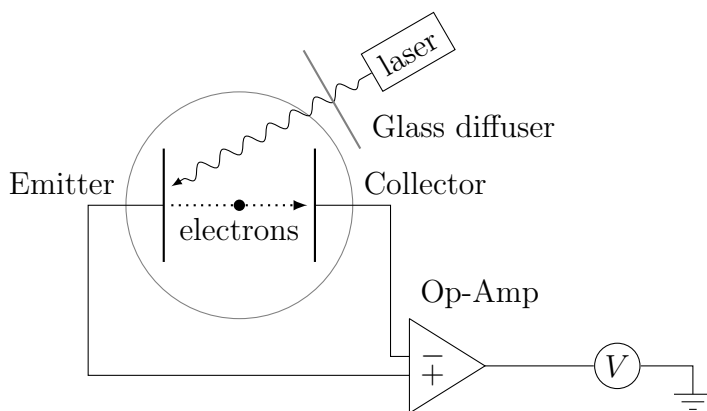
Figure 10.2: Photoelectric effect apparatus. A laser or LED provides photons of known wavelengths. An operational amplifier serves as a high-impedance voltmeter between the emitter and collector plates to measure V_s.

Procedure

- Prepare a spreadsheet table for your data which should look something like the following table, though your sources may differ from these examples.

Source	Color	λ(nm)	$\delta\lambda$(nm)	V_s(V)	δV_s(V)
Diode laser	Red	650	5		
Helium-neon gas laser	Red	632.8	0.1		
Helium-neon gas laser	Orange	612.0	0.1		
Helium-neon gas laser	Yellow	594.1	0.1		
Helium-neon gas laser	Green	543.5	0.1		
Diode-pumped YAG laser	Green	532.0	0.1		
Diode laser	Blue	405	5		
Light-emitting diode	UV	314	15		

- Use each of the available light sources, and measure the resulting V_s. Observe the variation of the meter readings for a long enough time to estimate each uncertainty δV_s.

- Use *WAPP+* to make a plot of V_s in volts versus $1/\lambda$ where λ is in nanometers.

- Does the $\overline{\chi^2}$ value indicate the expected function is a successful fit to your data?

- Print a graph of your data and the fitted function, with proper title and axis labels. Print an additional graph using an x scale option of Inverse. (Note: inverse wavelength is proportional to frequency so we expect this to be a straight line.)

- According to Eq. (10.6), the data is expected to fit a function of the form $y = A + B/x$, with $B \approx 1240$ V·nm. Does your result overlap the accepted value?

- The parameter A, also in volts, tells the value of the emitter work function ϕ expressed in electron-volts, because $\phi = e \cdot A$. Typical values tend to be in the range 1.25–1.75 eV. Does your result land somewhere in this range?

Final Considerations

1. Compare the fit equation to the photoelectric effect equation:

$$y \;=\; A + \frac{B}{x}$$
$$V_s \;=\; -\frac{\phi}{e} + \frac{hc}{e\lambda}$$

What is the relationship between B and h? What are the units for your B? (You will probably want to convert it to meters.) Calculate Planck's constant h (including error) from your B value.

2. Recall that the cutoff wavelength λ_c corresponds to the wavelength that would produce $V_s = 0$. Calculate λ_c (with error) for this phototube. Specifications on this kind of phototube usually list a cutoff around 700 nm. Does your result seem reasonable?

3. Make a Final Results Table properly reporting (units, significant figures, error) your values for: h, ϕ (in eV), and λ_c (in nm).

4. Put these same values on your Quick Report card.

5. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Table of sources, wavelengths and stopping potentials with uncertainties	
$WAPP^+$ fit report	
Interpretation of $\overline{\chi^2}$	
Graph with proper title, labels	
Linearized graph with proper title, labels	
Comparison of $B \pm \delta B$ with accepted value	
Comparison of $A \pm \delta A$ with expected value	
Calculation of λ_c and comparison with expected value	
Calculation of h and comparison with expected value	
Final Results Table: Proper units, significant figures, uncertainties	
Conclusions, Discussion, Critique	

11. Nuclear Radiation

Pre-Lab Questions

1. In Part 1, how many dice are expected to remain after 13.5 rolls? after 27 rolls? after 50 rolls?

2. If a Geiger counter registered 200 counts in one minute what would be the uncertainty in the count?

Purpose

Exponential functions are very common in physics. In today's experiment, you will be investigating phenomena that can be described with exponential functions, including the processes of nuclear decay and penetration of radiation.

Theory

Exponential functions

Exponential functions occur when the change in a quantity is a constant fraction or multiple. For example, an investment at a constant interest rate increases exponentially. A population growing at a constant percentage rate increases exponentially. In a similar manner, quantities that decrease at a constant percentage rate will decline exponentially.

Let us say that an investment principal N_0 increases at a constant annual percentage P (decimal equivalent). After one year the new principal will be $N_1 = N_0(1 + P)$. After two years, $N_2 = N_1(1 + P) = N_0(1 + P)^2$. In general, after t years,

$$N_t = N_0(1 + P)^t. \tag{11.1}$$

Now any positive number q and be written as an exponent: $q = e^r$; in fact since $\ln(x)$ is the the inverse function of $\exp(x) = e^x$, $r = \ln(q)$, i.e., $q = e^{\ln(q)}$. So

$$N_t = N_0 e^{Bt}. \tag{11.2}$$

where $B = \ln(1 + P)$.

Finally, consider what happens to $B = \ln(1 + P)$ if P is made very small, for example $\ln(1.001) = 0.9995 \times 10^{-3}$. With a bit of calculus one can show for small P:

$$\ln(1 + P) \approx P \tag{11.3}$$

so approximately:

$$N_t = N_0 \, e^{Pt}, \tag{11.4}$$

or for a declining investment, we have:

$$N_t = N_0 \, e^{-Pt}. \tag{11.5}$$

Note: in the case of interest rates, the value P may be made small simply by changing the 'compounding' period (i.e., when interest is paid) from annual to something smaller like weekly or daily. Bankers call the exponential relationship Eq. (11.4) 'continuously compounded' interest.

