

**PHYSICS 105**  
**LABORATORY MANUAL**

Physics for the Life Sciences I

Fall 2012



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# INTRODUCTION

When you can measure what you are speaking about, and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science. *Lord Kelvin*

## Purpose

Why does this course include a laboratory? In class you will be studying physical laws, many in the form of equations, and you will probably become proficient at manipulating the symbols and speaking the vocabulary of physics. Yet it is possible to do this without a clear understanding of what those symbols and formulas represent. For example, in the lecture part of this course you will study the equation

$$F = ma,$$

where  $F$  represents a force,  $m$  is a mass and  $a$  refers to the acceleration of the mass. These words may become familiar, and the symbols easy to use in solving problems from the text, but this does not assure that you understand force, mass, acceleration, or the relationship among them. Physics deals with quantities that are *measurable*, and one of the best ways to find meaning in a physical law is to perform measurements of the quantities it relates. When you have learned how to measure acceleration, you may have a better idea of what acceleration is. So one purpose of the lab is to reinforce some concepts through the experience of making measurements. Another main purpose of the lab is to teach techniques in taking and analyzing data, and in reporting results. There will be some emphasis on evaluating the amount of *uncertainty* that goes with measurements and results; measurements that are imperfect lead to results that are uncertain, and it is often so important to know the limits of their accuracy.

## Schedule

The experiments planned for the semester are synchronized as well as possible with the material covered in the Physics 105 course. There is also a Lab Practical Exam to be taken during the last lab cycle, and your grade will count as one more lab experiment. *This schedule may change during the semester. Your instructor will keep you updated.*

CYCLE	DATES	EXPERIMENT
1	8/29 – 8/31	Uncertainties
2	9/6 – 9/10	Data Analysis
3	9/14 – 9/18	Free Fall
4	9/24 – 9/26	Projectile Motion
5	10/2 – 10/8	Kinetic Friction
6	10/12 – 10/16	Ballistic Pendulum
7	10/22 – 10/24	Rotational Motion
8	10/30 – 11/1	No Lab
9	11/7 – 11/9	Archimedes' Principle
10	11/15 – 11/19	No Lab
11	11/28 – 11/30	Gas Behavior
12	12/6 – 12/10	Lab Practical

The equipment for each experiment will be set up for days 1–3 of a cycle. You should be enrolled in a Physics Lab section along with your lecture. It is your responsibility to attend your scheduled lab section. Switching lab sections without authorization will result in loss of credit.

## Lab Partners

You will typically perform the experiment with *one partner*, but each of you will be responsible for writing your own lab report. Identical lab reports will be considered evidence of plagiarism and will be handled accordingly.

## Laboratory Materials

You will need to purchase *two* laboratory notebooks; while one is being graded, the other is available for your use in the next lab. We recommend composition books with graph-paper pages. These are available in the campus bookstores. You should come prepared to do a complete lab during each 2-hour session. In addition to this laboratory manual, you will need your textbook as a reference book, one of your lab notebooks, writing materials, and a scientific calculator.

## Laboratory Apparatus

A list of apparatus is given with each experiment. Much of the apparatus is delicate and must be handled carefully. Please try to observe safe practice with the lab equipment. In particular,

- Do not attempt to adjust or work with the apparatus until you have been instructed in its use.
- Check each item of equipment you receive, making sure that you have all the items required and that all are in working order.
- Report any problems to your instructor at once.
- Do not, under any circumstances, make electrical connections to batteries or electrical outlets until your circuit has been checked and approved by the laboratory instructor.

Disregarding these simple precautions is not only dangerous, it can cost you time. Moreover, electrical instruments, in particular, are generally very sensitive. Many can be damaged in a fraction of a second if they are connected incorrectly.

## Laboratory Procedure

Since the laboratory is required, attendance will be taken. The time that you will have to perform each experiment is limited, so read the experiment before the beginning of the laboratory period. Many of the experiments have a pre-lab exercise associated with them. These pre-lab exercises must be turned in at the beginning of the lab and then approved by your lab instructor before you begin the experiment.

## Laboratory Reports

The notebooks are to be handed in at the end of each lab period. Please make sure that you hand in the notebook to your TA before you leave lab. *NOTE: Your TA cannot accept late lab reports.* If circumstances are extreme please see the Physics Lab Manager for possible extensions. It is likely that your score will be penalized under such circumstances.

A lab report should be written in such a way that someone who *did not* do the experiment can read it, together with the instructions in the lab manual, and understand exactly what happened. It certainly doesn't need to be long or wordy to be good. Ordinarily, it should include the following items.

**NAMES.** The title of the experiment, your name, and your partner's name.

**DATE.** The date the experiment was performed and handed in.

**PURPOSE.** A brief statement of the objective or purpose of the experiment.

**THEORY.** A brief explanation of the main ideas and formulas relevant to the experiment.

**PROCEDURE.** Similar to that given in the lab manual. You may just outline the procedure in the manual. However, be sure to note any modifications suggested by your lab instructor, or any changes in procedure that you devised.

**DIAGRAMS.** To anyone reading the lab report, a sketch of the apparatus is very helpful. A simple block diagram can often describe the experiment better than a great deal of written explanation.

**DATA.** All the important numbers you encounter, including *units* and *uncertainties*, plus any other relevant observations should be recorded as the experiment progresses. If you find it difficult to be neat and organized while the experiment is in progress, you might try using the left-hand pages of your notebook for doodles, raw data, rough calculations, etc., and later transfer the important items to the right-hand pages. Since eventually you will have to understand what the experiment is about, it's best to understand it in advance, and know what data are to be taken before you start. Often you can prepare a spreadsheet for recording data before lab. It's often helpful to graph the data as you acquire them in the lab; this allows you to see where more data are needed and whether some measurements are suspicious and should be repeated.

**CALCULATIONS.** A sample calculation should be included to show how results are obtained from the data, and how the uncertainties in the *results* are related to the uncertainties in the *data* (see Appendix A). By far the easiest way to do this is with a self-documented spreadsheet hardcopy taped into your lab notebook (see Lab 1).

**RESULTS/CONCLUSIONS.** One objective of laboratory work is to clarify theory and indicate when and how well it applies to practical situations. Give comparisons between your results and the theoretical or accepted values. Does your experimental range of uncertainty overlap the accepted value? Based on the Results, what does the experiment tell you?

**DISCUSSION.** This is sort of a catch-all category, but important. It should include any comments about what you learned, additional questions the experiment may have raised, the relevance of the lab to real life, and whatever else you want to say about the experiment. Critiques of the lab should be written here. We use constructive criticism to help improve things.

**QUICK REPORT.** As you leave lab, each group should turn in a  $3 \times 5$  "quick report" card. You will be told in lab what information belongs on your card, but generally it includes the numerical results (properly recorded: significant figures, units, uncertainty) from your Conclusion. Always include: names (you and your partner), table number (find the number on the computer base), and date. These cards go directly to the Lab Manager who will use them to identify problems.



## Grading

At the end of each experiment in this book there is a checklist which can serve as a guide for you so you won't forget some important part of the experiment. The TA will return your lab to you graded on a 100-point scale, and it may contain some helpful criticism, so please go over your labs carefully when they are returned. The TAs are expected to grade fairly and impartially, using a grading system something like the following. Please talk to your lab instructor if any problems arise.

Quality ⇒ ⇓ Component	Inadequate	Minimal	Adequate	Superior
Objective/ Purpose 0 – 5 pts	Objective omitted, or not related to lab 0 pts	Objective stated but not clearly related to the measured variables and calculations 1 pt	States an objective clearly related to some, but not all of the important variables 2 – 3 pts	Correctly states an objective related to all of the important variables 4 – 5 pts
Theory 0 – 10 pts	Theory omitted or unrelated to lab 0 – 1 pt	Unclear, or formulas not included 2 – 4 pts	Complete but with a few mistakes 5 – 7 pts	Clearly stated with all relevant formulas listed 8 – 10 pts
Procedure/ Diagrams 0 – 10 pts	Omitted, or incoherent 0 – 1 pt	Unclear, steps out of order, or diagrams not included 2 – 4 pts	Complete but not well stated or diagrammed 5 – 7 pts	Clear, concise and complete 8 – 10 pts
Data 0 – 10 pts*	Data missing or incomplete 0 – 1 pt*	Data listed but not labeled correctly or without units 2 – 4 pts*	Data listed with some units and labels 5 – 7 pts*	Data well organized with clear labels and all units 8 – 10 pts*
Analysis/ Calculations 0 – 30 pts*	Calculations and graphs missing or not relevant 0 – 3 pts*	Calculations and graphs incomplete 4 – 12 pts*	Calculations and graphs nearly complete and correct 13 – 21 pts*	Complete with clear example calculations 22 – 30 pts*
Conclusion/ Discussion 0 – 25 pts	Missing or does not relate to objective 0 – 2 pts	Conclusion not supported by data or analysis 3 – 9 pts	Written paragraph with supporting evidence but incomplete 10 – 16 pts	Written paragraph, logical arguments, included uncertainties 17 – 25 pts
Pre-Lab Exercise 0 – 10 pts	0 – 10 pts			
Total Points	60 – 69 = D	70 – 79 = C	80 – 89 = B	90 – 100 = A
* The point breakdown between Data and Analysis/Calculations may vary, depending on the experiment, but will total 40 points				

# 1. UNCERTAINTIES

## Procedure

This first-cycle lab may be completed by yourself online at any location with internet access. Feel free to come to the usual lab room, but you need not complete this lab at the scheduled time and location. You must complete the online lab report and spreadsheet exercise before the start of your scheduled Lab 2. A one-hour, 800 Mb video:

<http://www.physics.csbsju.edu/lab/105Lab1.mp4>

will substitute for your usual lab period, and the on-line lab report:

<http://www.physics.csbsju.edu/lab/105Lab1.html>

along with the **spreadsheet exercise** will substitute for the usual lab notebook report. Given the large size of the video, you are encouraged to view or download it on-campus (but bring your own earbuds).

## Measurements

### How much does a cat weigh?

When I put Tiger on a scale, the reading bounces around randomly: Tiger is not one to sit still. I'd guess that the average scale reading is more closely related to his weight than any particular reading, and indeed you'll learn this semester that the net impulse on an object determines the change in velocity of that object's center of mass ( $\Delta V_{cm}$ ). Since Tiger's center of mass isn't moving (much) we are assured that the time averaged scale reading is close to his weight. In particular, averaging over a time interval  $T$ , we exactly have:

$$\text{average scale reading} = \text{weight} + \frac{M\Delta V_{cm}}{T} \quad (1.1)$$

where  $M$  is Tiger's mass. So the longer we average those scale readings, the more accurately we'll know his weight, since the unknown (and likely small) quantity  $M\Delta V_{cm}$  will be divided by an ever larger number,  $T$ . Of course if I knew the value of  $M\Delta V_{cm}$ , I could include it in the equation and gain accuracy, but aside from guessing that it's "small," I know nothing about it, not even its sign. The best I can do is exclude times (like when Tiger jumps on or off) when his center of mass is in noticeable motion.

Randomly fluctuating measurements like this are known as random uncertainties or random errors. In the context of this document, "uncertainty" and "error" mean exactly the same thing. Use the word "blunder" to describe what is sometimes called "human error": incorrect or accidental human actions that result in inaccurate readings; for example not properly zeroing ("taring") the scale before putting Tiger on it.

At some point, it's not worth extending the averaging time because:

- The balance manufacturer only claimed a precision of  $\pm 0.02$  lbs, so there's little reason to reduce the random variation much below that level.
- If not level the scale will report only a component of weight. If off vertical by  $3^\circ$ , there will be in error of about  $-0.02$  lbs. How carefully did I level the scale?
- A living cat does not have a weight. The mass enclosed by his skin changes as fluids (air, water, urine) and solids (food, feces) are exchanged. Because of this it's hard to imagine any *practical* significance to weight difference of order  $\pm 0.1$  lbs.

The first two items on this list could be called “systematic errors” or “calibration errors” or “biases”. They result in incorrect measurements but do not signal their presence by a fluctuating answer. They can only be detected by testing with a standard, known mass. Since systematic errors are consistent, if we knew the value we could correct the reading, but as with random errors, at best we have some estimate of the magnitude of the error, but no idea of its precise value or sign.

Last item on this list is a “problem of definition:” the definition of the weight of a living being is ambiguous.

SO we can measure Tiger's weight and report guess-estimates for the random and systematic error, but the original questions was: How much does a cat weigh? A cat is a randomly selected cat, and of course it doesn't have a weight: different cats weigh different amounts. If we know the weight of the set of cats from which a cat was selected (or more likely: a collection of cats we judge to be similar), it is probable that a cat would be similar to the majority of cats. We need to report some typical weight and a range of variation that includes “most” cats. Notation: assume we have a set of  $N$  cat weights:  $\{w_1, w_2, w_3, \dots, w_N\}$  (or equivalently:  $\{w_i\}$  for  $i$  from 1 to  $N$ ).

The two most common measures of “typical” are average, a.k.a., mean (add up all the cat weights and divide by the number of cats):

$$\text{average of the weights} = \bar{w} = \frac{1}{N} \sum_{i=1}^N w_i \quad (1.2)$$

and median (the middle weight: half the cats weigh more than the median, half weigh less).

There are several common measures of “range of variation”. The most common is the standard deviation: subtract the average from each weight producing a list of deviations-from-average, some positive some negative; add up the square of these deviations-from-

average, divide by  $N - 1$ , and take the square root of the result:

$$\text{standard deviation of the weights} = \sigma_w = \sqrt{\frac{\sum_{i=1}^N (w_i - \bar{w})^2}{N - 1}} \quad (1.3)$$

**Remark:** You should never actually use this formula: calculators and spreadsheets have this function built in. We present it to you so you have a general idea of what the standard deviation is: The square root of the average of the squared deviations-from-average. Re-read the previous sentence and make sure you understand what it is saying. The standard deviation is a measure of typical deviation-from-average.

If the distribution of cat weights is “normal” you should expect that about  $\frac{2}{3}$  of the cat weights are in the interval:  $(\bar{w} - \sigma_w, \bar{w} + \sigma_w)$ . This likely interval is often marked with an “error bar” that displays the extremes of this interval as whiskers up and down from the average value. Leaving  $\frac{1}{3}$  of cats outside of this normal range is sometimes viewed as insufficiently inclusive; error bars that encompass 95% of the cats (typically this is nearly the same as the interval:  $(\bar{w} - 2\sigma_w, \bar{w} + 2\sigma_w)$ ) may be used and called 95% confidence limits. Some folks opt for ‘equality’ error bars that enclose half of the cats, which may be approximately the smaller range:  $(\bar{w} - 0.674\sigma_w, \bar{w} + 0.674\sigma_w)$ ; this is also called the interquartile range (the spread between the 25<sup>th</sup> percentile and the 75<sup>th</sup> percentile). Unfortunately there is no consistent meaning assigned to an error bar. In this this document, we will use the  $\pm 1\sigma$  meaning, which means it’s not hugely unlikely (i.e.,  $\frac{1}{3}$  of the time) for a datum to lie outside the error bar. You should recognize an unusual occurrence (5%) when a datum misses by 2 error bars.

Boxplots (a.k.a., box-and-whisker plots) are a standard, if less common, way to display the full range of variation which is encoded in the five-number summary: minimum, first quartile (25<sup>th</sup> percentile), median, third quartile (75<sup>th</sup> percentile), and maximum. A rectangle runs from first quartile to third quartile with a horizontal line indicating the median; ‘whiskers’ extend down to minimum and up to maximum. Unusual data points called outliers may also be specially marked.

Many have argued (the author included) that a better measure of variation would be the average absolute value of the deviations-from-average: while that is both a more robust and intuitive measure, the awkward mathematics of absolute value make it less commonly used by statisticians.

## How much does an average cat weigh?

If we actually weigh every cat in the world we could definitively answer this question, but usually we have weighted some subset of the world cat population. In the case of Tiger’s weight, Newton’s Laws prove that averaging over more weights improves the estimate of Tiger’s weight. It seems evident that averaging over larger collections of cats would

improve our estimate of the average cat’s weight. Indeed one can show that the likely inaccuracy of the average decreases with the inverse square root of the number of cats we weigh (and average). This **standard deviation of the mean** (SDOM) is given by:

$$\text{SDOM} = \sigma_{\bar{w}} = \frac{\sigma_w}{\sqrt{N}} \quad (1.4)$$

This formula is too good to be true: it claims we can have arbitrarily small uncertainties simply by collecting more data. Every time you use this formula you should ask yourself:

- Since systematic errors are not reduced by averaging, is this reduction of the random errors useful, given that total error is what really matters?
- Is the average really the quantity I want to know? In the case of Tiger’s weight I could show a direct relationship between the average and the actual weight. However that is not always the case. For example, the path followed by the average initial speed ball is not the average path of balls. The weight of an average radius steel sphere is not average weight of steel spheres.

It is important to recognize that the uncertainty in the weight of a cat is much larger than the uncertainty in the weight of the average cat.

## Average cat?

In 1947 the Cambridge statistician R.A. Fisher published weight data on 144 cats<sup>1</sup>. I’ve collected recent data on 25 Minnesota in-home cats. Which defines the average cat? Figure 1.1 displays the two distributions as a boxplot, as the mean with standard deviation error bars (STDEV), and as the mean with standard deviation of the mean error bars (SDOM). The boxplot shows that these samples overlap in part; STDEV shows no overlap between  $1\sigma$  errors bars, but clearly  $2\sigma$  errors bars (typically 95% inclusive) would again show a small overlap. SDOM shows distinctly different means. Clearly there is some systematic difference between well-fed MN home cats and cats used in post-war UK pharmaceutical research. Most Minnesotans would find it incredible that a  $6\frac{1}{2}$  lb cat was “significantly above average”. (More evidence that in Minnesota every cat is “above average”.) Here in lies the danger (and benefit) of SDOM: large datasets can reduce ‘errors’ to practically meaningless levels (in the UK data, less than 0.1 lb). If the average is exactly the quantity we seek, we can measure it with extraordinary accuracy. If the average is not exactly equal to the quantity of interest, or if the sample we use to define average is not representative, we risk humiliating reversals from future work.

