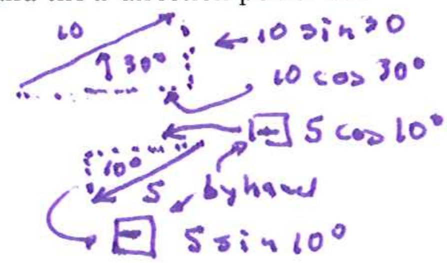
The following questions are worth 10 pts each

13. Consider a coordinate system in which the y direction points due north and the x direction points due east. The following vectors are given:

$$\vec{a} = 10 \text{ km } 30° \text{ north of due east}$$
$$\vec{b} = 5 \text{ km } 10° \text{ south of due west}$$
$$\vec{c} = 2 \text{ km due south}$$



Sketch (approximately) each of the above vectors and display how all four arrows can be arranged to find $\vec{d} = \vec{a} + \vec{b} - \vec{c}$ graphically. Resolve the vectors into (x, y) components and calculate $\vec{a} + \vec{b} - \vec{c}$ using the component forms. Express the resultant vector, $\vec{d}$, in magnitude and direction form. (Display on your drawing the angle you are reporting.)

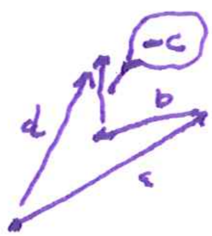
$\vec{a} = 10 \angle 30° \rightarrow (8.66, 5)$ or $(10 \cos 30°, 10 \sin 30°)$

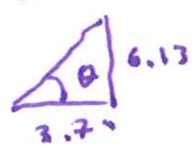
$\vec{b} = 5 \angle 190° \rightarrow (-4.92, -.868)$ or $(-5 \cos 10°, -5 \sin 10°)$

$a + b \rightarrow (3.74, 4.13)$ $\quad \sqrt{3.74^2 + 6.13^2}$

$-\vec{c} = (0, 2) \quad (0, 2)$

$(3.74, 6.13) \rightarrow 7.18 \text{ km} \angle 58.6°$



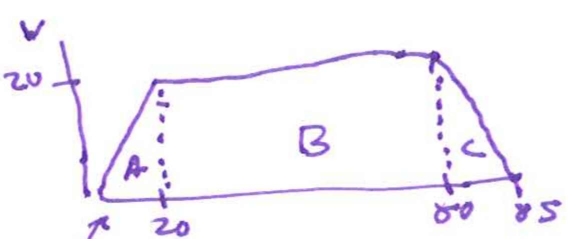


$\theta = \tan^{-1}\left(\frac{6.13}{3.74}\right)$

14. A long straight road in North Dakota stretches for 100 miles in a straight line with occasional stop signs. Starting from rest at a stop sign, you accelerate at 1 m/s^2 for 20 seconds, maintain your speed for the next 60 seconds, and then hit the brakes (de)accelerating at -4 m/s^2 until you are again stopped at a stop sign. Sketch a graph of your velocity vs. time. What distance separates the two stop signs? Convert your answer to miles where 1 mile = 1609 m.

$a = 1 \text{ m/s}^2$ for $20 s \rightarrow v = v_0 + at = 20 \text{ m/s}$ ($v_0 = 0$)

$a = -4 \text{ m/s}^2$ until $v = 0 \rightarrow v = v_0 - 4t$; $t = \frac{20}{4} = 5 s$ ($v = 0$)



$A: \Delta x = v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} \cdot 1 \cdot 20^2 = 200 \text{ m}$ ($v_0 = 0$)

$B: \Delta x = v_0 t = 20 \cdot 60 = 1200 \text{ m}$ ($v_0 = 20$)

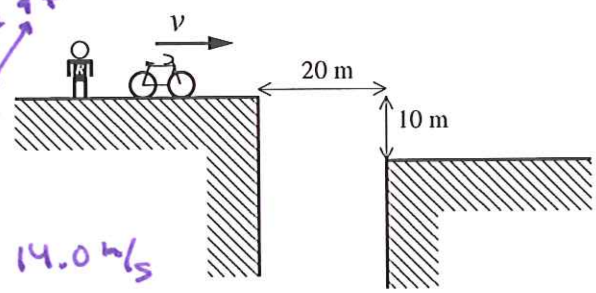
$C: \Delta x = v_0 t + \frac{1}{2} a t^2 = 20 \cdot 5 - \frac{1}{2} 4 \cdot 5^2 = 50$

1450 m

$\text{area} = \frac{h_1 + h_2}{2} b = \frac{60 + 85}{2} \cdot 20 = 1450 \text{ m}$

$1450 \text{ m} \cdot \frac{1 \text{ mile}}{1609} = .901 \text{ mile}$

15. In the movie *Godzilla vs. Terminator*, the Terminator — riding a bike at super human speed — is being chased by Godzilla. Tension builds as the pair approach a huge earthquake crack. Both crack rims are level but 20 m apart with a drop of 10 m. In the script the Terminator bike-sprints to the crack edge and just barely hits the far rim. In the real-world, stuntman Rupert decides that it would be wise to hire a physicist to calculate how fast he must be going horizontally on the bike to make the crack-jump. What is the slowest bike speed that would result in a safe landing on the far rim?