In the case of investment the rate of return was fixed: $N_t = N_0 e^{Pt}$; In the case of radioactive decay the *probability* of decay is fixed, so while $N_t \approx N_0 e^{-Pt}$, the exact amount of decay will vary. Consider the case of counting the number of Heads in a sequence of coin flips. If you flip the coin 10 times, it turns out the probability of getting exactly 5 Heads is about .25; 5 ± 1 Heads turns up about 66% of the time. On the other hand if you flip a coin a million times about 68% of the time the split will be between 49.95% and 50.05% Heads, i.e., 500000 ± 500 Heads. That is for large N the Heads-ratio will be quite close to the expected, even as the corresponding likely range of Heads-counts gets quite large. In the case of radioactive decays, Poisson statistics will provide an estimate for the random fluctuation in the number of decays: $\sqrt{N}$.

In today's experiment, you will be investigating radioactive nuclei that have a constant probability[1] of emitting alpha radiation (i.e., decaying). The constant probability of decay results in an approximate exponential decline in the number of radioactive atoms and a corresponding decline in the 'activity' or rate of emissions.

You will also be investigating the penetrating ability of beta radiation that has a distribution of energies. If the probability of beta radiation striking its target and being stopped or removed from an incident beam is nearly constant per unit distance of penetration, the result will be an exponential decline in radiation intensity with penetration distance. In both parts, you will be using $WAPP^+$ to see how well an exponential function fits the data.

When judging the results and your uncertainty, keep in mind the size of the numbers you are dealing with and the incremental change that is taking place. Low count rates will result in a greater percentage of uncertainty.

Radiation — The lifetime of Radon ^{220}Rn
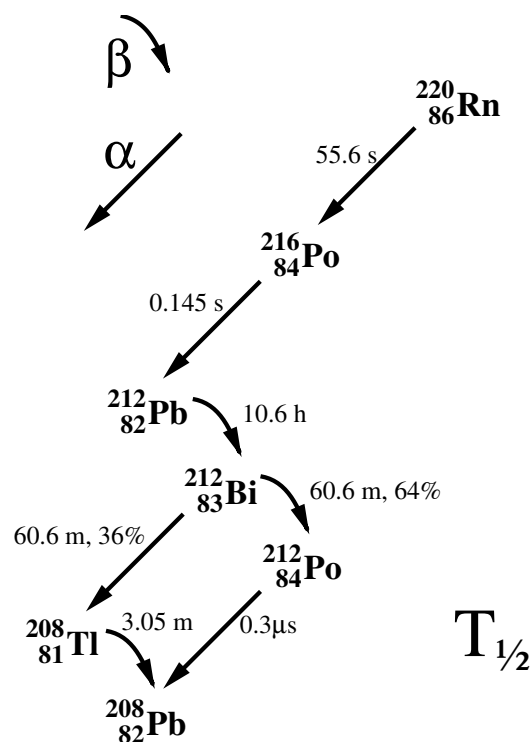
Most of your exposure to nuclear radiation comes from the alpha particles emitted by radon's decay products (also called progeny or daughters). These radionuclides

---

[1]This is an important point! Uranium, nuclei have the same chance of decaying today as they had in the first day after they were created. Nuclei do not develop in the way say chicken embryos do in an egg waiting to hatch.

stick to surfaces and easily lodge in your lungs when you breathe. When they decay the products, e.g., high speed alpha particles, can damage cells. Exposure to radon progeny is a recognized cause of lung cancer in humans.

Generally, it is difficult to isolate one pure source isotope, since usually the products of the decay are themselves unstable, so a sequence of reactions occur and an entire chain of decay products is present. However in this part of the experiment, you will be able to observe the decay of a single radon isotope — ^{220}Rn. We have filled a canister with lantern mantles that were manufactured using thorium-232 (it forms an oxide in the mantles, which makes them burn a nice white color). This eventually decays to radon-220, which attaches to the air molecules in the canister. We can then pump some the air out of the canister (and into an alpha detector) providing us with a sample of just one decaying isotope: ^{220}Rn.

While this airborne radon is continually generating isotopes of polonium, lead, and bismuth, these products will not have a noticeable effect on your results. In some cases, they are not alpha emitters and the radiations emitted have a high probability of escaping the detector without being detected. And when alpha emitters are produced either their half-life is much too long or the numbers much too small to have a statistical effect on your measurements.

β

α

$^{220}_{86}$Rn

55.6 s

$^{216}_{84}$Po

0.145 s

$^{212}_{82}$Pb   10.6 h

$^{212}_{83}$Bi   60.6 m, 64%

60.6 m, 36%

$^{212}_{84}$Po

$^{208}_{81}$Tl   3.05 m   0.3μs $T_{1/2}$

$^{208}_{82}$Pb

Procedure

For Part 1, two dice sets per room will be shared by the four groups in each room. The dice sets can be passed from group to group while the groups remain at their tables. Only one lab station per room is available for Part 2 of this experiment. The equipment is on a cart that can be moved from group to group. All four lab groups will have to rotate through this part of the experiment. Part 3 should be available at every table. The three parts of the experiment can be done in any order depending on the availability of the materials.

General Instructions for the LabQuest interface and Logger Pro software

Starting the Logger Pro program is done by clicking on the icon on the desktop. If you see the message **"No Device Connected"** in the upper left corner of the screen, the interface must be unplugged at the computer end of the cable. Then, after pausing a few seconds, it must be re-plugged into the computer. The message should then disappear.

For certain custom sensors, (like the ones used in this experiment) a confirmation window will open that asks you to connect the sensor. Click the **"Connect"** button.

For both files (parts 2 & 3), in the lower left corner of the screen, note a box labeled **"Total Counts"**. This box displays a running total of radiation counts each time that you perform a data collection run.