In summary: a measurement has no value if it lacks an estimate of its accuracy. In these labs you must always report both the measurement and its uncertainty (=“error”) that

<sup>1</sup>The data was actually produced as a part of a 1946 study on the lethal dose of digitalis in cats.

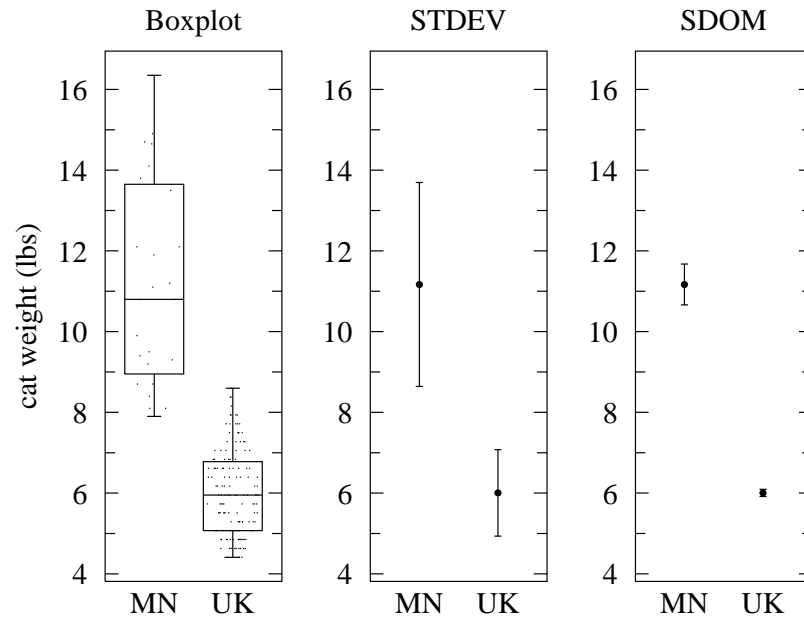


Figure 1.1: In search of the “average cat,” I present two datasets on cat weight. UK reports the weight of 144 cats used in a pharmaceutical study reported by R.A. Fisher in 1947; MN reports the weight of 25 MN in-home cats collected by the author. The boxplot’s rectangle shows the central 50% of cats with the median marked with a bisecting horizontal line and whiskers out the the group’s maximum and minimum. STDEV displays the mean with  $1\sigma$  error bars. SDOM displays the mean with error bars extending to the standard deviation of the mean. Clearly while there is some overlap between the distributions, the UK and MN means differ significantly. The difference between the datasets demonstrates that additional study is required to uncover the ‘average cat’; it would be foolhardy to accept either  $\text{mean} \pm \text{SDOM}$  as the measure of the average cat. While SDOM provides an interesting measure of a group mean, that group’s mean may not be exactly the quantity we seek.

may be a mix of reading variation, hard-to-estimate, pernicious systematic errors and problem of definition errors. The deviation of a hypothetical future measurement can be estimated by the standard deviation of set of previous measurements. The uncertainty in a average value can be estimated by SDOM of a set of previous measurements (if certain conditions are met).

## Judging Uncertainty

As significant fraction of your lab grade this semester will be based on your ability to judge the uncertainty of your measurements. The accuracy of most electronic instruments is recorded in the manufacturer's specifications; typically these are estimates of possible systematic error. (We will supply you with those estimates in lab.) Accuracies may be reported as 'absolute errors': a direct estimate of the likely standard deviation, or as 'percent errors', a.k.a., 'relative error' or 'fractional error': where the likely standard deviation can be calculated by taking the given percentage of the displayed value, or both.

For example the manufacturer reports that accuracy of the voltmeter you will be using next semester is  $\pm(0.1\% + 4\text{digits})$ , where one 'digit' is the place value of the rightmost digit on the display. So if the display showed 6.238, the uncertainty would be  $6.238 \times 0.1\% + .004$ ; if the display showed 71.49 the uncertainty would be  $71.49 \times 0.1\% + .04$ .

This semester many of your measurements will be made using rulers. Generally when using a ruler (or any device whose scale you read by eye) you can estimate the fractional bit between the marked gradations. Error estimates that are a third or a quarter (at most a half) of those marked divisions are then appropriate.

An important special case applies to counting events that have no preference over time or space. For example, the count of weeds growing in a uniform farm field, or the decay of a Uranium atoms which are just as likely to decay this year as in the year 4,000,000,000 B.C.. Whole number counts of such events (weeds in an acre or decays during a year) will be randomly different for different acres or years, but there is a simple estimate for the standard deviation of such measurements based on just one measurement: the square root of the number of events. For example, if I measure 121 weeds in one acre, I estimate that  $\frac{2}{3}$  of the acres will have between  $121-11=110$  and  $121+11=132$  weeds on them. (So  $\frac{1}{3}$  of the acres will have fewer than 110 or more than 132 weeds.) This special case is known as Poisson statistics.

In this semester's labs we will not be expecting precise error estimates. Because of the limited lab time you will often be called on to judge the typical range-of-deviation by eye rather than using the standard deviation formula. You should develop the ability to judge such deviations (accurate to maybe a factor of two) just by eye. The main point this semester is that every measure has uncertainty, and this needs to be factored into all your further calculations with the data.



## Reporting Measurements

You should record (in your lab book or Excel) every digit displayed by an instrument and every digit you estimate when reading a ruler. Don't round your raw data! Of course you must also report an uncertainty for those measurements. When it comes to reporting *final results* (in contrast to raw data or intermediate calculations) there are strict requirements as to exactly how you record those numerical results. Our uncertainty in the uncertainty is always large; hence round any uncertainties to one or two significant figures ("sigfigs"<sup>2</sup>). One sigfig is always easier and almost always as accurate as two sigfigs. (Recall: You may have estimated your range-of-deviation just by eye. Manufacturer's specification of errors almost always report just one sigfig (recall:  $\pm(0.1\% + 4\text{digits})$ ) so one sigfig would be appropriate for an error calculated from those numbers. One can show that in 'normal' cases, formula-calculated standard deviations will have uncertainty greater than 10% unless the number of available data points,  $N > 50$ .) Once you have properly rounded your errors, display all the digits of your measurement until the place-value of the least (rightmost) significant digit in the measurement matches the place-value of the least significant digit in the error. Some examples:

$$3.14159 \pm .00354 \implies 3.142 \pm .004 \text{ or } 3.1416 \pm .0035$$

$$2.71828 \pm .05472 \implies 2.72 \pm .05 \text{ or } 2.718 \pm .055$$

$$321456 \pm 345 \implies 321500 \pm 300 \text{ or } 321460 \pm 350$$

$$.1678 \pm 3.51 \implies 0 \pm 4 \text{ or } 0.2 \pm 3.5$$

Finally, remember to record the units of your measurement!

## Calculating with Uncertain Numbers

Measurements are usually followed by calculations, which make use of the measurements. Perhaps you are calculating the area of a rectangle. Imagine that your distance measurement of  $8.8 \pm 0.3$  units is to be multiplied by another uncertain distance measurement:  $2.1 \pm 0.2$  units. One way of determining the uncertainty of the calculated result is to use a "high-low" approximation.

### High-Low Method

The high-low method involves calculating the result three times: once without uncertainty — use your best values — and then two additional times: finding the highest possible result and the lowest possible result, as illustrated below and in Fig. 1.2. The answers to the three calculations are as follows,

<sup>2</sup>If you are unfamiliar with sigfigs, see page 86, or consult the textbook, or Wiki.

$$\begin{aligned} \text{Best: } & 8.8 \times 2.1 = 18.48 \\ \text{High: } & 9.1 \times 2.3 = 20.93 \\ \text{Low: } & 8.5 \times 1.9 = 16.15 \end{aligned}$$

The range of results could now be expressed as  $18.48 + 2.45$  to  $18.48 - 2.33$  units squared. This unsymmetric form is awkward, so the difference between the two extremes is split evenly ( $20.93 - 16.15 = 4.78$ , and  $4.78/2 = 2.39$ ), and the result expressed as  $18.48 \pm 2.39$ . Following our final value reporting rules this is:

$$18 \pm 2 \quad \text{or} \quad 18.5 \pm 2.4 \text{ units squared.}$$

Again, here is no point in expressing uncertainties with more digits (for instance,  $18.48 \pm 2.39$  or  $18.48 \pm 2$ ): at least in these labs our error estimates never deserve 3-digit accuracy and there is no point in reported digits in the base number that are overwhelmed by the likely errors.

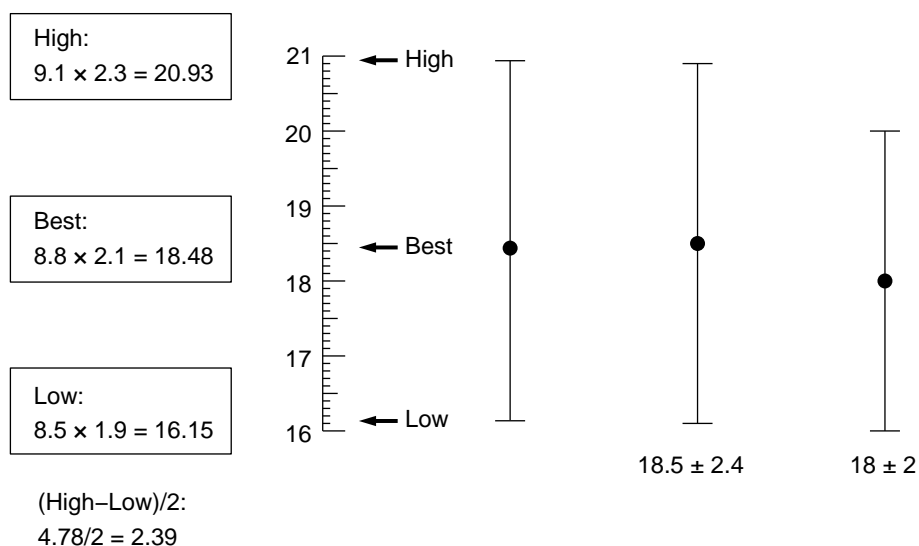


Figure 1.2: Example of calculating uncertainties by the high-low method.

It doesn't make any difference how complicated the calculations are when you use the high-low method. Always apply the uncertainties in such a manner as to maximize, and then minimize the result. In the case of division, maximize the numerator and minimize the denominator in order to maximize the result. Do the opposite to minimize the result.

One disadvantage to the high-low method is that several calculations are needed (the high, the low, their average and half their difference) to end up with a best estimate and its uncertainty. As a result it will be unclear which errors contribute most significantly to the final error; i.e., which measurements should be improved first.

## Fractional Uncertainty Method

There is a simpler approach we can sometimes use, known as the fractional uncertainty method, in which the uncertainty in a result is calculated directly from the measured values and their uncertainties. The fractional uncertainty formulas (see Appendix A) are often fast and easy calculations. For instance, in the case of multiplication or division, fractional uncertainties are additive. Going back to our two distance measurements,

$$\frac{\text{uncertainty in area}}{\text{area}} = \frac{\text{uncertainty in distance 1}}{\text{distance 1}} + \frac{\text{uncertainty in distance 2}}{\text{distance 2}},$$

or more briefly,

$$\frac{\delta A}{A} = \frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2},$$

where  $A$  represents the area,  $D_1$  and  $D_2$  are the measured distances, and their uncertainties are indicated using the Greek letter  $\delta$  (“delta”). We already know how to calculate the area  $A = D_1 \cdot D_2$ , and the uncertainty in the area can be found with one more calculation:

$$\delta A = A \left( \frac{\delta D_1}{D_1} + \frac{\delta D_2}{D_2} \right).$$

Using our measurements,

$$\delta A = 18.48 \times \left( \frac{0.3}{8.8} + \frac{0.2}{2.1} \right) = \pm 2.39$$

In this case the fractional uncertainty method and the high-low method produced exactly the same result; more generally they will differ, but never significantly. One advantage of the fractional uncertainty method is we can determine which terms are contributing the most to the final error. In this case  $0.3/8.8 = .034 < .2/2.1 = .095$ , so the the smaller absolute error (0.2) actually contributes more to final error.

In future labs, we’ll tell you which method works easier/better for the particular experiment in question. In general, the high-low method will be used for complicated, multi-step calculations while the fractional uncertainty approach will be used with fairly simple calculations. The rules for using fractional uncertainties are outlined in Appendix A.

For both the high-low method and the fractional uncertainty method the worst-case scenario is assumed, which may be unrealistic. If the actual deviations were random (or uncorrelated), as is often the case with measurement errors, one would expect some partial cancellation of the effects of uncertainty. Why, for example, would both distance measurements always be high or always be low? Why couldn’t one be a bit high while the other is a bit low? If a calculation involves many steps, shouldn’t there be at least some cancellation of random errors? These questions suggest that a statistical approach to estimating uncertainty is needed.

## Spreadsheet Descriptive Statistics

In many cases uncertainties will be given by manufacturers specifications (typically systematic errors) or by-eye estimates of deviations-from-average, e.g., Tiger's weight. For repeated measurements, spreadsheets allow easy calculation of average, standard deviation and standard deviation of the mean.

To find the average, select an used cell and enter the formula: `=AVERAGE(A1:A36)` where the list of cells you want averaged (here `A1:A36`—the first 36 cells in the first column which is called A) is enclosed by parenthesis. Typically you will get the list of cells by sweeping through them with left mouse button depressed or by clicking on the first cell and then the last cell in the list while also holding down the Shift key. After hitting Enter, your formula will be replaced by the appropriate value.

To find the standard deviation, use the formula: `=STDEV(A1:A36)`.

The standard deviation represents the uncertainty for any single measurement, e.g., a cat's weight.

If the mean is the value whose error you want to estimate, you need the SDOM (standard deviation of the mean). To find this, use the formula:

`=STDEV(A1:A36)/SQRT(COUNT(A1:A36))`, i.e.,

$$\text{SDOM} = \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \quad (1.5)$$

where  $N=\text{COUNT}(A1:A36)$ . In this case we know there are 36 data points, so we might simplify by typing `=STDEV(A1:A36)/SQRT(36)` or even `=STDEV(A1:A36)/6`

The standard deviation of the mean represents the uncertainty in the mean, e.g., the average cat's weight.

## Spreadsheet Self-Documentation

If you print out your spreadsheet, the formula used to calculate a cell will not be displayed; the grader will have no idea how the printed number was calculated. So in an adjacent cell you must display the formula used. This is easily done by copying and pasting the formula into an adjacent cell (which often results in an error or nonsense) and then editing out the '=' and hitting Enter. Excel will now display (and print if requested) the text of the formula used. Of course, you will also provide words ('headings') naming column and cell values (including units). Note: in documents and your lab notebook results (measurement/error/unit) are recorded in the form: '14.1±0.1 lbs'; don't do this in your spreadsheet! Text like '±' and 'lbs' make the numbers sharing that cell unavailable for further calculation. Instead put each number in its own cell and

in an adjacent cell report the name of the quantity and units (the units are typically enclosed in parenthesis):

14.9	Tiger's weight (lbs)
0.1	Error in Tiger's weight (lbs)

## Types of Measuring Instruments

We can enumerate two broad categories of measuring instruments: analog and digital.

The ruler and anything else that has a continuous scale where the operator can “read between the lines” is referred to as analog. Triple beam balances and meters with pointers are examples of analog instruments. The operator of an analog instrument must judge both the value and appropriate uncertainty. Quite often the uncertainty is estimated at plus or minus a fraction of the smallest scale division, but it always involves a judgment call by the operator.

Digital instruments are quite different. They are incremental rather than continuous and one can't “read between the lines.” In typical use they provide a steady reading with no obvious random fluctuation. The manufacturer usually provides the uncertainty specifications (typically systematic errors) of digital instruments. For example, a digital balance may read 10.3 grams where the last digit, according to the balance manufacturer, has an uncertainty of two (0.2 gram in this case). For other models it might be something different, such as 0.1 gram. Usually the uncertainty involves the last digit, but sometimes it is expressed as a percentage of the total value or a combination of percentage plus the last digit.

## Comparing Measurements with Uncertainty

It is unlikely that two measurements of the same quantity will exactly agree. However usually (i.e., approximately  $\frac{2}{3}$  of the time) the intervals spanned by their error bars will overlap. If they miss by more than 2 error bars, most folks would call it a significant disagreement. Occasionally measurements can be compared to precise predictions of theory (for example, the ratio of measured circumference to diameter should be precisely  $\pi$ ), in which case usually the theoretical prediction should lie within the error bars of a measurement.