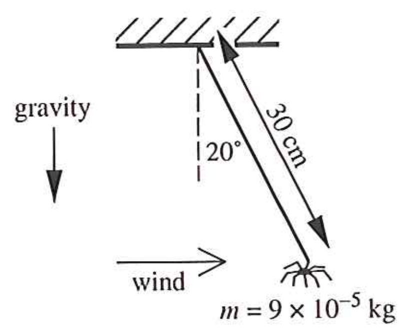
time to fall 10 m: $\Delta y = v_0 t + \frac{1}{2} a t^2$ ($v_0 = 0$, $a = +g$)

$t = \sqrt{\frac{2 \cdot \Delta y}{g}} = \sqrt{\frac{2 \cdot 10}{9.8}}$

$= 1.429\,s$

required x velocity: $\frac{20\,m}{1.429\,s} = 14.0\,m/s$

16. A spider of mass $m = 9 \times 10^{-5}$ kg hangs straight down on a 30 cm long thread she has just created. What is the tension in the thread? A steady breeze from the south pushes the hanging spider towards the north so that the thread makes an angle of 20° with the vertical. What is the tension in the thread now? Please draw a free body diagram for each situation!



$\uparrow T$, $\downarrow mg$

$T = mg = 9 \times 10^{-5} \cdot 9.8$
$= 8.82 \times 10^{-4}\,N$

T, 20°, wind, mg

$T \cos 20° - mg = 0$

$T = \frac{mg}{\cos 20°}$

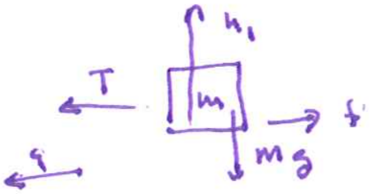
$= 9.38 \times 10^{-4}\,N$

17. A large slab ($M = 10$ kg) sits on frictionless surface. A block ($m = 1$ kg) rests on top of the slab. The surface between the slab and the block has a coefficient of static friction of $\mu_s = 0.4$ and a coefficient of kinetic friction $\mu_k = 0.3$. The block is pulled with a horizontal force T. If T is sufficiently small the block+slab will move together as one object; if T is larger, there will be slippage and the block will accelerate faster than the slab (and will eventually be pulled off the slab).

A. Draw free body diagrams for each mass separately. Show and name all forces acting each mass. Show the direction of the acceleration (if there is any).

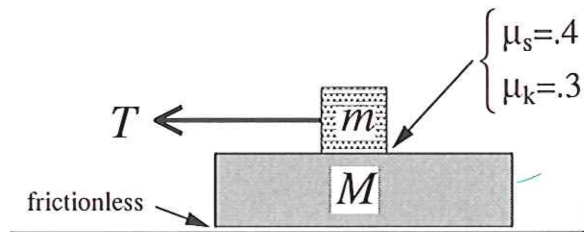
B. *For each mass separately* and for both the x and y directions, write down the equations that follow from Newton's second law ($F_{\text{net}} = ma$).

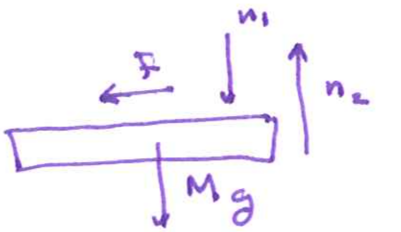
C. If $T = 15$ N, there will be slippage. Find the acceleration of each mass in this case.



$n_1 - mg = 0$

$T - f = ma$





$n_2 - n_1 - Mg = 0$

$f = Ma$

if slips - kinetic friction: $f = \mu_k n_1 = Ma$ ($n_1 = mg$)

slab: $a = \frac{\mu_k m}{M} g = .3 \cdot \frac{1}{10} \cdot 9.8 = .294 \text{ m/s}^2$

block $a = \frac{T - f}{m} = \frac{T - \mu_k m g}{m} = \frac{15 - .3 \cdot 1 \cdot 9.8}{1}$

$= 12.1 \text{ m/s}^2$