1. Dice: Exponentials in Theory and Practice

You will need to obtain a set of fifty twenty-sided dice and a tray for rolling the dice. They will be used to develop a simulation of radioactive decay. Each die has only one side marked with a number **1**. When a 1 comes up, we pretend that that 'atom' has decayed and remove it from the pot.

1. Roll the dice (all at once) and take out any dice showing a number **1** on the top. Continue this process until all dice have been removed or you have rolled the dice 50 times, whichever happens first. You will need to record the number of rolls R and the number of dice left D after each roll.

2. In your notes, construct a graph of D vs. R plotting the remaining dice D for every 5th roll R. Semi-log graph paper like the sample shown in Figure 11.1 will be provided.

3. On the same graph, generate a theoretical curve by using a calculator or spreadsheet to calculate a decay curve based on the probability being 19/20 for each die to remain after each roll. According to simple probability theory, $D_1 = \frac{19}{20}D_0$, $D_2 = \frac{19}{20}D_1 = \left(\frac{19}{20}\right)^2 D_0$, ..., or for any number R

$$D(R) = D_0 \left(\frac{19}{20}\right)^R.$$
(11.6)

4. **Q1:** Why does data taken from your measurements differ from the calculated data from Eq. (11.6)?
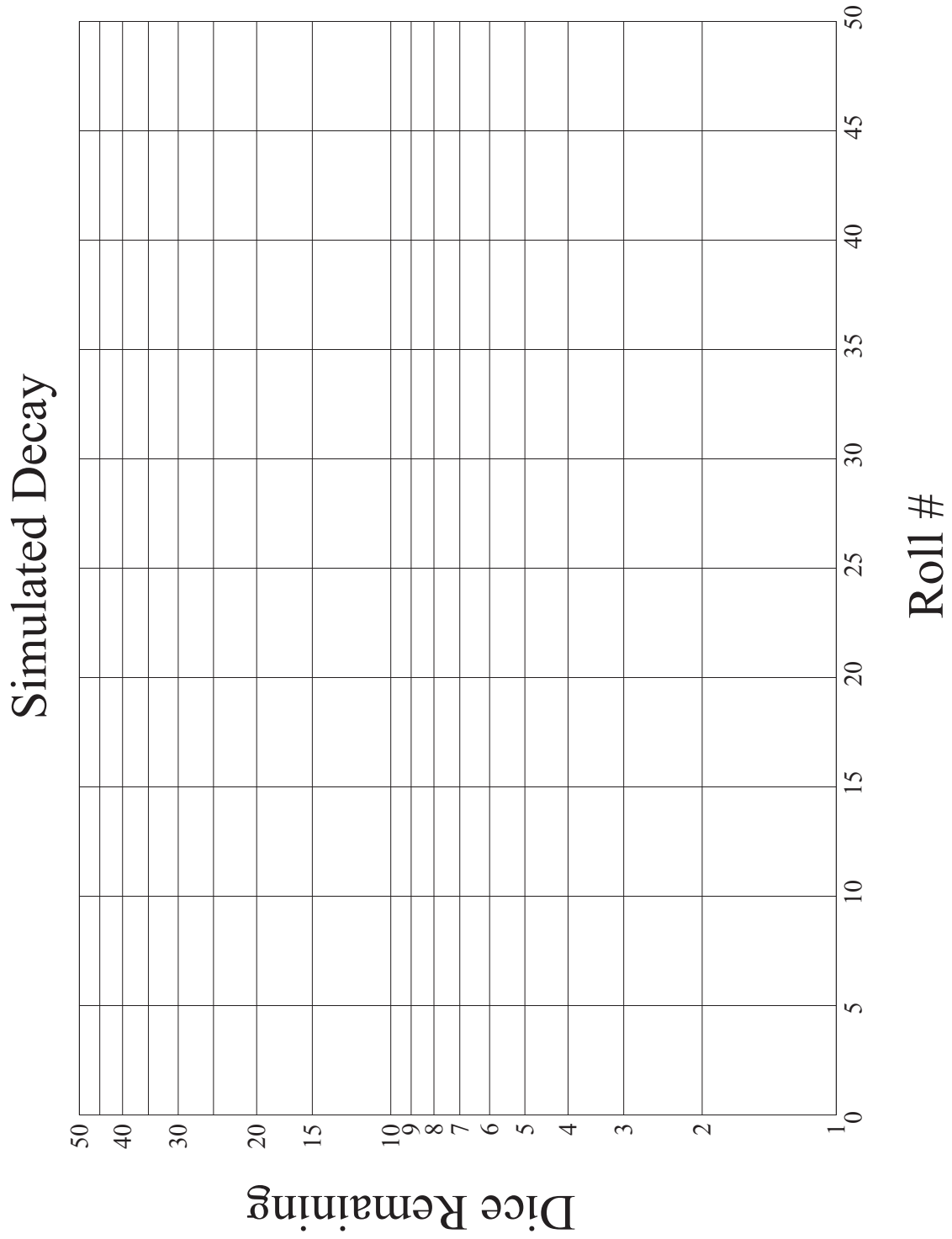
Figure 11.1: Sample semi-log paper for a simulated decay using dice.

2. Sample Activity and Half-life

1. Before the lab session begins, a thirty second "background" count will be determined by your instructor and posted on the board. That background count will be used by all of the groups. Once the background count is known, the air pump can be turned on by each group to pump radon into the detector chamber. The pump should be run for approximately thirty seconds.

2. When your group obtains the cart, insert the black phono plug, which is connected to a gray cord, into DIG 2 on the Lab Quest interface. You will notice that the pump is connected to a switched power strip that is used to turn the pump on and off. The power strip and the Ludlum counter are connected to a triple outlet adapter. Plug this adapter in any convenient outlet and turn the Ludlum unit on. Do not turn the pump on yet. You do not need to change any time or voltage settings on the Ludlum counter.