Clearly one can wash over disagreements simply by enlarging the error estimates. However the value of a measurement depends strongly on the error; technically the ‘weight’ of a measurement is proportional to the inverse square of the error. So a number with  $2\times$  bigger error bars has  $\frac{1}{4}$  the weight. Thus the worth of a measurement is judged by its

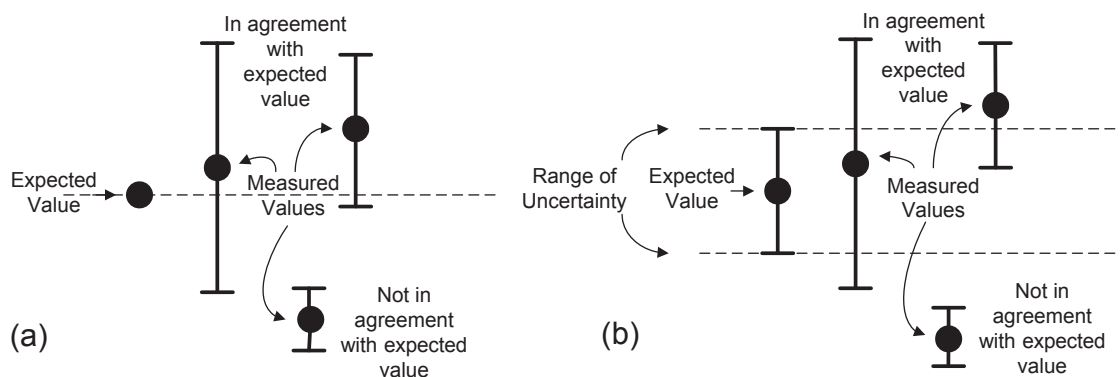


Figure 1.3: Examples of measured values with uncertainty, compared to an expected value (a) with negligible uncertainty, and (b) with the indicated range of uncertainty.

error; measurements with small systematic and random errors are considered the best measurements.

If results show significant disagreement, then either (A) we're in a statistically unlikely (but not impossible) case or (B) mistakes have been made or (C) something interesting is going on which makes two correctly measured quantities disagree.

While I haven't made a valid study of the frequency of these possibilities, my sense is that (B) is by far the most common situation, and that usually the mistake is undetected systematic errors in the measurement.

## Experiment

View the one hour, 800 Mb, on-line video: [www.physics.csbsju.edu/lab/105Lab1.mp4](http://www.physics.csbsju.edu/lab/105Lab1.mp4)  
 complete the questions: [www.physics.csbsju.edu/lab/105Lab1.html](http://www.physics.csbsju.edu/lab/105Lab1.html) For Part III, do your calculations in a spreadsheet; format the results in a form suitable for a final results report; self-document the spreadsheet; print out a hardcopy of the page and turn it in at the start of Lab 2.

## 2. DATA ANALYSIS

### Pre-Lab Exercises:

1. Estimate the slope and  $y$ -intercept (each with units) of the line graphed below. (Hint: the  $y$ -intercept is not 2.5; it needs to be calculated using the method shown in Fig 2.3.) Note: these pre-lab calculations should be recorded in your lab notebook before the start of lab.
2. View the video: [http://www.physics.csbsju.edu/lab/105\\_WAPPonline.mp4](http://www.physics.csbsju.edu/lab/105_WAPPonline.mp4) on this lab's use of the web site: *WAPP*<sup>+</sup>.
3. Briefly outline in your notebook the procedure you will be following in this experiment.

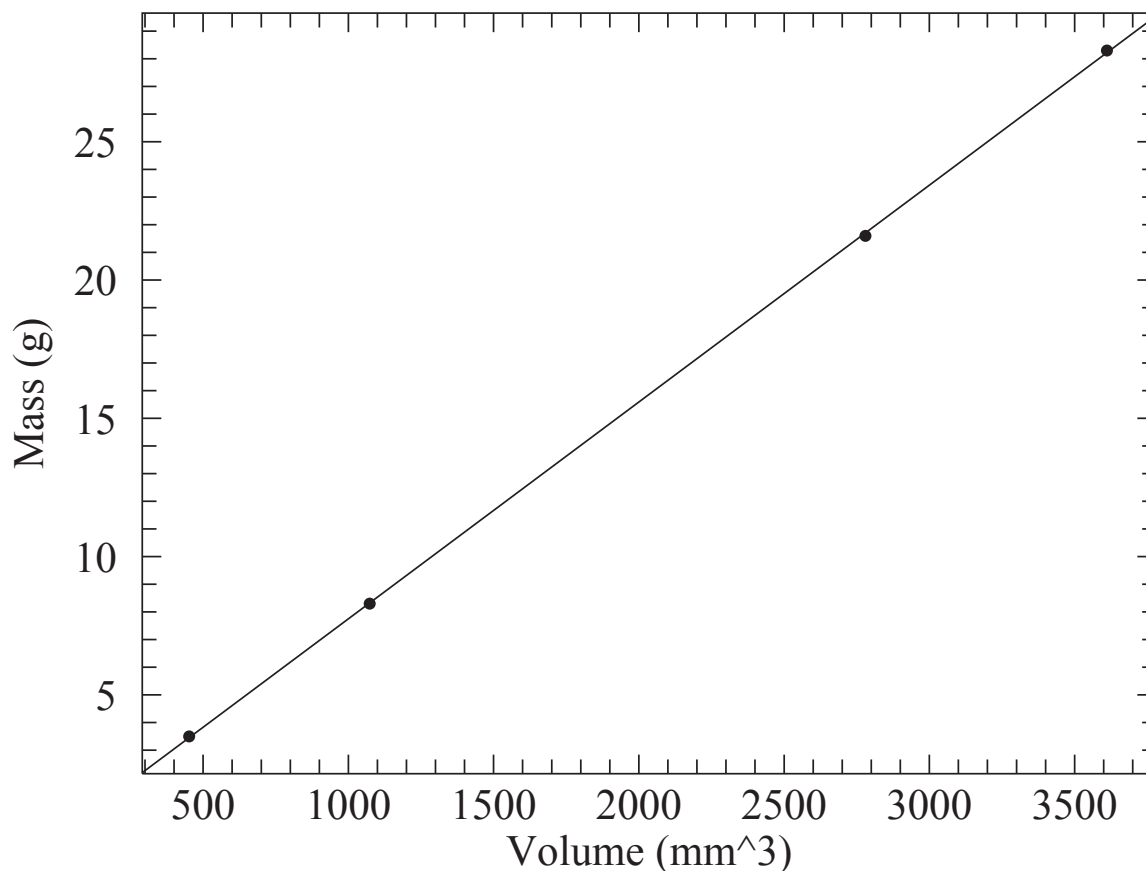


Figure 2.1: A sample straight line.

### Introduction

Physics involves making numerical measurements to test theories. With few exceptions, measurements have some uncertainty. The amount of uncertainty is related to the

methods and instruments used in making the measurement. The purpose of this lab is to acquaint you with techniques for handling experimental data and uncertainties, techniques that you will be expected to use throughout the semester. In particular, you will be:

1. estimating the uncertainty that goes with a measurement,
2. finding the uncertainty in calculations done with experimental data,
3. graphing data with uncertainties,
4. finding mathematical formulas to relate one set of data to another.

The measurements will be of the diameters and masses of several ball bearings made of some material. From the diameters you can calculate the volumes. The ratio of mass to volume is known as the *density* ( $\rho$ ), a property of the material, which you will be able to determine within some range of uncertainty.

## Apparatus

1. 4 ball bearings,
2. a digital caliper,
3. a digital balance (somewhere in the lab),
4. A PC with the programs Excel and WAPP<sup>+</sup>,

## Experiment

The experiment itself is a simple one:

1. Measure the diameters  $D$  and the masses  $M$  of several metal balls. There will be some uncertainty  $\delta D$  and  $\delta M$  with each measurement.
2. Calculate the volume  $V$  of each ball. There will also be some uncertainty  $\delta V$  in the calculated volumes.
3. Graph and analyze the  $M$  vs.  $V$  values, fitting the data to a linear function, and determine the density of the metal.
4. Graph and analyze the  $M$  vs.  $D$  values, fitting the data to a power function, and find the density from the parameters of the fitted function.



## Procedure

Log in to your network account and start Excel; you will be using Excel to list your data values as you go, and to perform calculations of results.

### 1. Measurements and Uncertainties

On the lab table you should find a collection of ball bearings and a caliper for measuring their diameters. Set the caliper for millimeter units, and be sure it reads 0.00 when the jaws are closed (if you're not sure how to use the caliper, get help from your lab instructor). Get a ball bearing between the jaws of the caliper to measure its diameter  $D$ . According to the manufacturer, measurements made with this device have an uncertainty of  $\delta D = 0.02$  mm. So, for example, if the caliper reads 6.00 mm, this means the diameter is somewhere between 5.98 and 6.02 mm; you should report a diameter  $D = 6.00 \pm 0.02$  mm.

Note on Significant Figures: The uncertainty determines how many digits are reported in a number, such as  $6.00 \pm 0.02$  mm. All digits to the right of the first uncertain digit are meaningless.

Measure  $D$  for each ball bearing, and enter your data into an Excel table, listing the  $D$  values in a column:

$D$ (mm)	$\delta D$ (mm)

A data table should always include the units of each quantity (mm in this case), and should make clear what the uncertainty is for each measured value, whether they are all alike or different for each measurement.

Next, use the digital balance to measure the masses  $M$  for the ball bearings, and list them with their units and uncertainties in the third and fourth columns of your table:

$D$ (mm)	$\delta D$ (mm)	$M$ (g)	$\delta M$ (g)

## 2. Calculations and Propagation of Uncertainties

You have measured the diameters and masses of the ball bearings — now calculate their volumes. You may recall that the volume of a sphere of radius  $r$  is  $V = \frac{4}{3}\pi r^3$ , or in terms of the diameter  $D$ :

$$V = \frac{\pi}{6}D^3 \quad (2.1)$$

The fifth column of your data table will have the calculated volumes of the ball bearings. To enter the formula, start with an equal sign, and where the formula requests the  $D$ , click on the cell that contains that value. For the cube hit  $\wedge 3$  (“caret”=Shift-6). The Excel name for  $\pi$  is `PI()`. The result should be something like: `= PI()/6*A2^3`. This should give the volume associated with the first-diameter ball, because the **A2** in the formula refers to the value in cell **A2** of the spreadsheet. Now copy it down to each row where it belongs. The volumes for each of the other balls should be generated automatically, using diameter values in **A3**, **A4** and **A5**.

$D$ (mm)	$\delta D$ (mm)	$M$ (g)	$\delta M$ (g)	$V$ (mm <sup>3</sup> )	$\delta V$ (mm <sup>3</sup> )

Now, what is the uncertainty  $\delta V$ ? That depends on  $\delta D$  of course, but how? Here are two methods (but we will use the second, to teach you a quicker, easier method):

1. One simple approach is to use the high and low values of  $D$  within its range of uncertainty, and find the high and low values of  $V$ . For example, if  $D = 6.00 \pm 0.02$  mm, then  $D_{min} = 5.98 \Rightarrow V_{min} = 112$  mm<sup>3</sup> (dropping the meaningless digits) and  $D_{max} = 6.02 \Rightarrow V_{max} = 114$  mm<sup>3</sup>; thus  $V = 113 \pm 1$  mm<sup>3</sup>. This method usually works fine, but requires two calculations for each uncertainty.
2. Alternatively, consider the following lines of algebra, where  $C$  is a short-hand for the constant  $\pi/6$  and  $\delta D$  is small compared to  $D$ :

$$V = C \cdot D^3 \quad (2.2)$$

$$V \pm \delta V = C \cdot (D \pm \delta D)^3 \quad (2.3)$$

$$V \pm \delta V = C \cdot (D^3 \pm 3 \cdot D^2 \delta D + 3D \cdot \delta D^2 \pm \delta D^3) \quad (2.4)$$

Dividing by  $V$  on the left, and  $C \cdot D^3$  (which is the same thing) on the right we get:

$$1 \pm \frac{\delta V}{V} = 1 \pm 3 \frac{\delta D}{D} + 3 \left( \frac{\delta D}{D} \right)^2 \pm \left( \frac{\delta D}{D} \right)^3 \quad (2.5)$$

Since  $\delta D/D$  is a small number, when squared or cubed it becomes negligibly small and is simply dropped, leaving

$$\frac{\delta V}{V} = 3 \frac{\delta D}{D}. \quad (2.6)$$

So the *fractional uncertainty* in  $V$ , that is  $\frac{\delta V}{V}$ , depends in a simple way on the fractional uncertainty in  $D$ ; the 3 comes from the fact that  $D$  is cubed in calculating  $V$ .

Note: Throughout the semester it will often be the case that fractional uncertainties are the simplest to handle. For your convenience some of the rules for propagation of uncertainties, similar to Eq.(2.6), are summarized in Appendix A of the lab manual.

Finally, for  $\delta V$  we have:

$$\delta V = 3 \cdot V \cdot \frac{\delta D}{D} \quad (2.7)$$

This method will agree quite well with the method of using the high and low values, and requires only one calculation instead of two for each  $D$  value. In the spreadsheet, again assuming you have a row of titles, type in the second row of column F the formula =3\*E2\*B2/A2, then ENTER, and copy this formula into the rows beneath.

After you have determined the volumes and their uncertainties, with due attention to significant figures again, go on to the next part of the procedure.

### 3. Graphing and Analysis of the Data

In any experiment only a limited number of measurements are made. The task of analyzing the data is to determine a relationship between one set of measurements and another, which allows us to find their values for cases we have not measured. Graphs are a good way of displaying data, and they are usually helpful in determining relationships between sets of data. Throughout the semester, you will be doing your graphs and analysis using a computer, but it is important that you understand what you are doing in order to avoid elementary blunders.

A graph should have a title to identify it, such as “Mass vs. Volume of Ball Bearings”, and it should have labels on the axes to identify the data and units (for example, “ $M$  (grams)” on the vertical axis, “ $V$  (mm<sup>3</sup>)” on the horizontal axis). Error bars  $\bar{\updownarrow}$  should be included to cover the range of uncertainty. The graph is very important in the analysis because, with some experience, the shape of the curve suggested by the data points will indicate what mathematical function relates the quantities on the graph. Here are two important functions you’ll use often:

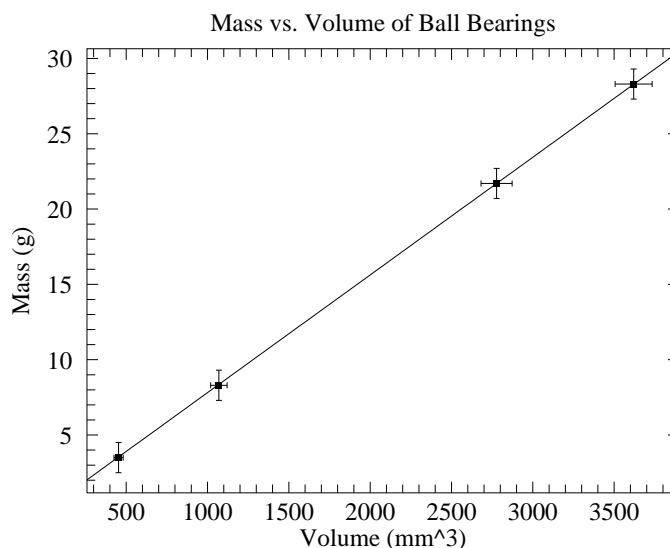


Figure 2.2: A sample graph. Note: the error bars here have been exaggerated so they are visible. Actual lab results would have error bars a factor of ten smaller and hence they would not be visible.

### a. Linear Relationship

Often a straight line could be drawn to fit the data points fairly well; we hope this will be the case with your  $M$  vs.  $V$  data, for instance. A line has the general equation

$$Y = A + B \cdot X \quad (2.8)$$

where  $X$  and  $Y$  are generic names for the horizontal and vertical coordinates, corresponding in this case to  $V$  and  $M$ , respectively. The constants  $A$  and  $B$  are the *parameters* to be determined.  $B$  represents the slope of the line and  $A$  is the  $Y$ -intercept — the point where the line crosses the  $X = 0$  vertical axis. When  $A$  and  $B$  have been determined, you can use the formula to calculate values of  $Y$  for  $X$  values you didn't measure. For a line through two points,  $(X_1, Y_1)$  and  $(X_2, Y_2)$ , the slope is the rise/run, or  $B = \frac{Y_2 - Y_1}{X_2 - X_1}$  and the intercept satisfies the formula  $B = \frac{Y_1 - A}{X_1 - 0}$ , therefore  $A = Y_1 - B \cdot X_1$ . As shown in Figure 2.3, when the points have error bars, a range of lines approximate the data, so the “best” line will have parameters with uncertainties  $\delta A$  and  $\delta B$ .

### b. Power Relationship

Suppose the experimental data fits a power law of the form

$$Y = A \cdot X^B. \quad (2.9)$$

as expected for a graph of  $M$  vs.  $D$  data. On such a curve how would you find the parameters  $A$  and  $B$ ? It turns out that by changing the scales of the graph you can

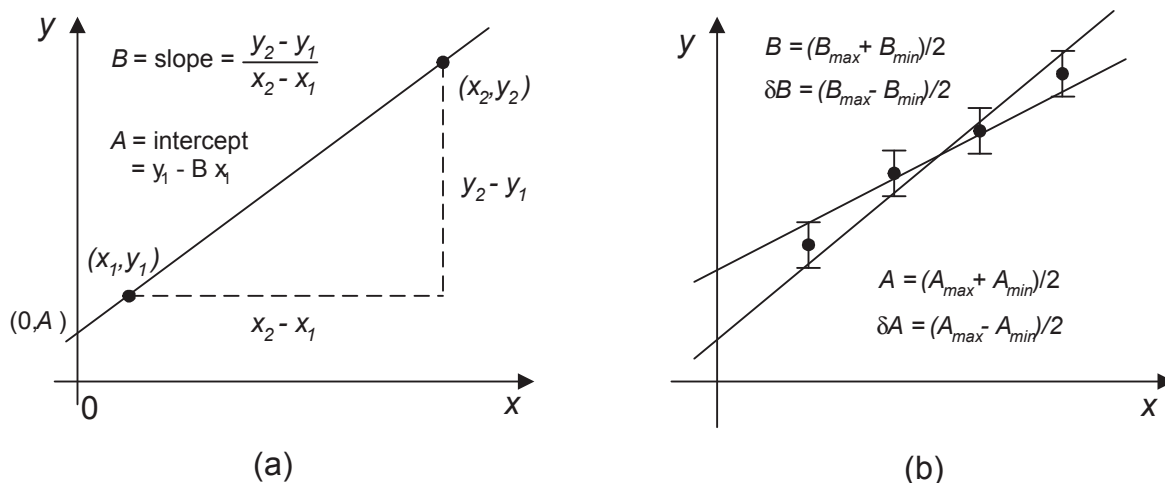


Figure 2.3: (a) Linear fit parameters. (b) Uncertainties in fit parameters

make the data fit a straight line and then use the techniques of the previous section. The change of scale is accomplished by the use of logarithms, as follows:

$$\log(Y) = \log(A \cdot X^B) \quad (2.10)$$

$$= \log(A) + \log(X^B) \quad (2.11)$$

$$\log(Y) = \log(A) + B \cdot \log(X). \quad (2.12)$$

So according to the rules of logarithms, if you take the logarithm of your  $Y$  values, and plot these versus the logarithm of your  $X$  values, you should get a straight line whose slope is  $B$  and whose intercept is  $\log(A)$ . Again, if there are more than two data points or the points have uncertainties, then the parameters will also have uncertainties  $\delta A$  and  $\delta B$ .