3. Open the **Logger Pro** file named Radon. The file will display a graph of Count Rate versus Time and a table with the same data. The data from the table will be used with $WAPP^+$. When you are ready to start counting, run the pump for approximately thirty seconds. Immediately after turning the pump off, click on the **"Collect"** button. The computer will determine the count rate or activity (counts per thirty seconds) in thirty second intervals for five minutes. You may stop counting before five minutes is up if you believe that the count rates are becoming too low to be meaningful.

4. Once you have your data, unplug the phono plug and power adapter. Transfer the cart to any group still waiting to do this part.

5. Paste the count rate vs. time data from the *Logger Pro* table into an Excel spreadsheet. The uncertainty in time will be negligible and can be ignored. Consequently, you will not be including any X uncertainty. The uncertainty in activity A can be estimated by using the square root of the activity:

$$\delta A = \sqrt{A}, \tag{11.7}$$

Make a column in Excel containing the uncertainty, e.g., '=SQRT(B2)'.

6. Using Excel, make a new column in which you subtract the background from each count. Transfer this background-subtracted column, δA and t to $WAPP^+$. Use $WAPP^+$ to check for an exponential relationship, $y = A \exp(B\ x)$, representing activity (counts per thirty second time interval) vs. elapsed time.

$$A = A_0 e^{-Pt}, \tag{11.8}$$

where the $WAPP^+$ parameter A corresponds to A_0 (the initial activity) in Eq. (11.8) and $B = -P$. The B parameter determined by $WAPP^+$ represents the probability of decay per second. (FYI: our textbook names P the 'decay constant' and denotes it with the letter λ.) Print out a fit report and plot from $WAPP^+$.

7. A commonly used characteristic of exponential decline is 'half-life' which, as we show below, is inversely related to the B parameter. The half-life is the time t required for the activity to be halved, i.e., the time at which $A = A_0/2$;

$$\frac{A_0}{2} = A_0 e^{Bt} \tag{11.9}$$

$$\frac{1}{2} = e^{Bt} \tag{11.10}$$

$$\ln\left(\frac{1}{2}\right) = Bt \tag{11.11}$$

$$\frac{-\ln(2)}{B} = t \tag{11.12}$$

Thus the half-life is given by:

$$T_{1/2} = \frac{-\ln(2)}{B} \tag{11.13}$$

and its uncertainty, by the rules of fractional uncertainties, turns out to be

$$\delta T_{1/2} = T_{1/2}\frac{\delta B}{|B|} \tag{11.14}$$

Properly record (units, error, significant figures) your value of $T_{1/2}$.

3. Radiation Penetration

Once emitted, radiation will interact with matter causing ionizations. In this part, you will be using a beta emitter ^{204}Tl. Each ionization reduces the kinetic energy of the beta particle (electron) until it is no longer energetic enough to cause further ionizations, i.e., it is stopped. The betas that start with the lowest kinetic energy will be stopped first.

To see how the number of beta radiations is diminished as it passes through matter; the activity of a ^{204}Tl sample will be measured after passing through several sheets paper placed between the sample and the detector. Each layer of paper will be considered one 'thickness'.

1. To measure the intensity of the beta radiation (number of counts per minute), you will be using a Geiger counter connected to the computer through DIG 1 on the LabQuest interface. Open the *Logger Pro* file named "Geiger". Two displays will be seen. One is a graph which shows the counts as they accumulate with time and the other, a table with the same data.

2. The ^{204}Tl sample is permanently mounted in a plastic cylinder that has a Velcro patch attached to the underside. The Geiger tube is mounted in a protective metal cylinder directly above a matching Velcro patch. A shield on the cylinder can be rotated to expose the Geiger tube. This must be done prior to taking any counts. The tube is very delicate and should not be touched.

3. Make sure the Geiger counter is on and set to ×100. With the radioactive sample removed and set aside, take a one minute background count. The computer is set to automatically stop one minute after you click on the **"Collect"** button. Don't be surprised if you see the count jump by two or more during a single time interval. The radiations are coming in randomly at any time, but the computer only checks the total count every 10th of a second. More than one radiation could be detected in one time interval.

4. The number you want for your data is found in the "Total Counts" display on your computer. Record that value in your lab notebook or directly in the Excel spreadsheet.

5. After you have the background count, place the cylinder with the radioactive sample on the Velcro patch. Do not move the cylinder during the counting process.

6. You will take a series of one minute counts (enough to plot a $WAPP^+$ graph) starting with one note card placed on top of the cylinder, covering the sample. Add note cards one at a time for each of the subsequent counts. After you have finished, turn the Geiger counter off and rotate the protective shield to cover the tube.

7. List your results in an Excel spreadsheet and subtract background from each count. The statistical variations in the number of counts will be the Y uncertainty:

$$\delta I = \sqrt{I}. \qquad (11.15)$$

Time errors are insignificant. The physical thickness of paper cards turns out to have some uncertainty, which we've measured as the equivalent of about 2% of the number, n, of cards. use that as the X uncertainty. Transfer the data to $WAPP^+$; check for an exponential relationship between the counts and the number of cards. The $WAPP^+$ B constant can be related to half-thickness $n_{1/2}$ in the same way that it was related to half-life in Part 2: $B = -\ln 2/n_{1/2}$. We then have intensity of radiation I:

$$I = I_0 \, e^{-B \cdot n} = I_0 \, e^{-\frac{n \ln(2)}{n_{1/2}}} = \frac{I_0}{2^{(n/n_{1/2})}} \qquad (11.16)$$

Properly record (units, error, significant figures) your value of $n_{1/2}$.

8. In these physics labs, we have made use of the reduced chi-squared value to judge the validity of empirical fits to the data. If a few points were slightly off the curve, but the error bars overlapped the curve generated by the fitting function, the fit was considered acceptable. Here we go step further and ask ourselves "Where are the points located that do not fit the curve so well?" Are the points in question scattered randomly throughout the entire range of the data or are they found primarily at one or both extremes? In other words, do they show a systematic deviation from the fit? $WAPP^+$ doesn't provide an answer to this question. You must use your own judgment.

In the case of an exponential function, a semi-log plot of the function will yield a straight line. If the data points show a tendency to curve away from the straight line, it suggests that the relationship may not be exponential after all. This could be true even if the reduced chi-squared value seems acceptable. **Q2:** Check the semi-log plot of the data from this part of the experiment and make up your own mind as to whether or not the relationship is truly exponential.

9. Print a fit report and semi-log graph of radiation intensity vs. paper thickness (# of cards).

Final considerations

1. Quick Report: $T_{1/2}$ of ^{220}Rn (units, error, significant figures), $n_{1/2}$ of ^{204}Tl (units, error, significant figures)

2. Following the suggestions given in the Introduction to this Lab Manual, record in your notebook a brief critique of this lab.

CHECKLIST

Pre-Lab Questions	
Theory, with relevant equations and quantities defined	
Brief Methods & Materials	
Semilog graph of D vs. R (theory and experiment)	
Q1: Comparison of Dice Rolling and Exponential Theory	
Measurements of activity minus background for ^{220}Rn	
Calculated uncertainties in activity for ^{220}Rn	
$WAPP^{+}$ plot and fit report of A vs. t for ^{220}Rn	
Final result: $T_{1/2}$ of ^{220}Rn (units, error, significant figures)	
Measurements of activity minus background for ^{204}Tl	
Calculated uncertainties in activity for ^{204}Tl	
$WAPP^{+}$ semi-log plot and fit report of I vs. n for ^{204}Tl	
Final result: $n_{1/2}$ of ^{204}Tl (units, error, significant figures)	
Q2: on semi-log plot of I vs. n	

APPENDIX A — Uncertainties

Introduction

All *measured* quantities have some uncertainty associated with them. It follows that anything you *calculate* using this measurement will also have some uncertainty associated with it. Any time you report a quantity, for example a quantity called Y, you should also report the amount of uncertainty δY it has, in the form $Y \pm \delta Y$. This appendix will assist you in determining the uncertainties for measured and calculated quantities.

Measurements

Quantity Y measured once

The amount of uncertainty in a measurement usually depends on how the measurement was made. For instance, if you measure the temperature in a room using a mercury-column Celsius thermometer that is marked in 1° increments, you should be able report the temperature to the nearest degree. You might report that the temperature is $T = 25°$C, and mean that the temperature is in the range $24.5°$C $\leq T \leq 25.5°$C; if this is the case, you should report it as $T = 25.0 \pm 0.5$ °C. It is common when reporting length data to take the uncertainty as $1/2$ the smallest unit on the measuring device. Often you can do a bit better than this. You might, for example, be able to estimate where the mercury column ends between two degree markings, and report a temperature of, say, $25.3°$C. If you mean that the temperature is certainly between $25.25°$C and $25.35°$C, then say so by reporting $T = 25.30 \pm 0.05$ °C. However, if you only mean that it's certainly between 25.1 and 25.5 degrees, you should report $T = 25.3 \pm 0.2$ °C. The idea is to report a measurement as accurately as possible, but to also state clearly its range of possible values.

It can be very useful to put the uncertainty into a fractional or percentage form. As we shall see, the percent uncertainty in a calculated quantity is often related in a simple way to the percent uncertainty of the measured quantity. In the example above, where $T = 25 \pm 0.5$ °C, we could have said $T = 25°$C $\pm 2\%$, since $0.5/25 \times 100\% = 2\%$.

In summary, when only a single measurement of quantity Y is performed, express it as $Y \pm \delta Y$, meaning you're reasonably sure the value lies in the interval $(Y - \delta Y, Y + \delta Y)$. Note the following terms:

$$\text{Absolute uncertainty in } Y = \delta Y \tag{A.1}$$

$$\text{Fractional uncertainty in } Y = \frac{\delta Y}{Y} \tag{A.2}$$

$$\text{Percentage uncertainty in } Y = \frac{\delta Y}{Y} \times 100 \tag{A.3}$$

Quantity Y measured N times

Sometimes repeated measurements of a quantity give different results. This may be because the quantity actually varies; for example, your weight may vary from day to day. On the other hand, it may be purely a consequence of how the measurement was made; for instance, you may be measuring the time for a pendulum to swing once, and get a different result each time, even though the pendulum always behaves the same. In either case, repetition can tell us something useful. We can take the average, or mean, $\overline{Y}$ as the best overall value of the quantity Y. We can also find how much the measurement typically varies from the average. This typical variation is called the standard deviation, σ_Y. So, for example, it is likely that the next pendulum swing will fall in the range $(\overline{Y} - \sigma_Y, \overline{Y} + \sigma_Y)$. The best estimate of the uncertainty in $\overline{Y}$ (called the standard deviation of the mean), is $\sigma_{\overline{Y}} = \sigma_Y/\sqrt{N}$. The idea is that the <u>mean</u> of many measurements is less uncertain than the measurements themselves. If we have N measurements of quantity Y, call them $(Y_1, Y_2, \ldots, Y_N)$, and they are all taken to be equally trustworthy, then:

$$\text{Mean of } Y \quad = \quad \overline{Y} \quad = \quad \frac{Y_1 + Y_2 + \ldots + Y_N}{N}$$

$$\text{Standard deviation of } Y \quad = \quad \sigma_Y \quad = \quad \sqrt{\frac{(Y_1 - \overline{Y})^2 + (Y_2 - \overline{Y})^2 + \ldots + (Y_N - \overline{Y})^2}{N-1}}$$

$$\text{Standard deviation of } \overline{Y} \quad = \quad \sigma_{\overline{Y}} \quad = \quad \sigma_Y/\sqrt{N}$$