## Using WAPP<sup>+</sup>

A computer can calculate these fitting parameters and their uncertainties very quickly — learn how in the next section of the procedure, which makes use of a web-based program called WAPP<sup>+</sup>.

### 1. Start WAPP<sup>+</sup>:

Find WAPP<sup>+</sup> either by typing its URL

<http://www.physics.csbsju.edu/stats/WAPP2.html>

into Internet Explorer's Address Bar, or else by searching on Google for "wapp+".

You should probably bookmark it for convenience during this and future labs.

### 2. Report $y$ and $x$ uncertainties: The default setting for WAPP<sup>+</sup> is "Enter $y$ error for each data point" and "Enter $x$ error for each data point". In this lab we in

fact have separate columns for the uncertainties ( $\delta D$ ,  $\delta M$ ,  $\delta V$ ). Confirm that the above selection has been made and then click the **Do Bulk** button. A “Data Entry” page should appear.

3. Copy the numbers from Excel: Highlight the 6-column array of numbers (no column headings, just numbers) in Excel. Copy those values using your favorite method (e.g., right click, control-C, or clicking the appropriate button on the top ribbon).
4. Paste those numbers into WAPP<sup>+</sup>: Using your favorite method (right click, control-V, or clicking Paste from the Edit menu) paste those values into the large data entry box that has a background image of spreadsheet cells. There is no reason for the columns to line up with either the cell-image or the column headings. As far as the browser is concerned this is just a big lump of numbers. Don’t be concerned if the numbers you paste in do not align with anything. WAPP<sup>+</sup>’s only concern is that there is some sort of separator between numbers (space, comma, tab, new-line, etc.); it is not concerned about the column alignment visible to you.

## Linear Fit: Mass vs. Volume

1. Tell WAPP<sup>+</sup> which column is which: If you ordered your columns as shown above, volume (the  $x$  variable) is in column 5 and mass (the  $y$  variable) is in column 3. The diameter data (columns 1 and 2) should be ignored for now. Tell WAPP<sup>+</sup> this by making the column selections below:

column1	column2	column3	column4	column5	column6
Ignore	Ignore	Y	Y Errors	X	X Errors

2. Tell WAPP<sup>+</sup> which function should fit the data: Select “Linear:  $y = A + Bx$ ” (which should be the default).
3. Compute fit: click the **Submit Data** button. The resulting page includes the ‘fit report’. There is an important quantity called “reduced chi-squared” ( $\bar{\chi}^2$ ) which tells how well the line fits the data:
  - if  $\bar{\chi}^2$  is approximately 1 then the line is a good fit within the error bars;
  - if  $\bar{\chi}^2$  is much larger than 1, the line misses the error bars by too much to be considered reliable;
  - if  $\bar{\chi}^2$  is much less than 1, the line misses the points by distances that are unexpectedly small compared to the error bars — your error bars are probably larger than necessary. WAPP<sup>+</sup> may give you a warning about this, but don’t feel bad, just think about whether the error bars could have been smaller, and fix them if necessary; otherwise go on.

So if the  $\bar{\chi}^2$  value is far from 1 either direction, you ought to check your data or error estimates.

4. Print the fit report: The bottom part of the fit report page involves plotting the data and need not be printed. Rather than print the entire page, you can save some trees by highlighting (left-click sweep) the report area, down to and including the line Data Reference, and then print just the selected area. Tape this fit report into your notebook.
5. Request a plot: For this linear (normal) plot select “X Scale Options” Linear and “Y Scale Options” Linear (which should be the default). For the final version of this plot you will want to enter appropriate  $x$  axis labels,  $y$  axis labels, and plot title, but for this check-plot just leave those as the default. (If you want you may alternatively by-hand write the proper labels on hardcopy plots). Click the Make Plot button. On the following page click PDF File. If everything looks OK, hit the Internet Explorer back arrow a couple of times to return to the fit report page. Now enter appropriate  $x$  axis labels, etc. and remake a final pdf plot. Once the final plot is displayed in the browser, print out hardcopies and tape them into your notebook.

## Density

As you may know, the density  $\rho$  of a material is the ratio Mass/Volume. The slope of the  $M$  vs.  $V$  line is therefore the density  $\rho$ , in units of grams/mm<sup>3</sup> unless you used other units. Can you identify the material? Here is a short list of some metals and their densities:

Metal	$\rho$ (g/mm <sup>3</sup> )
Magnesium	$1.74 \times 10^{-3}$
Aluminum	$2.70 \times 10^{-3}$
Titanium	$4.54 \times 10^{-3}$
Chromium	$7.19 \times 10^{-3}$
Iron	$7.86 \times 10^{-3}$
Nickel	$8.90 \times 10^{-3}$
Molybdenum	$10.2 \times 10^{-3}$
Tungsten	$19.3 \times 10^{-3}$

## Power-law Fit: Mass vs. Diameter

Next, using the measured masses and diameters of the balls, try a non-linear fit.

1. Return to the data entry page: Simply hit Internet Explorer’s back arrow enough times to reach the page into which you copied your six columns of data.

2. Tell WAPP<sup>+</sup> which column is which: If you ordered your columns as shown above, diameter (the  $x$  variable) is in column 1 and mass (the  $y$  variable) is in column 3. The volume data (columns 5 and 6) should be ignored. Tell WAPP<sup>+</sup> this by making the column selections below:

column1	column2	column3	column4	column5	column6
X	X Errors	Y	Y Errors	Ignore	Ignore

3. Tell WAPP<sup>+</sup> which function should fit the data: Select “Power:  $y = Ax^B$ ”.
4. Compute fit: click the  button. The resulting page includes the fit report. Check the value of reduced chi-square ( $\bar{\chi}^2$ ). Print out the appropriate part of the page and tape it into your notebook.
5. Request a plot: Power law data should produce a straight line if plotted on Log-Log paper. For a Log-Log plot select “X Scale Options” Log and “Y Scale Options” Log. Follow the  button to the PDF File option and see what the data with fitted function looks like on Log-Log paper. If it looks good, then everything went fine. Return to the fit report page and make a normal plot by “X Scale Options” Linear and “Y Scale Options” Linear. Follow the  button to the PDF File option and see what the data with fitted function looks like on normal graph paper. Make final plots (with appropriate  $x$  and  $y$  labels) of both the Log-Log and the normal version.

## Fixed-Power Fit

Suppose WAPP<sup>+</sup> reported  $B = 3.018 \pm .003$  with a warning that  $\bar{\chi}^2$  (reduced chi-square) was unbelievably small (so the reported uncertainty is suspect). However the expected theoretical relationship between mass and diameter (Eq. 2.2):

$$M = \frac{\pi\rho}{6} D^3$$

is a power-law that requires  $B$  to exactly equal 3. Thus if we are checking the reliability of that theory, we really should be fitting to  $B \equiv 3$  not  $B$  as some freely adjustable parameter. If you look at the bottom of the WAPP<sup>+</sup> data entry page you’ll find these words:

Occasionally the value of  $B$  is set by theory to a known value. If that situation applies to this data, enter here that fixed value of  $B$ . (Usually you will leave this blank.)

$B =$



Do a fixed-power fit required by theory:

1. Return to the data entry page
2. Use WAPP<sup>+</sup> to compute a  $B = 3$  (held fixed) fit
3. Print out the resulting fit report: tape it into your notebook. A graph shouldn't be necessary for this case; it should look about the same as your power-law result.

## Conclusions and Discussion

Your lab notebook at this point should include a table of your data, your graphs, and WAPP<sup>+</sup> results, and an estimate of the density  $\rho$  of the ball material. To clarify what it all means, collect the WAPP<sup>+</sup> results for  $A$ ,  $\delta A$ ,  $B$ ,  $\delta B$  (each with appropriate units and sigfigs) together in a table such as the one below:

Data	$A$	$\delta A$	$B$	$\delta B$	$\bar{\chi}^2$
$M$ vs. $V$					

Data	$A$	$\delta A$	$B$	$\delta B$	$\bar{\chi}^2$
$M$ vs. $D$					
$M$ vs. $D$ , $B \equiv 3$			3	0	

Are the WAPP<sup>+</sup> results reasonable for the  $M$  vs.  $V$  fit? Is the value of  $\bar{\chi}^2$  reasonable?

According to Eq. (2.1) the volume of a ball is proportional to  $D^3$ , and your experiment has shown (we hope) that  $M$  is proportional to  $V$ . Thus you should expect that  $M$  is proportional to  $D^3$ :

$$M = \frac{\pi\rho}{6}D^3, \quad (2.13)$$

which was the reason it was fitted to the function

$$Y = A \cdot X^B. \quad (2.14)$$

Are your results for the  $M$  vs.  $D$  parameters consistent with this formula? That is, does your value for  $B$  with its uncertainty agree with the expected value 3? And does  $\pi\rho/6$  land within the range  $A \pm \delta A$ ? Does the  $\bar{\chi}^2$  value indicate a good fit to this kind of function?

Fill out a “Quick Report” card properly listing (with units, significant figures, and uncertainties)

1. your value for the ball-material density
2. the power-law parameter  $B$  from  $M$  vs.  $D$ .
3. the fixed-power parameter  $A$  from  $M$  vs.  $D$ .

and give it to your lab instructor. Include any other discoveries, conclusions, discussion, etc. in your lab report.

#### CHECKLIST

Pre-Lab Estimates of line paramters	
Brief outline of Procedure	
$D$ and $M$ Measurements with Uncertainties	
$V$ Calculations with Uncertainties	
Graph of $M$ vs. $V$	
WAPP <sup>+</sup> analysis of $M$ vs. $V$	
Identification of ball material using measured $\rho$	
Graphs of $M$ vs. $V$ , $M$ vs. $D$ (Log and Linear)	
WAPP <sup>+</sup> analysis of $M$ vs. $D$ as a power law	
WAPP <sup>+</sup> analysis of $M$ vs. $D$ as a fixed-power law	
Comparison of $M$ vs. $D$ parameters with Eq.(2.13)	
Conclusions, Discussion	

## 3. FREE FALL

### Pre-Lab Exercises:

1. Read through the whole experiment so you can use your time well.
2. Calculate and record in your notebook the accepted value of  $g$  in units of  $\text{cm}/\text{wink}^2$ .
3. Briefly outline in your notebook the procedure you will be following in this experiment.

### Introduction

*Time* and *distance* are concepts familiar to all of us; we have plenty of direct experience in measuring or estimating them. You probably can estimate a person's height fairly accurately in feet and inches, and you know about how many minutes it takes you to get to your classes. The concept of *speed* is probably well understood, but most of us have less direct experience with measuring or estimating speeds. Some runners develop a very good sense for their speed, but you may have to look at your speedometer to know how fast your car or bicycle is going. The concept of *acceleration*, which is of great importance in physics, is frequently misunderstood by beginning physics students. This experiment is intended to give you some insight into the relationship of acceleration to distance, time, and speed as you measure these quantities for a falling object. One of the objectives is to determine the value of  $g$ , the acceleration of an object in free fall near the surface of the Earth. You will also get some good practice in methods of analyzing data, including the use of a spreadsheet and the use of a *weighted average* to combine several uncertain estimates into a single best estimate.

### Apparatus

1. Free fall tower, spark generator, power supply, plummet, paper tape
2. Meter stick
3. PC with Excel spreadsheet and a link to the WAPP<sup>+</sup> program.

### Experiment

The experiment is done using the Behr free-fall apparatus shown schematically in Fig. 3.1. **CAUTION:** *This apparatus produces very high voltages at very low currents. It is unlikely to be dangerous, but it is possible to get a painful shock.* Be sure to follow the lab

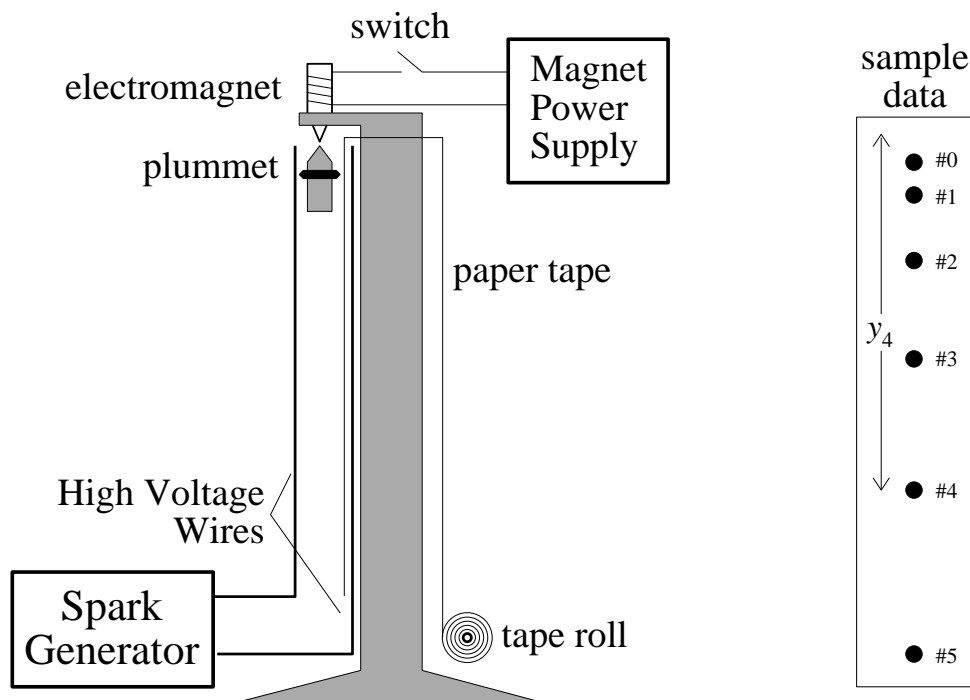


Figure 3.1: Behr Free Fall apparatus with sample data.

instructor's instructions before operating it. An electromagnet holds an iron plummet at the top of a tower; when the magnet is turned off the plummet falls. High-voltage wires produce a spark from the plummet every  $1/60$  second. The sparks make little marks on a paper tape, showing where the plummet was at intervals of  $1/60$  second. By measuring these positions you can derive the speed and acceleration of the plummet during its fall, using *WAPP*<sup>+</sup> to analyze the data.

## Theory

Newton's law of gravitation gives the force of attraction between two masses. If the Earth were a uniform, non-rotating sphere, Newton's law would give  $GM/R^2$  for the acceleration of a small mass arriving at the Earth's surface;  $G$  is the gravitational constant,  $M$  is the Earth's mass, and  $R$  is the Earth's radius. But the Earth is not spherical, its mass is not distributed uniformly, and it rotates; in addition, a falling object may be at some altitude above the ground. Any measurement of the acceleration  $g$  of a freely falling object is affected by these factors, but we expect these to be small effects, and that  $g$  should be quite close to the customary value of  $9.8 \text{ m/s}^2$ .

A measurement of  $g$  can be based on the positions of a falling object at different times. Since the motion is downward, let the  $y$  direction be positive downward. Assuming

constant acceleration  $g$ , the vertical position  $y$  at a time  $t$  is

$$y(t) = y_0 + v_0t + \frac{1}{2}gt^2 \quad (3.1)$$

measured from the starting position  $y_0$  with an initial velocity  $v_0$ . Experimental  $y$  vs.  $t$  data should correspond to this equation, and by curve-fitting you can determine the parameters  $y_0$ ,  $v_0$  and  $g$ .

The velocity  $v$  varies with time according to

$$v(t) = v_0 + gt, \quad (3.2)$$

a linear equation. Instead of measuring *instantaneous* velocities directly, *average* velocities over a time interval can be found from the  $y(t)$  data. The graph of  $v$  vs.  $t$  will have a slope of  $g$  and intercept of  $v_0$ .

## Procedure

1. Learn from the lab instructor how to use the spark apparatus so as not to alter your personality. Make a couple of trial runs (without the paper tape) to see that the plummet drops all the way without hitting the wires.
2. Make a tape and check to see that it has visible spark marks along it.
3. Circle the spark marks and number them for convenience in tabulating your data. They represent times  $t = 0, 1, 2, \dots$  in units of 1/60 second. This is obviously a more convenient time unit than the second for this experiment, so let us define 1/60 second = 1 wink. The uncertainties in  $t$  are negligibly small because the timing of the sparks is very well regulated. Note: the mark labeled ' $t = 0$ ' is not literally the location when the fall begins; not only are the positions directly adjacent to the magnet likely to be affected by residual magnetic force (so don't start data collection directly adjacent to the magnet) but additionally it is unlikely that the release occurred at an integer wink moment.
4. The repetitious calculations to come are much easier to do using a spreadsheet such as Excel.
  - (a) Start Excel. Make a table for  $t$  and  $y$  values like the one below,

$t$ (winks)	$y$ (cm)
0	$y_0$
1	$y_1$
2	$y_2$
...	...