Standard deviations can be tedious to calculate if N is large; many hand calculators have a button for computing σ_Y; on spreadsheets try `STDEV(...)`. If $N = 2$, the above formulas take on simple and reasonable forms:

$$\overline{Y} \quad = \quad \frac{Y_1 + Y_2}{2}$$

$$\sigma_Y \quad = \quad \frac{|Y_1 - Y_2|}{\sqrt{2}}$$

$$\sigma_{\overline{Y}} \quad = \quad \frac{|Y_1 - Y_2|}{2}$$

Quantity Y measured N times with different uncertainties

Suppose we somehow get several measurements of some quantity Y, and the measurements have different uncertainties associated with them. There is a way to get a "best" estimate of Y by taking all the measurements into consideration, but counting most heavily on the more precise ones; it's called the *weighted average*. Furthermore, we can attach an uncertainty to it that is less than the smallest of the individual uncertainties. If the measurements of Y are $(Y_1, Y_2, \ldots, Y_N)$ and their uncertainties are $(\delta_1, \delta_2, \ldots, \delta_N)$, we calculate weighting factors $W_1, W_2, \ldots, W_N$ given by:

$$W_i = \frac{1}{\delta_i^2}$$

and use them to compute

$$Y_{best} = \frac{W_1 Y_1 + W_2 Y_2 + \ldots + W_N Y_N}{W_1 + W_2 + \ldots + W_N}$$

Notice that this is like an average of the Y values, but with each one weighted by a factor that increases as its uncertainty decreases. So the ones that are most reliable influence the average the most. The weighted average has a "best" uncertainty associated with it, given by

$$\sigma_{Y_{best}} = \frac{1}{\sqrt{W_1 + W_2 + \ldots + W_N}}$$

Calculations

This important section gives formulas for finding uncertainties in calculations. Suppose quantities $A, B, C, \ldots$, are measured and used to compute quantity Y. The uncertainty δY in the value of Y depends on the uncertainties $(\delta A, \delta B, \delta C, \ldots)$ in the measurements and the way the measurements are used in the calculation. A little calculus gives approximate formulas to use for a "worst-case" estimate. Some statistical analysis gives similar but slightly better formulas to use when the uncertainties are all independent and random.

When the measurements $A, B, C, \ldots$ have estimated uncertainties $\delta A, \delta B, \delta C, \ldots$, the uncertainty δY depends on the formula for Y in the following ways.

$$\begin{aligned} \text{Addition and Subtraction: If} \quad Y &= A \pm B \pm C \pm \ldots, \\[2mm] \text{then} \quad \delta Y &\leq \delta A + \delta B + \delta C + \ldots \\[2mm] \text{Multiplication and Division: If} \quad Y &= \frac{AB}{D}, \\[2mm] \text{then} \quad \frac{\delta Y}{Y} &\leq \frac{\delta A}{A} + \frac{\delta B}{B} + \frac{\delta D}{D} \\[2mm] \text{Exponentiation: If} \quad Y &= A^a B^b D^d, \\[2mm] \text{then} \quad \frac{\delta Y}{Y} &\leq a\frac{\delta A}{A} + b\frac{\delta B}{B} + d\frac{\delta D}{D} \\[2mm] \text{Logarithms: If} \quad Y &= \ln A, \\[2mm] \text{then} \quad \delta Y &= \frac{\delta A}{A} \end{aligned}$$

General remarks on calculated uncertainties

Note how fractional uncertainties like $\delta A/A$ appear in formulas for fractional uncertainty in Y. When measured quantities are multiplied, divided, or raised to a

power in calculating Y, the easiest way to find δY is to use fractional or percentage uncertainties.

In exponent formulas, the exponents don't have to be integers. For example, the period T of a pendulum depends on the square root of its length L; the formula is $T = (2\pi/\sqrt{g})L^{1/2}$, so measuring $L \pm \delta L$ gives $\delta T/T = \delta L/2L$. The significance of a measured quantity in determining δY obviously increases with higher exponents. As another example, the volume of a cube is $V = L^3$, where L is the length of a side. Measuring $L \pm 10\%$, we obtain $V \pm 30\%$. To get V within 10%, we must measure L within about 3%.

The formula for obtaining Y from measurements $A, B, C, \ldots$ may be more complicated than the simple operations listed above. Rather than learn the calculus and statistics to find the correct uncertainty relationships, you can just plug in the maximum and minimum values of $A, B, C, \ldots$ and find the resulting range of Y values by the "high-low" method.

Significant Digits

The rules for significant digits are important in determining where to truncate values when using a computer or calculator. *As a rule, the last significant figure in any stated number should be of the same order of magnitude (in the same decimal position) as the uncertainty.* In the physics lab, you should use the following rules in addition to reporting uncertainties:

1. *Nonzero integers.* Nonzero integers always count as significant figures.

2. *Zeros.* There are three classes of zeros:

   (a) **Leading zeros** are zeros that *precede* all the nonzero digits. These do not count as significant figures. In the number 0.0025, the three zeros simply indicate the position of the decimal point. This number has only two significant figures.

   (b) **Captive zeros** are zeros *between* nonzero digits. These always count as significant figures. The number 1.008 has four significant figures.

   (c) **Trailing zeros** are zeros at the *right end* of the number. They are significant only if the number contains a decimal point. The number 100 has only one significant figure, whereas the number 1.00×10^2 has three significant figures. The number one hundred written as 100. also has three significant figures.

3. *Exact numbers.* Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 apples, 8 molecules. Such numbers are called *exact numbers.* They can be assumed to have an infinite number of significant figures. Other examples

of exact numbers are the 2 in $2\pi r$ (the circumference of a circle) and the 4 and 3 in $4/3\pi r^3$ (the volume of a sphere). Exact numbers also can arise from definitions. For example, one inch is defined as *exactly* 2.54 centimeters. Thus in the statement 1 *in* = 2.54 *cm*, neither the 2.54 nor the 1 limits the number of significant figures when used in a calculation.

For calculations in the lab it is a good idea to carry at least one extra significant figure until the final result is obtained, then round to the proper number of digits as the last step. The rule of thumb is to drop all but one extra significant figure when multiplying/dividing (do this first). Lastly, for addition and subtraction, line up decimal places and add. In your final result round to the left-most decimal place in the added (subtracted) numbers. For example, adding: $1.05 + 3.797 + 6.71236$

Lining up decimals:

$$
\begin{array}{r}
6.71236 \\
3.797 \\
+\quad 1.05 \\
\hline
11.55936
\end{array}
\;=\; 11.56 \text{ because 1.05 has the left-most decimal place}
$$

At some point you will likely find a number reported *without* a value for its uncertainty. In this case the author has assumed some value of uncertainty that you can only guess. A reasonable guess would be $\pm$ the last decimal place (e.g. 15.5 would be assumed to be 15.5 ± 0.1).

Reporting Measurements

You should record (in your lab book or Excel) every digit displayed by an instrument and every digit you estimate when reading a ruler. Don't round your raw data! Of course you must also report an uncertainty for those measurements. When it comes to reporting *final results* (in contrast to raw data or intermediate calculations) there are strict requirements as to exactly how you record those numerical results. Our uncertainty in the uncertainty is always large; hence round any uncertainties to one or two significant figures. One significant figures is always easier and almost always as accurate as two significant figures. (Recall: You may have estimated your range-of-deviation just by eye. Manufacturer's specification of errors almost always report just one significant figures so one significant figures would be appropriate for an error calculated from those numbers. One can show that in 'normal' cases, formula-calculated standard deviations will have uncertainty greater than 10% unless the number of available data points, $N > 50$.) Once you have properly rounded your errors, display all the digits of your measurement until the place-value of the least (rightmost) significant digit in the measurement matches the place-value of the least significant digit in the error. Some examples:

$3.14159 \pm .00354 \Longrightarrow 3.142 \pm .004$ or $3.1416 \pm .0035$

$2.71828 \pm .05472 \Longrightarrow 2.72 \pm .05$ or $2.718 \pm .055$

$321456 \pm 345 \Longrightarrow 321500 \pm 300$ or 321460 ± 350

$.1678 \pm 3.51 \Longrightarrow 0 \pm 4$ or 0.2 ± 3.5

Finally, remember to record the units of your measurement!

APPENDIX B — Data Handling

Tables

Experimental data are usually recorded first in a tabular form, as shown below. The table should have labeled columns for the controlled quantity and the resultant quantities. Each column label should indicate the units for that quantity. In each column, a representative estimate of the uncertainty should be made and recorded for the first entry and any other entries where there is a significantly different uncertainty. Leave room for more columns if other quantities will be calculated from the direct measurements. A sample calculation should also be shown at the bottom of the table for those quantities which are calculated.

Point #	L (cm)	$\pm\delta L$ (cm)	T (sec)	$\pm\delta T$ (sec)	Other Quantities (units)
1	1.50	0.05	0.25	0.01	...
2	1.60	"	0.40	0.02	...
...	...	...	...	...	...

Graphs

It is much easier to recognize a pattern from a graph than a table, so we will use graphs extensively in this laboratory. Appendix C will explain more about how functional relationships may be determined using graphs. An initial graph can be used to monitor an experiment as data is being collected. As each measurement is made, put it on the graph. After a few points are plotted, a pattern may become apparent which can serve to tell whether subsequent measurements fit the pattern. Although we may not know the exact pattern the data will follow, large deviations from any pattern can be seen easily and you can catch some mistakes in time to correct them. This initial graph will also serve as the starting point for data analysis. A refined and more complete final graph should be made for the lab report. Here are some guidelines for proper graphing:

- Put a title on each graph.

- Label each axis with the name and units of the quantity.

- Select the graph scale such that the data spans most of the full page.

- Indicate the uncertainty of each data point using error bar symbols, unless the error bars are smaller than the dots representing the data.

- Draw a smooth curve through the data; DO NOT simply connect the dots.
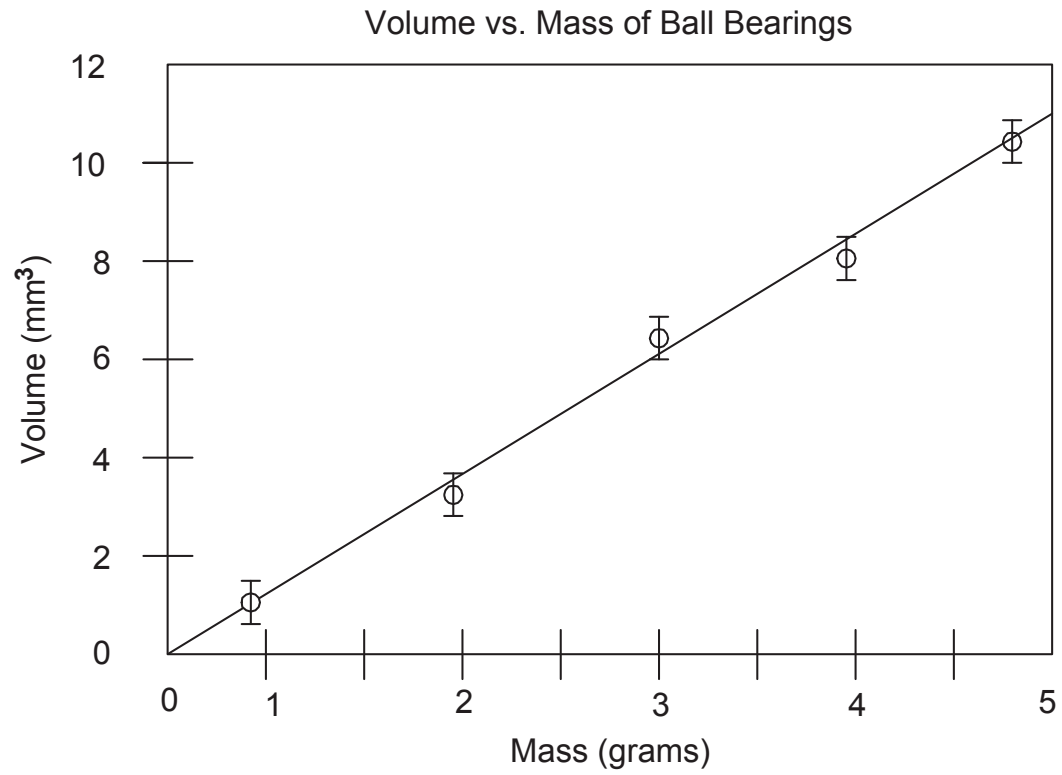
A sample graph is shown below.