- (b) Enter your data:  $t$  values down the first column (column **A**), and  $y$  values down column **B**. Be sure to measure the  $y$  distances from some unchanging beginning, not from the previous dot. The uncertainties associated with  $y$  will depend on your measuring device — for a meter stick marked in millimeters a conservative estimate is  $\delta y = 0.05$  cm, but with care you should be able to estimate the dot locations to a fraction of a mm. You should save your data to a disk file for safekeeping.
- (c) Now put a heading  $v(\text{cm/wink})$  on column **C** and calculate the velocities of the falling plummet. The spreadsheet makes this simple to do. The average velocity at each spark mark is the distance the plummet fell during two time intervals, divided by the time (2 winks). So at time  $t = 1$ ,  $v_1 = (y_2 - y_0)/2$ ; at  $t = 2$ ,  $v_2 = (y_3 - y_1)/2$ , etc. You can't do this at time  $t = 0$ , so leave cell **C2** blank, skip down to cell **C3** and type the formula for calculating this velocity, namely  $=(\mathbf{B4}-\mathbf{B2})/2$ . When you hit the **Enter** key, the result of the formula should appear in the cell. Does it look right? If not, get your lab instructor to check what you've done.
- (d) Now the fun part. Click on cell **C3** then the **COPY** icon (two little pages). The formula is copied. Now paste it into all the cells where you want it — from **C4** downward — by dragging the mouse to highlight these cells and clicking on the **Paste** icon. All your velocities should appear. Do they all look right? Click on any cell to see the formula used in that cell. You can't get a velocity at your first and last time points, so these cells should be left empty.

5. Analysis of  $v(t)$ :

- (a) Start  $WAPP^+$  .
- (b) Set the uncertainties for  $WAPP^+$  . Mark
- “*Enter a formula for y errors*” is probably what you want, if all your measurements were accurate to  $\pm 0.05$  cm. You will get a chance to indicate this uniform uncertainty. Incidentally, later, the  $v$  values would have the same  $\pm 0.05$  cm uncertainty because of the way they are computed from the  $y$  values [because  $(0.05 + 0.05)/2 = 0.05$ ].
  - “*No x errors*”. This is appropriate because the timing of the sparks is highly precise, and also necessary for fitting a quadratic function to the  $y(t)$  data.
- (c) Highlight and transfer the  $t$ ,  $y$  and  $v$  columns of data to  $WAPP^+$  for analysis and graphing. Don't copy your heading text, and don't copy the  $t = 0$  or final rows either (they don't have a  $v$  value;  $WAPP^+$  will not be happy).
- (d) Tell  $WAPP^+$  which columns to fit. You should first fit  $v$  vs.  $t$ , so give the columns the right labels ( $x$ , ignore,  $y$ ). Fit with a linear function  $y = A + Bx$ , in accordance with Eq.(3.2):

- $A$  here corresponds to the “initial speed”, i.e., the speed at the time you called  $t = 0$ , which occurred after the true beginning of the fall, but nevertheless should still be small.
  - $B$  here corresponds to  $g$  in units of  $\text{cm/wink}^2$ .
- (e) Do the fit, paying attention to the  $\bar{\chi}^2$  outcome and the parameters  $A$  and  $B$ . Think about what the parameters represent:  $A$  corresponds to the initial speed (is it near zero?), and  $B$  to the acceleration (is it near  $g$  in  $\text{cm/wink}^2$ ?).
  - (f) Look at the graph of the data with the fitted line. Good? Then enter labels and a title for the graph and print it out.
  - (g) Get back to the Fit Report showing the results for  $A$  and  $B$  and their uncertainties  $\delta A$  and  $\delta B$ . Print out the useful area (left-click sweep) of the screen. The  $WAPP^+$  result for  $B \pm \delta B$  gives you your first experimental value of the acceleration,  $g_1 \pm \delta g_1$ .
6. Analysis of  $y(t)$ :
- (a) Go back to the  $WAPP^+$  screen which contains your data:  $t$  in the first column,  $y$  in the second.
  - (b) Mark the columns again ( $x, y, \text{ignore}$ ) so you can fit  $y(t)$ . Make sure  $WAPP^+$  has the appropriate uncertainty for the  $y$  values.
  - (c) Choose the fitting function: As Eq. (3.1) suggests, you should select the quadratic function,  $y = A + Bx + Cx^2$ , where
    - $A$  corresponds to the initial position of the object, approximately zero.
    - $B$  corresponds to the initial velocity, also approximately zero.
    - $C$  corresponds to  $g/2$ , in units of  $\text{cm/wink}^2$ , which you can calculate.
  - (d) Do the fit, paying attention to the  $\bar{\chi}^2$  outcome, and the expected values of the parameters.
  - (e) Look at the graph of the data with the fitted curve. If it looks good, enter labels and a title for the graph and print it out.
  - (f) Print out the fit report showing results for  $A, B, C$  and their uncertainties  $\delta A, \delta B$ , and  $\delta C$ . You will need  $C \pm \delta C$  to get your second experimental value of the acceleration,  $g_2 \pm \delta g_2$ . Because  $C$  corresponds to  $g/2$ ,  $\delta C$  corresponds to  $\frac{\delta g}{2}$ .

## Conclusions and Discussion

1. At this point, your lab report should have a table of your data, the graphs of  $v(t)$  and  $y(t)$ , and fit reports for each. And, even though they came from only one set of measurements, you should have two measured values of  $g$ , probably very nearly the same but with ranges of uncertainty that may differ. Convert them both into standard units of  $\text{cm/s}^2$ . Since  $60 \text{ winks} = 1 \text{ s}$ , an acceleration in  $\text{cm/wink}^2$  must be multiplied by  $(60 \text{ wink/s})^2 = 3600 \text{ wink}^2/\text{s}^2$  to put it into these standard units.

	from $y(t)$	from $v(t)$
$g$ (cm/s <sup>2</sup> )		
$\delta g$ (cm/s <sup>2</sup> )		

Do the two  $g$  values agree with each other; that is, do their ranges of uncertainty overlap?

2. Instead of two results, reduce them down to a single best estimate of  $g$  using the method of weighted averaging. A simple average  $(g_1 + g_2)/2$  may not be so good because it treats all the values as equally trustworthy. The ones with the smallest uncertainty should be more influential.

In a weighted average, each  $g$  value gets a *weighting factor*: the smaller the uncertainty, the larger the weighting factor and the more influential that  $g$  value will be. For your results,  $g_1$  and  $g_2$ , with uncertainties  $\delta g_1$  and  $\delta g_2$ , they are:

$$w_1 = 1/(\delta g_1)^2, \text{ and } w_2 = 1/(\delta g_2)^2.$$

The weighted average is:

$$g = \frac{w_1 g_1 + w_2 g_2}{w_1 + w_2}$$

and its uncertainty is smaller than either of the  $\delta g_i$  values, namely

$$\delta g = 1/\sqrt{w_1 + w_2}$$

Perform these calculations to get a final experimental result for the local value for  $g$  and its uncertainty  $\delta g$ . Does it agree with the expected result of 980 cm/s<sup>2</sup>?

3. Fill out a “Quick Report” card for your lab instructor listing your two individual values of  $g$  and the weighted average value, all with units, significant figures, and uncertainties.
4. Get all your tables, graphs, printouts, calculations and comments organized into a nice lab report and deliver it to your TA.

#### CHECKLIST

Pre-Lab Exercise		
Brief outline of Procedure		
Tables of $t$ , $y$ , $v$ with units and uncertainties		
Graph of $v(t)$		
WAPP <sup>+</sup> analysis of $v(t)$		
Graph of $y(t)$		
WAPP <sup>+</sup> analysis of $y(t)$		
Comparison of two measured values of $g \pm \delta g$		
Weighted average result for $g$		
Conclusions, Discussion		



# 4. PROJECTILE MOTION

## Pre-Lab Exercises:

1. Calculate the time it takes a ball to fall to the ground from a height of 1 meter. Then find the horizontal distance it would fly if it was moving horizontally at 2 m/s.
2. Briefly outline in your notebook the procedure you will be following in this experiment.

## Introduction

Galileo was greatly concerned with motion of all kinds, particularly projectile motion. In his work Two New Sciences, he wrote:

On the Motion of Projectiles

PROPOSITION I. THEOREM I.

When a projectile is carried in motion compounded from equable horizontal and from naturally accelerated downward [motions], it describes a semi-parabolic line in its movement.

The purpose of this experiment is to demonstrate these conclusions about projectile motion, i.e., that it is compounded of two independent motions—a horizontal component and a vertical component, and that the path followed is parabolic. You'll also be learning some useful techniques of data analysis.

## Apparatus

1. Projectile launcher,
2. Paper and carbon paper,
3. Board on vertical stand,
4. Meter stick,
5. Tape,
6. PC with the programs Excel and *WAPP*<sup>+</sup>.

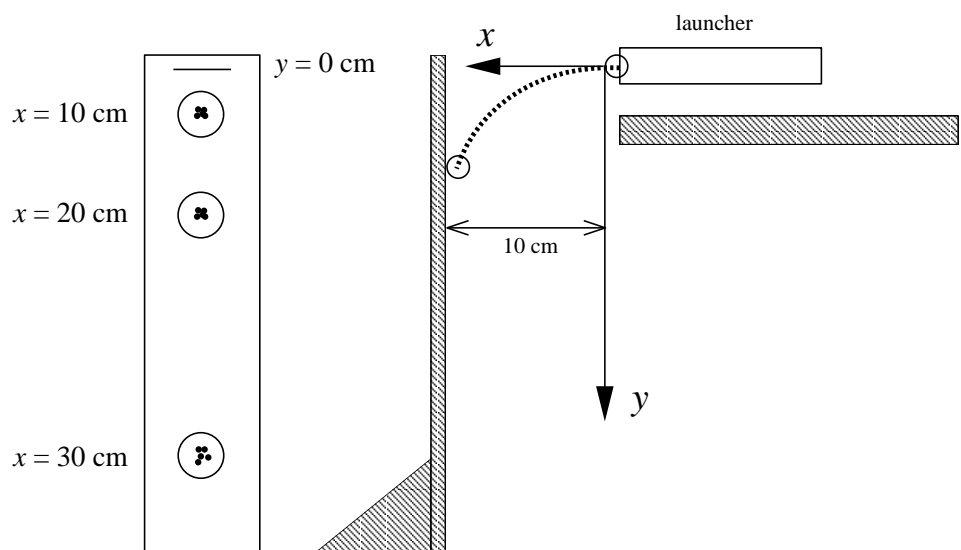


Figure 4.1: Projectile motion apparatus.

## Experiment

The apparatus for this experiment is simple but can yield reasonably good data if careful measurements are made. Essentially, you will be launching ball bearings as projectiles, and measuring their travel in the vertical and horizontal directions. A spring-loaded launcher will shoot the ball horizontally off a table. Carbon paper marks the ball's point of impact on a vertical board. The horizontal distance  $x$  it travels is controlled by moving the board further away, and the vertical distance  $y$  it travels will be measured. The relationship between these distances determines the shape of the ball's flight path.

## Theory

The important equations should be familiar by now. If the projectile has an initial velocity  $v_{0x}$  in the  $x$  direction, and is in free-fall in the  $y$  direction,

$$x = v_{0x}t \quad (4.1)$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}gt^2 \quad (4.2)$$

where  $t$  is the time for the ball to fall distance  $y$  and move distance  $x$  horizontally. Solving Eq. (4.2) for  $t$  we obtain  $\pm\sqrt{\frac{2y}{g}}$ , and the negative solution is not relevant here, so the time of flight is

$$t = \sqrt{\frac{2y}{g}}. \quad (4.3)$$

Substituting  $t$  into Eq. (4.2) we can write the horizontal velocity

$$v_{0x} = \frac{x}{t} = x \sqrt{\frac{g}{2y}} \quad (4.4)$$

and the relationship of  $y$  to  $x$  can be written in the form

$$y = \frac{g}{2v_{0x}^2} x^2 \quad (4.5)$$

This is the equation of a parabola with its vertex at the origin. You will be testing whether the horizontal velocity does in fact remain constant by determining whether the path is parabolic.

## Procedure

1. Examine the apparatus to find out how it works. The ball is to be launched horizontally, and hit the board at some point. A level is available for you to make the board vertical, and the launcher horizontal.
2. Load the ball into the launcher and push it in with a stick or pencil (you won't want your finger stuck in there) until it "clicks" one time.
3. Attach white paper to the board and then a layer of carbon paper so the carbon is facing the white paper and will leave a mark where the ball hits it. Position the board at the end of the launcher and fire the ball to get a mark corresponding to  $y = 0$ . Then move the board 10 cm away from the end of the launcher; this means your first horizontal distance is  $x = 10$  cm. Try to give a reasonable estimate of the uncertainty  $\delta x$  too. Now fire the ball three times, and it will leave three carbon marks on the paper. You can peek under the carbon to be sure there are three identifiable marks.
4. Continue these measurements for horizontal distances of 20, 30, 40, ..., 100 cm. Move your wall to a table top, and remove the carbon paper. Make a table of your  $x$  and  $y$  data with their uncertainties. For each trial, estimate (by eye) the average vertical ( $y$ ) location of the ball hits and accurately measure the corresponding fall distance  $y$ . Take half the distance between the outermost points as an estimate of  $\delta y$ . It's a good idea to graph the data in your notebook as you go, so you can tell immediately if a point seems out of place and check your measurements.
5. Since there are some repetitive calculations to be done, open the Excel spreadsheet program; that way you'll only have to type each formula once and copy it. You may want to set up the spreadsheet columns like this:

$x$ (cm)	$\delta x$ (cm)	$y$ (cm)	$\delta y$ (cm)	$v_{0x}$ (cm/s)
10				
20				
...				
100				

Use Eq. (4.4) to calculate the  $v_{0x}$  values in each row; as a spreadsheet function it will look something like: `= A1*SQRT(980/2/C1)` (be sure you understand whatever you do here; the 980 assumes units of  $\text{cm/s}^2$ ). It is expected that the velocity will turn out fairly constant; if it doesn't, consult your lab instructor for help. Do your values show any systematic trend, or do they appear to be randomly varying? The values of  $v_{0x}$  that result from smaller values of  $x$  and  $y$  are probably the most uncertain and may differ significantly from the others.

6. Also use the spreadsheet to calculate the average value of  $v_{0x}$  by placing a formula “`= AVERAGE(E2:E11)`” (or whatever cells are appropriate) somewhere in the spreadsheet. Then compute the standard deviation of the mean using “`= STDEV(E2:E11)/SQRT(10)`”. This would give you a best estimate of  $v_{0x}$  and its uncertainty. Remember to self-document your spreadsheet!
7. Now analyze the experimental data using *WAPP+*.
  - (a) Your data probably has significant uncertainties in both  $x$  and  $y$ , so use this default setting on the *WAPP+* screen.
  - (b) Copy and paste your columns of data into *WAPP+*.
  - (c) Identify which column is which: probably (x, x errors, y, y errors, ignore).
  - (d) Select a power-law fit:  $y = Ax^B$  in accordance with Eq. (4.5). The first thing to check is whether  $y(x)$  is a parabola, which would have  $B = 2$ . When you have fitted this function, check whether  $B \pm \delta B$  is near 2, and whether  $\bar{\chi}^2$  is reasonably good (not too far from 1). Record these results, or ask your instructor for help if things don't look right.
  - (e) If the data appear to fit a parabola, then set  $B = 2$  on *WAPP+* and re-do the power-law fit. Again, examine the Fit Report results to be sure that  $\bar{\chi}^2$  is reasonably good (not too far from 1). Print the useful area of the Fit Report screen.
  - (f) Make and print a nice graph of the  $y(x)$  data with the fitted curve.
  - (g) Now the value of  $A$  will be of interest, because according to Eq. (4.5) it should correspond to  $g/(2v_{0x}^2)$ , hence an estimate of  $v_{0x}$  can be obtained from  $\sqrt{g/2A}$ . And because  $A$  is uncertain, your calculated  $v_{0x}$  will have a corresponding uncertainty:

$$\delta v_{0x} = v_{0x} \frac{1}{2} \frac{\delta A}{A}. \quad (4.6)$$

So calculate your estimate of  $v_{0x} \pm \delta v_{0x}$  based on your *WAPP+* results. Does it agree fairly well with the value calculated from your table of data? It should be reasonably close, but because *WAPP+* has taken into account the uncertainties of your data points, this result should be a more accurate estimate.

8. Fill out a “Quick Report” card for your lab instructor, listing your calculated  $v_{0x}$ , and your *WAPP+* parameters  $A$  and  $B$ , all with proper significant figures, units and uncertainties.

9. Finish your lab report and hand it in to your TA.

## CHECKLIST

Pre-Lab Exercise	
Brief outline of procedure	
Table of $x, y$ data with uncertainties	
Calculations of $v_{0x}$	
Comments on constancy of $v_{0x}$	
WAPP <sup>+</sup> analysis of $y(x)$	
Graph of $y(x)$	
Comparison of $B$ with expected value 2	
Results of $v_{0x} \pm \delta v_{0x}$ calculated from $A \pm \delta A$	
Conclusions, Discussion	



# 5. KINETIC FRICTION

## Pre-Lab Exercises:

1. In your lab notebook do the algebra to derive Eq. (5.2) from Eq. (5.1).
2. In a few words, describe what is meant by the coefficients of static and kinetic friction.
3. Briefly outline the procedure for this experiment in your notebook.