Figure B.1: A linear function fitted to a data set.

APPENDIX C — $WAPP^+$

Using $WAPP^+$

1. Find $WAPP^+$: $WAPP^+$ is a web-based program that can be found at

   `http://www.physics.csbsju.edu/stats/WAPP2.html`.

   However it's often faster just to Google "`wapp+`". You should bookmark this page for fast future reference. Also,

   `http://www.physics.csbsju.edu/lab/105_WAPPonline.mp4`
   or on youtube: `http://youtu.be/9sJWetyzPdM`

   provides a video introduction to this program.

2. Start Excel: It is recommended to use Excel for all data analysis this semester.

3. Enter data into Excel: Enter your data values and formula-based calculations in spreadsheet columns. For example, generically calling a controlled variable x, with uncertainties δx, you may be measuring a related variable y with uncertainty δy, and then perhaps computing a third quantity z that depends on the variables somehow, and has an uncertainty δz that depends on the other measurements and their uncertainties. Use the spreadsheet columns to record data and perform calculations:

x	δx	y	δy	z	δz

4. $WAPP^+$ y and x error: The default setting for $WAPP^+$ is "Enter y error for each data point" and "Enter x error for each data point".

   Other options are available. For instance, if the uncertainties are uniform for δx and/or δy, or readily expressed with a formula, you can avoid transferring those columns and instead enter just once the uniform or formula-based error. (A box will appear on the following page so you can enter those values/formulas.) Not uncommonly the error in one coordinate is so small it can be neglected; you may also make that selection now. Make your desired selection and then click the **Do Bulk** button. A "Data Entry" page should appear.

5. Copy the numbers from Excel: Either by using:

   (a) Block Mode:
       Highlight an array of up to six columns of numbers (no column headings, just numbers) in Excel. Copy those values using your favorite method (e.g., right click, control-C, or clicking the appropriate button on the top ribbon). Paste those numbers into the large $WAPP^+$ data entry box that has a background image of spreadsheet cells, using your favorite method

(right click, control-V, or clicking Paste from the Edit menu). There is no reason for the columns to line up with either the cell-image or the column headings. As far as the browser is concerned this is just a big lump of numbers. The numbers you paste in will almost certainly not align with anything. *WAPP*⁺ is only concerned that there is some sort of separator between numbers (space, comma, tab, new-line, etc.); it is not concerned about the column alignment visible to you.

(b) Column Mode:
If appropriate, you can use the "Column Copy & Paste" region and, in separate copy & paste actions, move the appropriate columns of data into the appropriate *WAPP*⁺ columns.

6. Tell *WAPP*⁺ which column is which: In Block Mode, you must tell *WAPP*⁺ the identity of each column you pasted in, so it can perform the data analysis you wish to do. In Column Mode, the headings were already in place when you entered the data.

7. Tell *WAPP*⁺ which function should fit the data: Select from the choices:

- Linear: $y = A + Bx$ (which should be the default).
- Power: $y = Ax^B$
- Exponential: $y = A \exp(Bx)$
- Natural Log: $y = B \log(x/A)$
- Inverse x: $y = A + B/x$
- Inverse y: $1/y = A + Bx$
- Inverse x & y: $1/y = A + B/x$
- Arrhenius: $y = A \exp(B/x)$
- Inverse-Natural Log: $1/y = B \log(x/A)$
- Quadratic: $y = A + Bx + Cx^2$ (only available if there is no x uncertainty)
- Resonance: $1/y^2 = A + Bx^2 + Cx^4$ (only available if there is no x uncertainty)

If you look at the bottom of the *WAPP*⁺ data entry page you'll find these words:

Occasionally the value of B is set by theory to a known value. If that situation applies to this data, enter here that fixed value of B. (Usually you will leave this blank.)
$B=$ ☐

8. Compute fit: click the ☐ Submit Data ☐ button. The resulting page includes the 'fit report' that gives the best values for parameters A and B and the uncertainties for each.

There is an important quantity called "reduced chi-squared" ($\overline{\chi}^2$) which tells how well the computed curve fits the data:

- if $\overline{\chi}^2$ is approximately 1 then the curve is a good fit within the error bars;
- if $\overline{\chi}^2$ is much larger than 1, the curve misses the error bars by too much to be considered reliable;
- if $\overline{\chi}^2$ is much less than 1, the curve misses the points by distances that are unexpectedly small compared to the error bars — your error bars are probably larger than necessary. *WAPP*⁺ may give you a warning about this, but don't feel bad, just think about whether the error bars could have been smaller, and fix them if necessary; otherwise go on.

So if the $\overline{\chi}^2$ value is far from 1 either direction, you ought to check either your data or error estimates. By looking at the plot of the curve you may see a problem you should correct.

9. Request a plot: For a linear (normal) plot select "X Scale Options" Linear and "Y Scale Options" Linear (which should be the default). Power law data should produce a straight line if plotted on Log-Log paper. For a Log-Log plot select "X Scale Options" Log and "Y Scale Options" Log. For the final version of a plot, enter appropriate x axis labels, y axis labels, and plot title. Click $\boxed{\text{Make Plot}}$ button. On the following page click <u>PDF File</u>. If everything looks OK, make a final pdf plot.

10. Print the fit report: Hit the Internet Explorer back arrow a couple of times to return to the fit report page. The bottom part of the fit report page involves plotting the data and need not be printed. Rather than print the entire page, you can save some trees by highlighting (left-click sweep) the report area, down to and including the line Data Reference, and then print just the selected area.