## Introduction

The force of friction opposes the slipping of surfaces that are in contact, and the amount of friction depends on the nature of the surfaces. This experiment is to allow you to study some of the basic factors that do or do not influence the amount of friction. You will also get experience using a computer as a measuring device.

## Apparatus

1. Pasco interface box and computer,
2. Smart Pulley (the pulley with wire attached),
3. Table clamp,
4. Mass and hanger set,
5. Blocks with hooks,
6. String,
7. Digital balance (somewhere in the lab),
8. Students should bring a scientific calculator.

## Experiment

The force of friction can be determined by measuring the acceleration of a body when all the other forces are known. You will be doing this for a block of some material as it accelerates across the table top, pulled forward by a hanging mass, and with friction opposing the motion of the block. Fig. 5.1 shows the experimental setup. The string

connecting the hanging mass to the wooden block turns a pulley as the block moves. The computer keeps track of the pulley's rotations and quickly calculates the block's displacement, velocity, and acceleration. You will be making use of the acceleration to compute the coefficient of kinetic friction  $\mu_k$  of the two sliding surfaces. You can change the area of contact between the surfaces by turning the block on edge to study whether this affects  $\mu_k$ .

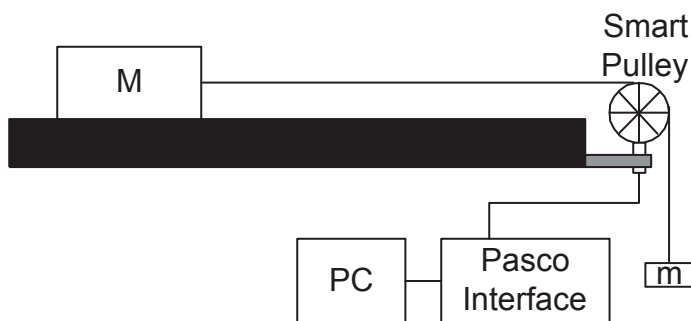


Figure 5.1: Kinetic Friction Apparatus

## Theory

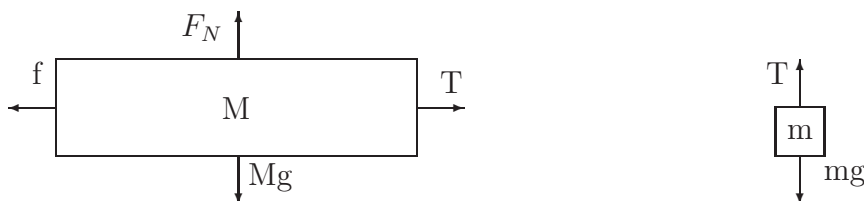


Figure 5.2: Free-body diagrams.

Figure 5.2 is a free-body diagram of the sliding block, showing the force  $T$  exerted by the string, the weight  $Mg$  of the block, the normal force  $F_N$  of the table acting on the block, and the frictional force  $f = \mu_k F_N$ .

The vertical forces must cancel, so  $F_N = Mg$ . Newton's second law gives

$$T - \mu_k Mg = Ma \quad (5.1)$$

The hanging mass  $m$  is also shown in the free-body diagram, with tension  $T$  upward and the weight  $mg$  downward. Since it accelerates downward,  $mg - T = ma$ , hence  $T = m(g - a)$ . Substituting for  $T$  in Eq. (5.1) we can solve for  $\mu_k$ :

$$\mu_k = \frac{m}{M} - \frac{a}{g} \left(1 + \frac{m}{M}\right). \quad (5.2)$$

From this equation we can calculate  $\mu_k$  for each measured  $a$  if  $m$ ,  $M$  and  $g$  are known.



## Procedure

1. Set up the experiment as shown in Fig. 5.1, checking to see that the string pulls horizontally on the block; raise or lower the pulley as needed. Plug the Smart Pulley into the Pasco Interface box, make sure the interface box is turned on, and then turn on the computer.
2. At this point you're ready to run the *Data Studio* program. Open the file named **kin\_fr.ds** When it starts running you'll see a tool bar with four buttons and a digital timer. The **Start** button will be used to start and stop your measurements. There will also be a graph display. Displays, whether they are graphs or tables, will feature a number of buttons or icons. If you place the mouse cursor temporarily on each of the icons, brief descriptions will be displayed that describe their functions. Pay particular attention to the icons used for statistics, zooming in or out, and the smart tool which displays coordinates of selected data points. In any given experiment the particular tools that prove useful may be different.
3. Now try it. Place enough mass on the hanger such that the block will slide on the table without needing any initial push.
4. Pull the block ( $M$ ) back until the mass ( $m$ ) is raised to the pulley. Hold the block at rest until you have clicked on the **Start** button, then release the block. Hit **Stop** when the block stops. Your first data set, **Run #2** (because Run #1 is always blank), will be used to calculate a mean acceleration. The value of the mean will be displayed in the box near the top of the graph. The number of data points (the "count") will also be displayed.
5. In case there are oddities at the beginning or end of the data, you can select a smaller portion of the data by holding down the left mouse button while dragging the cursor to enclose the desired data points on the graph. A new mean and count will appear for this group of data points. You only need to record the mean acceleration for each data run.

When you make subsequent data runs, a new curve will be plotted with the old curve(s), but only the new data will be used to calculate the new mean.

Helpful Hint: To rid the graph of the clutter caused by previous curves, click on the **Data** button at the top of the graph display prior to making a new run, and select **No Data**. This will erase the old data from the graph but keep it saved if you want to call it back at a later time. To recall old runs, click the **Data** button and select the appropriate run.

6. Make a table for calculating  $\mu_k$  from Eq. (5.2), based on different sets of  $M$  and  $m$ , with the different resulting  $a$  values. An Excel spreadsheet might be set up something like the following to guide you through the data-taking and calculations:

$M$ (g)	$m$ (grams)	$\bar{a}$ (m/s <sup>2</sup> )	$\mu_k$		
$M_1$	$m_1$	...	...	Extra masses	
$M_1$	$m_2$	...	...	Large Side	
$M_1$	$m_3$	...	...	Average	SDOM
$M_1$	$m_4$	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
$M_1$	$m_5$	...	...		

$M$ (g)	$m$ (grams)	$\bar{a}$ (m/s <sup>2</sup> )	$\mu_k$		
$M_2$	$m_1$	...	...	No extra masses	
$M_2$	$m_2$	...	...	Large Side	
$M_2$	$m_3$	...	...	Average	SDOM
$M_2$	$m_4$	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
$M_2$	$m_5$	...	...		

$M$ (g)	$m$ (grams)	$\bar{a}$ (m/s <sup>2</sup> )	$\mu_k$		
$M_2$	$m_1$	...	...	No extra masses	
$M_2$	$m_2$	...	...	Small Side	
$M_2$	$m_3$	...	...	Average	SDOM
$M_2$	$m_4$	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
$M_2$	$m_5$	...	...		

$M$ (g)	$m$ (grams)	$\bar{a}$ (m/s <sup>2</sup> )	$\mu_k$		
$M_1$	$m_1$	...	...	Extra masses	
$M_1$	$m_2$	...	...	Small Side	
$M_1$	$m_3$	...	...	Average	SDOM
$M_1$	$m_4$	...	...	$\overline{\mu_k}$	$\sigma_{\overline{\mu_k}}$
$M_1$	$m_5$	...	...		

As outlined in the table, you should do 20 trials:

- Start with the wood block and some additional masses (total mass  $M_1$ ), with the block lying on its large face, and using 5 different hanging masses  $m$ .
- Next, remove the additional masses and just test the block (mass  $M_2$ ) using your 5  $m$  values (use the same  $m$  values as before).
- Third, turn the block (mass  $M_2$ ) onto a side with a smaller area for 5 more runs (**make sure to correct the pulley height to keep the string horizontal, as required in the theory**).
- Finally, replace the extra masses (total mass  $M_1$ ) with the block lying on the small face.

Doing all of these different arrangements will help you decide whether mass, surface area, or velocity affects the kinetic friction coefficient.

7. The  $\mu_k$  values should come out reasonably close to each other. Find the mean  $\overline{\mu_k}$  and the standard deviation of the mean  $\sigma_{\overline{\mu_k}}$  for each of the four groups of

data. You can calculate the mean of a list of numbers by placing in a cell the formula = **AVERAGE**( ); inside the parentheses goes the list of cells containing the numbers to be averaged, such as **D2:D6** or whatever (you can also do this by dragging the mouse to highlight these cells). The standard deviation of the mean of these values,  $\sigma_{\overline{\mu_k}}$ , would be obtained by placing in some cell the formula = **STDEV(D2:D6)/SQRT(5)**. Print out a copy of your spreadsheet results for in your lab report.

8. Finally, get an estimate of the STATIC coefficient of friction  $\mu_s$  using the materials provided. According to the free body diagram Fig. 5.2,  $f = \mu_s Mg = mg$  as long as  $M$  hasn't begun to move. Find the largest  $m$  for which  $M$  remains at rest, and use it to compute  $\mu_s$ . (Note: it is easiest to begin with too much mass on the block and remove it gradually until motion begins.)
9. In your lab report summarize your results and comment briefly on them:
  - Did the coefficient of friction appear to vary with surface area?
  - Did it vary with the mass of the block (compare results for  $M_1$  and  $M_2$ )?
  - Did it vary with the speed of the block (compare results for some  $M$  when pulled by large  $m$  and small  $m$ )?
  - Is it a reasonable assumption that the coefficient of kinetic friction is constant?
  - Is  $\mu_s$  higher or lower than  $\mu_k$ ?
10. Fill out a "Quick Report" card for your lab instructor, listing your four results for  $\mu_k$  and your estimate of  $\mu_s$ , all with uncertainties and significant figures.
11. Turn in your lab report to your TA.

## CHECKLIST

Pre-Lab Exercise	
Brief outline of Procedure	
Table of $M, m, a, \mu_k$ with units	
Calculation of average and standard deviation of the mean of $\mu_k$	
Measurement of static coefficient of friction $\mu_s$	
Summary of results	
Conclusions, Discussion	



# 6. BALLISTIC PENDULUM

## Pre-Lab Exercises:

1. In your notebook, do the algebra to obtain Eqs. (6.5), (6.7), and (6.8). Be ready to show all this to your lab instructor.
2. Briefly outline the Procedure in your notebook.

## Introduction

In any collision the total momentum of the colliding bodies is unchanged. In the study of collisions, two extremes are generally defined. A *perfectly elastic* collision is one in which the total kinetic energy of the system doesn't change. Collisions of microscopic objects such as atoms, for example, may fit into this category. Other collisions are inelastic. A *perfectly inelastic* collision is one in which the colliding objects stick together. In this experiment you'll be using a device known as a ballistic pendulum to study inelastic collisions and to measure the speed of a projectile. You will also get practice with some basic statistical analysis techniques.

## Apparatus

1. Projectile launcher, steel ball;
2. Ball catcher suspended from a supporting rod;
3. Plastic rail with tinfoil marker;
4. Tape measure and scale (somewhere in the lab);
5. Plumb line;
6. Meter stick.

## Experiment

In this experiment you'll be shooting a steel ball horizontally from a launcher, as illustrated in Fig. 6.1. The ball will be caught by a container suspended as a pendulum on strings. By measuring the distance the pendulum swings you determine the ball's initial speed  $v$ . With this information you should also be able to determine how far the launcher can shoot the ball.

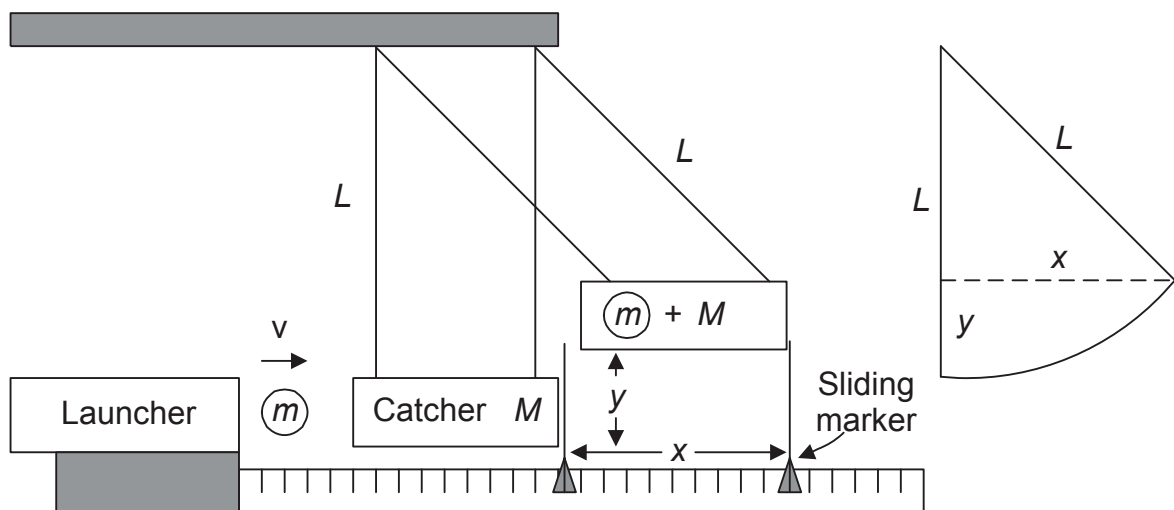


Figure 6.1: Ballistic pendulum experiment.

## Theory

When the ball collides with the catcher, the forces they exert on each other are opposite in direction but equal in magnitude, by Newton's third law. As a result, the momentum  $\mathbf{p}_i$  before the collision equals  $\mathbf{p}_f$ , the momentum after the collision. In equation form,

$$\mathbf{p}_i = m\mathbf{v} = \mathbf{p}_f = (m + M)\mathbf{V}, \quad (6.1)$$

so the ball's initial speed  $v$  is related to the speed  $V$  of the ball and catcher after the collision by

$$v = \frac{m + M}{m}V, \quad (6.2)$$

where  $m$  is the ball's mass and  $M$  is the catcher mass. In principle you could measure the masses and  $V$  to determine  $v$ . But there is another way. Since the impact of the ball with the catcher is inelastic the kinetic energy of the system is not conserved in the collision. But after the collision, the swinging motion of the catcher is nearly frictionless, so mechanical energy is nearly conserved. The pendulum's kinetic energy changes to gravitational potential energy as it swings, rising a distance  $y$  so that

$$\frac{1}{2}(m + M)V^2 = (m + M)gy. \quad (6.3)$$

Solving for  $V$  in this expression gives

$$V = \sqrt{2gy} \quad (6.4)$$

Combining Eqs. (6.4) and (6.2), the initial speed of the ball is related to the height  $y$ :

$$v = \left(1 + \frac{M}{m}\right) \sqrt{2gy}. \quad (6.5)$$

Instead of measuring  $y$  directly, it is simpler to measure the horizontal distance  $x$  the pendulum swings. From Fig. 6.1, it is clear that

$$L^2 = x^2 + (L - y)^2 = y^2 - 2Ly + L^2 + x^2, \quad (6.6)$$

which is a quadratic equation for  $y$  in terms of  $x$  and  $L$ , with the solutions

$$y = L \pm \sqrt{L^2 - x^2}. \quad (6.7)$$

The minus sign gives the solution which is relevant here. (Can you explain what the other one means?) So by measuring  $x$ , the initial speed  $v$  of the ball can be calculated.

Mechanical energy will be lost in *any* non-elastic collision. In the *perfectly* inelastic case, writing the ratio of kinetic energies before ( $\frac{1}{2}mv^2$ ) and after [ $\frac{1}{2}(m + M)V^2$ ] the collision, and then using Eq. (6.1), it is easy to obtain the ratio

$$\frac{E_{before}}{E_{after}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}(m + M)V^2} = \frac{m + M}{m}. \quad (6.8)$$

Your measured quantities,  $x$ ,  $L$ ,  $m$ , and  $M$  all have some uncertainty about them, leading to uncertainty in the calculated  $v$ . The formulas connecting  $v$  with these variables would make the error analysis very complicated. But a major source of uncertainty is the behavior of the launcher, and this is not possible to address in the theory. You will find variations in  $x$  partly because  $v$  in fact varies from shot to shot. The best way to handle this is simply to fire the ball repeatedly, and use statistical analysis to find the best estimate for  $v$  and its uncertainty; this will be outlined in the Procedure section.

## Procedure

1. Be sure that the launcher on your table is mounted so that it shoots horizontally. Get it positioned very close to the catcher. The launcher and catcher should be aligned along the same direction. The catcher should be at the same level as the launcher, and both should be level. Check with your lab instructor if you don't see how to do this.
2. Use the digital balance to measure the ball mass  $m$ . The catcher mass  $M$  should be labeled on it. Use the tape measure to measure the string length  $L$ . Record these values in your notebook.

3. Notice that there are three settings for the launch speed — it is necessary to push the ball in until the third click, so that it can shoot past the end of the ruler later in the experiment. Push the ball into the launcher with a pen or pencil and listen for three clicks. Take some practice shots to make sure everything works correctly, each time removing the ball from the catcher using a pen or pencil. Make sure the tinfoil marker slides with hardly any resistance on the rail. Make sure the catcher swings upward without rotating. Stop the catcher from hitting the launcher when it swings back. If there are problems check with your lab instructor. When everything is ready load the launcher again.
4. Now carefully find the zero position  $x_0$  of the tinfoil marker by pushing it against the catcher. Take an initial measurement of  $x$  by firing. The new position minus the original position of the marker indicates the distance  $x$  moved. For subsequent measurements, start with the marker about 1 cm short of the expected final position — this will reduce the effects of the small amount of friction.
5. Start recording the distances  $x$  that the tinfoil marker moves with each shot. Carefully put the marker back about 1 cm each time. Make a list of the  $x$  values. Record  $x$  for 5 good (smooth, no-twist) shots. In making your Excel spreadsheet table, recall the functions: `SQRT()` (square root), `AVERAGE()` (mean or average), and `STDEV()` (standard deviation). A sample data table could look like the following example:

Trial	$x$	$y = L - \sqrt{L^2 - x^2}$	$v = \left(1 + \frac{M}{m}\right) \sqrt{2gy}$
1			
2			
...			
5			
Average speed			$\bar{v}$
Standard Deviation			$\sigma_v$
Standard Deviation of Mean			$\sigma_{\bar{v}}$

6. Add a column to your spreadsheet for computing  $y$  and then  $v$ , using Eqs. (6.7) and (6.5) as indicated in the table above, along with your values of  $m$ ,  $M$ ,  $L$ , and  $g$ . You can place any of these quantities in spreadsheet cells too if you wish, and use their fixed cell addresses (such as **\$A\$15** or whatever) in your formula rather than the numbers themselves. Remember to pay attention to the units of  $g$ . Get the formula right once, then copy it to the other cells in a column.
7. Now you should have 5 measurements of  $v$ . What is the best estimate of  $v$ ? The best we can do is to calculate the average, or mean, of all the speeds. So on your spreadsheet, add a formula to the cell at the bottom of the  $v$  data to compute the average,  $\bar{v}$ , of all 5 measurements, something like `=AVERAGE(D2:D6)`. What uncertainty should be attached to this mean? It turns out that the best estimate is the so-called “standard deviation of the mean,”  $\sigma_{\bar{v}}$ . Use the spreadsheet to do this too, by placing in some cell the formula `=STDEV(D2:D6)/SQRT(5)`. Self-document please!



8. Print a copy of the spreadsheet with calculated mean and SDOM of  $v$  for your notebook.
9. A good way to verify these measurements of  $v$  is to predict the range  $R$  of the ball, the distance to the place it hits the floor, according to the equations of projectile motion. Review the physics that yielded Eq. (4.5) which relates the vertical and horizontal distances traveled by a projectile. Show that in this case the range will be

$$R = v\sqrt{\frac{2h}{g}}, \quad (6.9)$$

where  $h$  is the initial height of the ball above the floor. Position the launcher at the edge of the table, measure  $h$  and calculate the expected limits of  $R$ . **DO NOT REMOVE THE PLASTIC RAIL FROM THE LAUNCHER.**

10. Measure and mark on the floor the region where you expect the ball to land. (A plumb line may be helpful for determining where to measure from on the floor.) Don't shoot until you notify your lab instructor when you are ready to test your prediction.
11. In your lab report, compare the kinetic energy before the collision ( $KE_{before} = \frac{1}{2}mv^2$ ) with the potential energy at the top of the swing:  $PE = (m + M)gy$ . Do this using your  $m$  and  $M$  values, with some pair of  $v$  and  $y$  values in your table of data. Do the energies differ? What do you think accounts for the difference? Check whether the ratio  $E_{before}/E_{after}$  turns out approximately to equal  $(M + m)/m$  as predicted by Eq. (6.8). What does this suggest about the collision?
12. Make any other comments on collisions, the ballistic pendulum, the equipment, etc.
13. Fill out a "Quick Report" card with your value for  $v$  (units, significant figures, uncertainty), and whether you hit the mark on your first try.
14. Get your lab report finished and turn it in.

## CHECKLIST

Pre-Lab Exercise	
Brief outline of Procedure	
Measurements of $m, M, L$	
Spreadsheet calculations of $v, \bar{v}, \sigma_{\bar{v}}$	
Range calculation and success of prediction	
Comparison of energies before and after collision, and comments	
Conclusions, Discussion	



# 7. ROTATIONAL MOTION

## Pre-Lab Exercises:

Calculate the moments of inertia  $\mathcal{I}$  for the following objects:

1. A disk of diameter = 12.7 cm, mass  $M = 1356$  g.
2. A solid bar of length  $L = 12.7$  cm, square cross-section of width  $w = 2.25$  cm, and mass  $M = 489$  g.

## Introduction

In this experiment you will gain some familiarity with rotational motion by working with the rotational form of Newton's second law. The well-known quantities used to describe linear motion have analogous quantities related to rotation, some of which are listed in the table below.

LINEAR MOTION	ROTATIONAL MOTION
Displacement $x$	Angular displacement $\theta$
Velocity $v = \Delta x / \Delta t$	Angular velocity $\omega = \Delta \theta / \Delta t$
Acceleration $a = \Delta v / \Delta t$	Angular acceleration $\alpha = \Delta \omega / \Delta t$
Force $F$	Torque $\tau$
mass $m$	Moment of inertia $I$

In the 19<sup>th</sup> century physicists were careful to distinguish two potentially different types of mass: inertial mass (which could be measured by finding the acceleration caused by a force:  $m = F/a$ ) and a gravitational mass (which was defined by the force of gravity on the object, i.e., the way most every balance measures mass). Experimentally<sup>3</sup> the two seemed to be identical. In 1908 Einstein postulated his Equivalence Principle which states these that two masses are exactly identical, so as a result today we just talk about the mass.

Analogously we can distinguish two different types of moment of inertia. We will denote with  $I$  the moment of inertia measured by finding the angular acceleration caused by a torque:  $I = \tau/\alpha$ . We will denote with  $\mathcal{I}$  the moment of inertia calculated from the mass and shape of the object. In this lab you will see if these two things seem to be identical.

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<sup>3</sup>According to Wiki: the earliest experiments were done by Isaac Newton and improved upon by Friedrich Wilhelm Bessel. Roland von Eötvös famously carried out extremely accurate experiments starting around 1885.

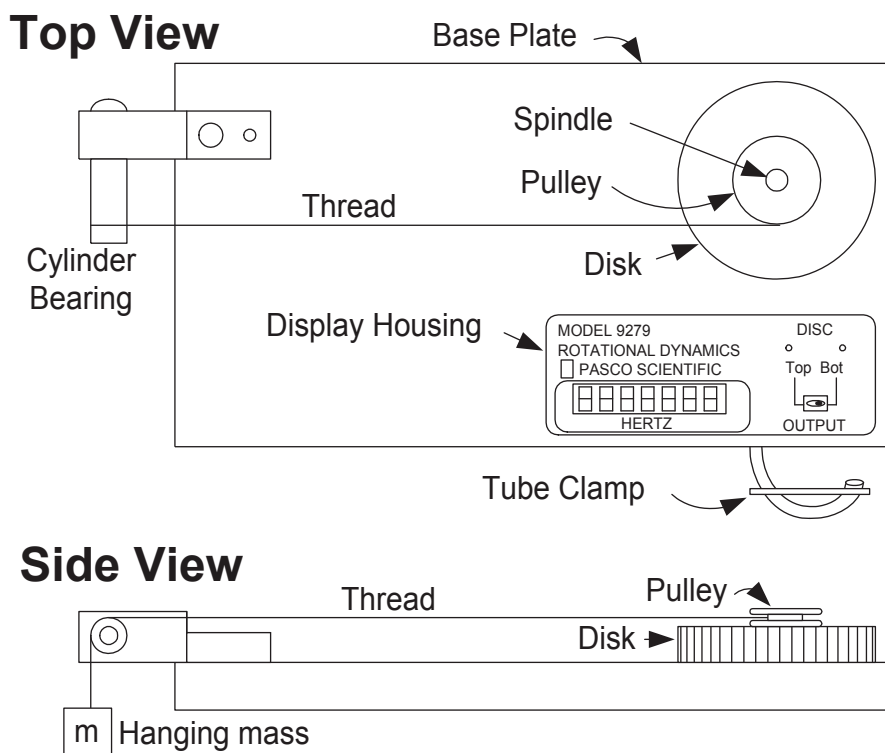


Figure 7.1: Rotational motion apparatus

## Apparatus

1. Pasco rotation apparatus, interface box and PC
2. Compressed air source with pressure control valve,
3. Digital caliper (recall its uncertainty is 0.02 mm) and digital balance (somewhere in the lab).

## Experiment

In this experiment you will be applying a torque  $\tau$  to a steel disk to get it to rotate. Friction is nearly eliminated by a cushion of air supporting the moving disk, and the resulting angular acceleration  $\alpha$  is measured by a computer attached to the apparatus. By relating  $\tau$  and  $\alpha$  you find the moment of inertia of the rotating disk, here called  $I$ . This will be compared with a value  $\mathcal{I}(\text{disk})$  calculated from the mass and radius of the disk. If all goes well, the two values should agree. Next, a rectangular metal bar of mass  $M$ , length  $L$  and width  $w$  will be rotated; both  $I$  (based on angular acceleration) and  $\mathcal{I}(\text{bar})$  (based on shape) will be determined. Again they should agree. The apparatus is illustrated in Fig. 7.1. A mass hanging on a string from a pulley provides the torque. The

first object to be rotated is a steel disk. The disk circumference is marked with stripes which the computer detects as the disk rotates, so the angular speed can be calculated. Measurements of the mass and radius of the disk will be used to get an independent estimate of the moment of inertia for comparison. The process will be repeated with a metal bar.

## Theory

According to Newton's second law, a net force  $F$  acting on a mass  $m$  gives it an acceleration  $a$ , where

$$F = ma \quad (7.1)$$

The analogous form of this law for rotational motion says that a net torque  $\tau$  acting on a body gives it an angular acceleration  $\alpha$  such that

$$\tau = I\alpha, \quad (7.2)$$

where  $I$  is the moment of inertia for the body. So if  $\tau$  and  $\alpha$  can be measured,  $I$  can be calculated from these measurements.

With a mass  $m$  hanging on the string in Fig. 7.1, the tension  $T$  in the string satisfies:

$$mg - T = ma = mr\alpha, \quad (7.3)$$

since the acceleration  $a$  and the angular acceleration of the pulley are related by  $a = r\alpha$ , where  $r$  is the inner radius of the pulley where the string winds. Thus  $T = m(g - r\alpha)$ . The torque on the rotating disk is then

$$\tau = Tr = m(g - r\alpha)r \quad (7.4)$$

So if you measure  $m$ ,  $r$  and  $\alpha$  you can get  $I$  from Eq. (7.2):

$$I = \frac{mgr}{\alpha} - mr^2. \quad (7.5)$$

Alternatively a moment of inertia  $\mathcal{I}$  can be calculated from the mass and shape of the object. Tables of  $\mathcal{I}$  for simple shapes are found in most physics texts, with diagrams such as in Fig. 7.2. For a solid disk with the rotation axis through its center, the inertia depends on the mass  $M$  and radius  $R$  of the disk, as follows:

$$\mathcal{I}(\text{disk}) = \frac{1}{2}MR^2. \quad (7.6)$$

For a rectangular bar rotated around an axis through its center as shown the moment of inertia is

$$\mathcal{I}(\text{bar}) = \frac{1}{12}M(L^2 + w^2), \quad (7.7)$$

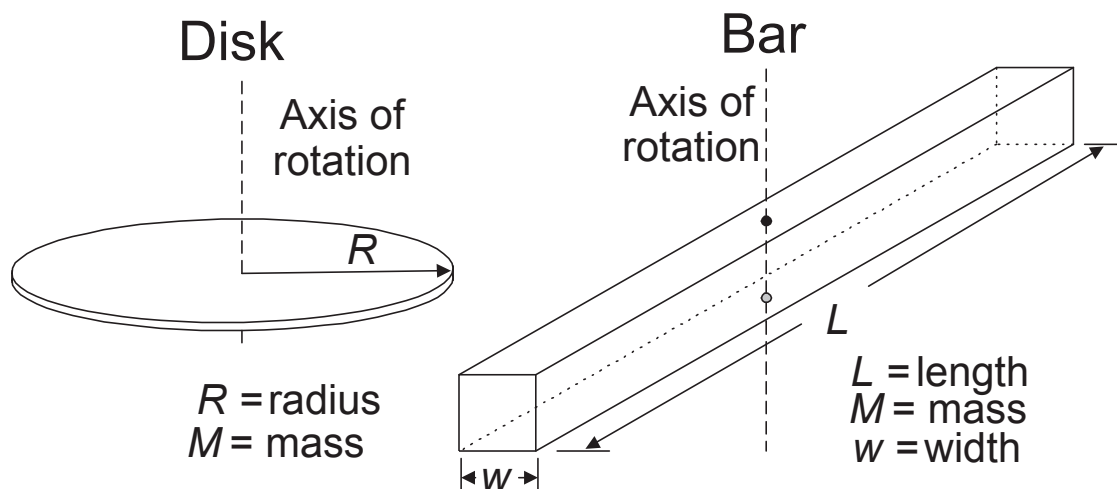


Figure 7.2: A solid disk and a solid bar rotated about axes through their centers.

where  $M$  is the mass of the bar, and  $L$  and  $w$  are its length and width.

The uncertainty of your  $\mathcal{I}(\text{disk})$  and  $\mathcal{I}(\text{bar})$  values depends on the uncertainties of your mass and shape measurements. Use the high-low method to determine these uncertainties. For the disk, calculate the highest value and the lowest value of  $\mathcal{I}(\text{disk})$  from Eq. (7.6). For the bar, calculate the highest and lowest values of  $\mathcal{I}(\text{bar})$  using Eq. (7.7). Half the difference between the high and low values will be your estimate for uncertainty.

The uncertainty in  $I$  depends strongly on the uncertainty in  $\alpha$ , which is not simple to evaluate; the best way to estimate  $\delta I$  will be to take the standard deviation of the mean of several  $I$  measurements.

Including appropriate uncertainties with your measured values, your  $\mathcal{I}(\text{disk})$  and  $I(\text{disk})$  ought to agree. Similarly, when the bar is used, the expected value of the inertia,  $\mathcal{I}(\text{bar})$  should agree with its measured inertia  $I(\text{bar})$ .

## Procedure

1. The apparatus is delicate, and must be treated properly. The steel disk is designed to rotate on a thin cushion of compressed air that flows underneath it and along the spindle. *DO NOT remove the disk from the spindle, and DO NOT rotate the disk unless the air is turned on at a minimum pressure of 9 psi (some units require 12 psi).*
2. Use the bubble level to make sure the disk is level; if it needs leveling you can adjust it yourself or ask your lab instructor for help.
3. Next, measure and record the radius  $r$  of the pulley where the string winds (with uncertainty estimates!). You will need the radius to calculate the applied torques.

4. The pulley should attach to the top disk with the black thumbscrew. The string is anchored with a washer inside the pulley and it should come out the slot on the rim of the pulley. It then passes over the cylindrical bearing, and a mass  $m$  of 25 grams hangs on the end of it.
5. Turn on the air supply and adjust the supply so that the air pressure allows the top disk to turn freely. (Typically around 10 psi.) The bottom disk should not freely rotate.
6. Make sure the 25g mass  $m$  on the string can drop without interference. If the string rubs on the table, move the apparatus to correct the problem. Make sure that the air pressure is still at the proper level (keep monitoring the pressure throughout the experiment). By *gently* turning the top disk, wind the thread around the pulley until the top of the 25 g mass is level with the bottom of the cylindrical bearing bracket. Hold the top disk stationary for a moment and then release it. The falling mass  $m$  should accelerate the disk. The thread should unwind from the pulley and, before the mass hits the ground, the thread should begin winding back on the pulley. If the mass reaches the ground, shorten the thread. Make a couple of test runs for practice.
7. Get acquainted with the computer and take your data. The Data Studio program (`rot_dyn.ds`) will provide a plot of angular velocity versus time. Wind up the thread and hold the disk steady. Tell the computer to begin taking data and *then* release the disk. The computer will pause for a while as it takes the data and performs calculations. It does a least-squares curve-fitting routine, similar to what you have done with *WAPP+*. As the the mass moves up and down a triangular wave appears on the graph (which will confuse the curve-fitting routine).
8. Begin a new data collection run; let it proceed for at least three full cycles. On the resulting velocity vs. time graph select a linear portion of your data for analysis, Data Studio will report the slope of that section. Record the absolute value of the slope in your spreadsheet as  $\alpha$ . Collect consecutively upward segments and downward segments of the graph, a total for six slopes from three complete cycles. (It often turns out that the slope from an upward segment of the graph,  $\alpha_{\text{up}}$ , is slightly different from  $\alpha_{\text{down}}$  from a downward segment. By combining an equal number of up and down slopes we end up with a more unbiased estimate.)
9. Make a data table in your spreadsheet, so you can record 6 angular accelerations for the disk and have columns for calculating  $\tau$  and  $I$  from Eq. (7.5) as shown below. **Be sure to also record your units for each column, because the units are critical to getting agreement of the two approaches to measuring  $I$ .** Remember to use correct units for  $g$ ; if distances are in centimeters, you will need  $g = 980 \text{ cm/s}^2$ ; if meters are used, then  $g = 9.8 \text{ m/s}^2$ .

$\alpha$ ( $\text{s}^{-2}$ )	$\tau = mgr - m\alpha r^2$ ( $\text{g}\cdot\text{cm}^2/\text{s}^2$ )	$I = \tau/\alpha$ ( $\text{g}\cdot\text{cm}^2$ )
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...
Mean		...
Standard Deviation of Mean		...

10. Work out the values of  $\tau$  and  $I$  for each of the 6 trials, if you have not already done so in a spreadsheet.
11. Now you should have 6 measurements of the moment of inertia  $I$  for the disk. Calculate the mean and standard deviation of the mean. Self document all spreadsheet calculations!
12. Record mass  $M$  and radius  $R$  of the top steel disk. (To measure a radius, use the caliper to determine the diameter, then divide by 2.)
13. Calculate the moment of inertia  $\mathcal{I}(\text{disk})$ , and also calculate the uncertainties using the high-low method, as mentioned previously; use your maximum and minimum values of  $M$  and  $R$  in Eq. (7.6).
14. Repeat steps 8–11 after attaching the bar to the top of the pulley with the red thumbscrew.
15. To find  $I$  for the bar, subtract  $I(\text{disk})$  from  $I(\text{bar}+\text{disk})$ :

$$I(\text{bar}) = I(\text{disk} + \text{bar}) - I(\text{disk}) \quad (7.8)$$

Find the uncertainty of this result by taking

$$\delta I(\text{bar}) = \delta I(\text{disk} + \text{bar}) + \delta I(\text{disk}) \quad (7.9)$$

16. Calculate the moment of inertia  $\mathcal{I}(\text{bar})$  from the mass and shape of the bar, and also calculate the uncertainties using the high-low method, as mentioned previously; use your maximum and minimum values of  $M$ ,  $w$  and  $L$  in Eq. (7.7). The inertia  $\mathcal{I}(\text{bar})$  from measurements of the bar shape should agree (within uncertainties) with the result  $I(\text{bar})$  that you got using angular acceleration.
17. Make a final results table that properly records (units, errors, sigfigs) your four moments of inertia. (This is often easier to do by hand than in a spreadsheet.)

object	$I$ units	$\mathcal{I}$ units
disk	$\pm$	$\pm$
bar	$\pm$	$\pm$



Put an identical table on your “Quick Report” card and give it to your instructor.

18. Comment on your results. Do the two ways of determining  $I$  agree (within error)? If they differ, think about factors that were neglected and whether they may be significant. What would happen to  $I$  if friction were not quite eliminated? What would happen to  $\mathcal{I}(\text{disk})$  and  $\mathcal{I}(\text{bar})$  if the pulley was included? Finish your lab report and hand it in to your TA.

## CHECKLIST

Pre-Lab Exercise	
Brief outline of Procedure	
Mass $M$ and radius $R$ of disk	
Mass $M$ and dimensions $L$ and $w$ of bar	
Radius $r$ of pulley recorded	
Table of $\alpha$ of disk; 6 trials	
Calculation of mean and SDOM for $I(\text{disk})$	
Calculation of $\mathcal{I}(\text{disk})$ with uncertainty for disk	
Table of $\alpha$ for disk+bar; 6 trials	
Calculation of mean and SDOM for $I(\text{disk} + \text{bar})$	
Calculation of $I(\text{bar})$ and $\delta I(\text{bar})$	
Calculation of $\mathcal{I}(\text{bar})$ with uncertainty	
Comparison of results, comments	
Conclusions, Discussion	



## 8. ARCHIMEDES' PRINCIPLE

### Pre-Lab Exercises:

1. Suppose you have a container of water sitting on a scale, and the scale reports the weight of the water and container. Now if you lower a rock on a string into the water, what effect will this have on the scale? Think about Newton's Third Law, and explain briefly in your notebook what will happen.
2. As a particular example of the above, consider a rock whose mass is known to be 50 g. When the rock (on a string) is lowered into water, the scale reading increases by 20 g. Calculate the density of the rock.
3. Briefly outline the Procedure in your notebook.

### Introduction

An object submerged (partially or fully) in a fluid experiences an upward buoyant force that counteracts part or all of the force of gravity on the object. For instance, a large rock seems to weigh less under water than in air. Many objects are found to float in water, such as ice cubes, life jackets, and huge ships made mostly of steel. Some time ago, Archimedes (287–212 B.C.) deduced the following principle: *the buoyant force on an object is equal to the weight of the fluid it displaces*. The principle can also be expressed in terms of the densities of the object and the fluid. In this experiment you will be testing Archimedes' principle by measuring the buoyant forces experienced by objects in water, and comparing with the weight of the displaced water.

### Apparatus

1. Unmarked cylinder containing water,
2. Digital balance,
3. Digital height gauge,
4. Objects that float, and objects that sink.

### Experiment

The experiment consists of measuring the buoyant force on objects that sink and objects that float in water, and comparing this force with the weight of the displaced water. We

measure the buoyant force by the change in the reading of a digital balance. And we know the density of water,  $\rho_w = 1 \text{ g/cm}^3$ , so we can determine the weight of the displaced water by finding its volume.

## Theory

### A. Sinking Objects

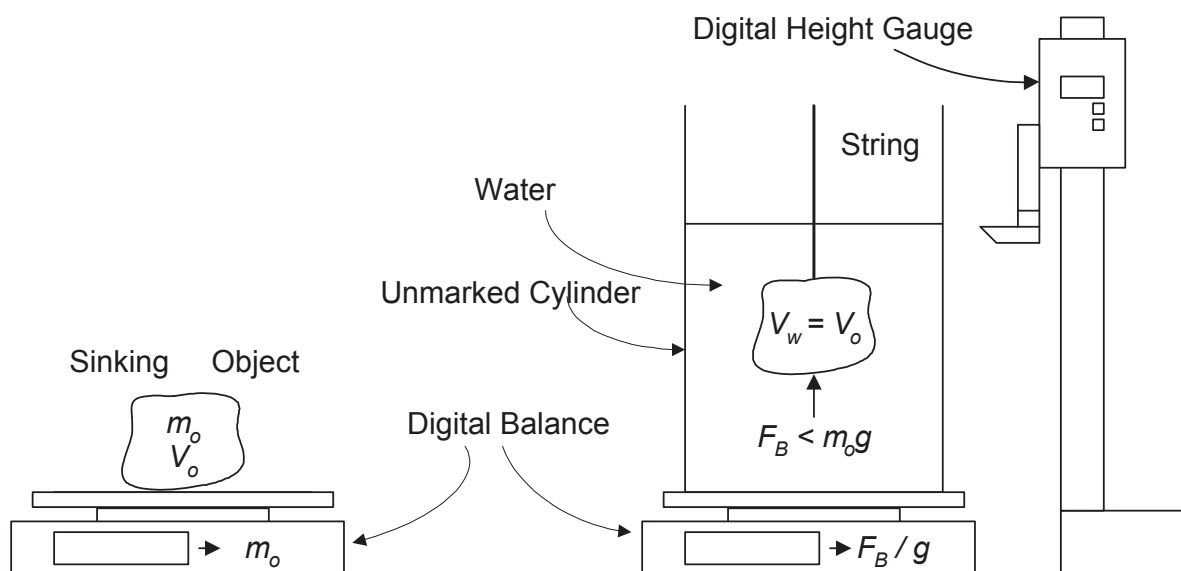


Figure 8.1: Testing Archimedes' principle for a sinking object.

For the first part of the experiment, we test Archimedes' principle for objects that *sink* in water, by measuring the buoyant force  $F_B$  and the volume  $V_w$  of water displaced. The uncertainties are probably going to be significant in this method, so you will need to pay attention to them. First, we use the digital balance to measure the mass,  $m_o$ , of the object. Then, as illustrated in Figure 8.1, the object is suspended in water on a string. It may feel noticeably lighter than in air, because the water exerts a buoyant force upward on the object. So according to Newton's Third Law, the object exerts an equal downward force on the water. This is conveniently measured with the digital balance; we use the TARE button to set the balance to read zero before the object is suspended, then the buoyant force will be given by the scale (actually, the scale gives the force divided by  $g = 980 \text{ cm/s}^2$ , as if it were a mass instead of a force). So your measurements of buoyant forces will be:

$$F_B = g \cdot \text{scale reading} \quad (8.1)$$

with an uncertainty of

$$\delta F_B = g \cdot 2 \quad (8.2)$$

because the uncertainty of the scale is  $\pm 2$  grams.

Because the object sinks, the object's volume,  $V_o$ , equals the the volume of water it displaces  $V_w$ , which can be calculated from the change in the water level of the cylinder, denoted by  $h$ :

$$V_w = \pi r^2 h, \quad (8.3)$$

where  $r$  is the radius of the cylinder. The uncertainty in  $V_w$  can be approximated by

$$\delta V_w = \pi(r + \delta r)^2(h + \delta h) - V_w. \quad (8.4)$$

The weight of the displaced water is:

$$\text{Water weight: } W_w = V_w \rho_w g \quad (8.5)$$

where  $\rho_w = 1 \text{ gram/cm}^3$  is the density of water. The uncertainty in the water weight is simply

$$\delta W_w = \delta V_w \cdot \rho_w g \quad (8.6)$$

where  $\delta V_w$  is your estimate of the uncertainty in the volume [see Eq. (8.4)].

Now, according to Archimedes, the buoyant force  $F_B$  should equal the water weight  $W_w$ , and you can compare your measurements of  $F_B$  and  $W_w$  to see if they agree within their uncertainties.

From your data you can also compute the sinking object's density,

$$\rho = \frac{m_o}{V_o} \quad (8.7)$$

because the volume  $V_o$  of the sinking object is the same as the volume of water  $V_w$  it displaces. The uncertainty associated with this density would be

$$\delta \rho = \rho \left( \frac{\delta m_o}{m_o} + \frac{\delta V_o}{V_o} \right). \quad (8.8)$$

## B. Floating Objects

For the second part of the experiment, test Archimedes' principle using objects that *float* in water, as illustrated in Fig. 8.2. The buoyant force  $F_B$  in this case equals the weight of the object, and you will get it from the scale reading as before:

$$F_B = g \cdot \text{scale reading} \quad (8.9)$$

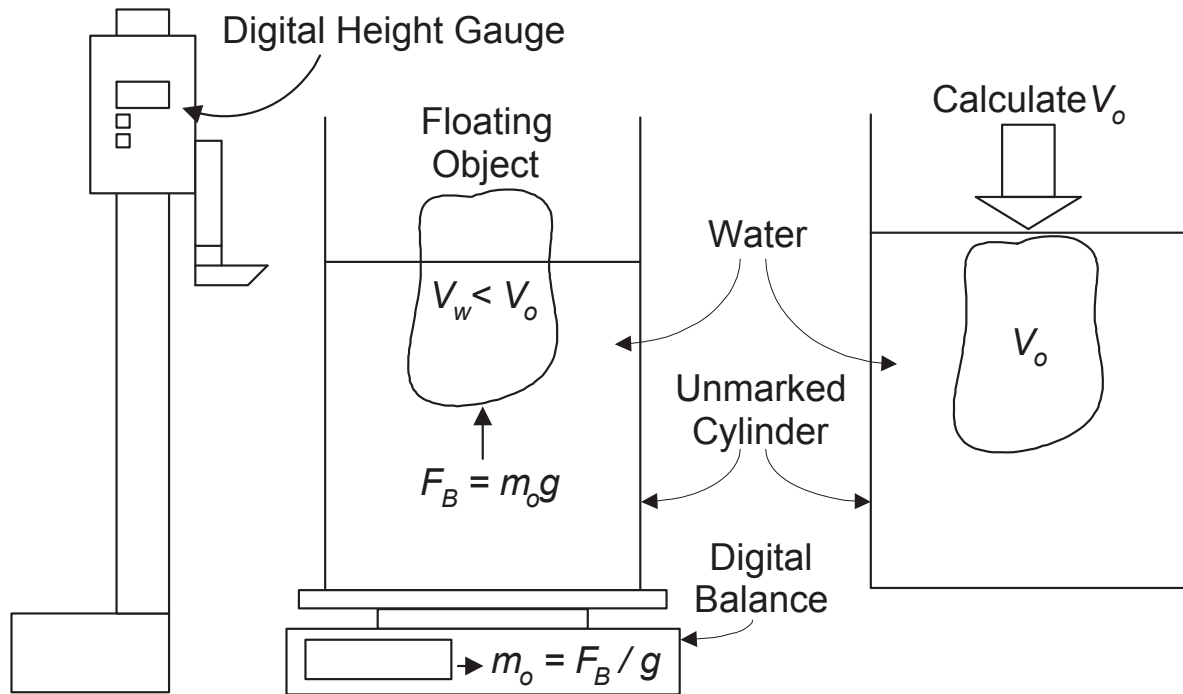


Figure 8.2: Testing Archimedes' principle for a floating object.

with an uncertainty of

$$\delta F_B = g \cdot 2 \quad (8.10)$$

because the uncertainty of the scale is  $\pm 2$  grams.

But a floating object displaces a water volume  $V_w$  which is something less than the object's volume,  $V_o$ . As before, the weight of the displaced water is

$$\text{Water weight: } W_w = V_w \rho_w g \quad (8.11)$$

where  $\rho_w = 1 \text{ gram/cm}^3$  is the density of water. And the uncertainty in the water weight is again simply

$$\delta W_w = \delta V_w \cdot \rho_w g \quad (8.12)$$

where  $\delta V_w$  is your estimate of the uncertainty in the volume [see Eq. (8.4)].

According to Archimedes, the buoyant force again should be equal to the weight of the displaced fluid.

After measuring the volume  $V_w$  displaced by the object when it floats, you should also measure the total volume  $V_o$  of the object by pushing it down so it is completely submerged. Then you will be able to calculate the density of the floating object:

$$\rho = m_o / V_o \quad (8.13)$$

as before, and the uncertainty in density will again be

$$\delta\rho = \rho \left( \frac{\delta m_o}{m_o} + \frac{\delta V_o}{V_o} \right). \quad (8.14)$$

## Procedure

1. BEFORE YOU COME TO LAB: Find something you'd like to bring to lab as floating or sinking objects of study. There will be some in the lab, but your favorites are welcome too (sorry...we can't fit your lecture instructor in the cylinder — it only has a diameter of 5 cm).
2. IN LAB: Familiarize yourself with the digital balance. Figure out what units it is calibrated in, and try weighing some known masses to be sure the scale is accurate; the TARE button lets you set it to read zero when there is a load on the scale. Also, estimate the uncertainty of forces measured with this scale. Next, get to know your digital caliper and height gauge, and make an estimate of the uncertainty  $\delta V$  for volumes calculated with them. Note that  $1 \text{ mL} = 1 \text{ cm}^3$ .
3. Find two different objects that sink in water. Measure their masses  $m_o$ , and the buoyant forces  $F_B$  and water volumes  $V_w$  displaced when they are submerged. Be sure to estimate uncertainties.
4. To test Archimedes' principle, a table like the following will help. A convenient force unit is the dyne:  $1 \text{ dyne (dyn)} = 1 \text{ g}\cdot\text{cm/s}^2$ , so a mass of 1 gram has a weight of 980 dyn.

Object	Scale Reading (g)	$F_B$ (dyn)	$\delta F_B$ (dyn)	$h$ (cm)	$V_w$ (cm <sup>3</sup> )	$\delta V_w$ (cm <sup>3</sup> )	$W_w$ (dyn)	$\delta W_w$ (dyn)	Overlap? Yes/No
Chunk									
Hunk									

5. Figure out the densities of the two sinking objects. Note that the masses  $m_o$  should have been measured separately — they are not the same as the scale readings of  $F_B$  when the masses were suspended on a string. The volumes  $V_o$ , however, are the same as  $V_w$ s which you measured in the previous step.

Object	Mass $m_o$ (g)	$\delta m_o$ (g)	Volume $V_o$ (cm <sup>3</sup> )	$\delta V_o$ (cm <sup>3</sup> )	Density $\rho$ (g/cm <sup>3</sup> )	$\delta\rho$ (g/cm <sup>3</sup> )
Chunk						
Hunk						

6. Find two objects that float in water. For each one, the mass  $m_o$  is given by the scale reading when you place the object into the cylinder, and the displaced volume  $V_w$  can be calculated when the object is floating. Also find the object's volume  $V_o$  by pushing the object down until it is fully submerged and measuring the water height. Include uncertainties, of course.
7. To test Archimedes' principle for floating objects, make another table like this:

Object	Mass $m_o$ (g)	$F_B$ (dyn)	$\delta F_B$ (dyn)	$h$ (cm)	$V_w$ (cm <sup>3</sup> )	$\delta V_w$ (cm <sup>3</sup> )	$W_w$ (dyn)	$\delta W_w$ (dyn)	Overlap? Yes/No
Blob									
Glob									

8. Then figure out the densities of the two floating objects.

Object	Mass $m_o$ (g)	$\delta m_o$ (g)	Volume $V_o$ (cm <sup>3</sup> )	$\delta V_o$ (cm <sup>3</sup> )	Density $\rho$ (g/cm <sup>3</sup> )	$\delta \rho$ (g/cm <sup>3</sup> )
Blob						
Glob						

9. Does Archimedes' principle appear to be correct in all the cases you studied?
10. How do the densities of the floating and sinking objects compare to the density of water?
11. Use what you have learned to measure the density  $\rho_\ell$  of an unknown liquid, and its uncertainty  $\delta\rho_\ell$ . Your instructor will show you the equipment. Explain your procedure.
12. Fill out a "Quick Report" card listing (units, significant figures, uncertainties) the densities of one floating object, one sinking object, and the unknown liquid. Then finish up your lab report and give it to your TA.

## CHECKLIST

Pre-Lab Exercise		
Brief outline of Procedure		
Table for $F_B$ and $W_w$ comparison for sinking objects		
Table for densities of sinking objects		
Table for $F_B$ and $W_w$ comparison for floating objects		
Table for densities of floating objects		
Comments on results for Archimedes' principle		
Comments on densities of floating and sinking objects		
Measurement of density $\rho_\ell$ with uncertainty $\delta\rho_\ell$ of unknown liquid		
Conclusions, Discussion		



# 9. GAS BEHAVIOR

## Pre-Lab Exercises:

1. Calculate the volume in mL ( $\text{cm}^3$ ) and its uncertainty for a cylinder with a diameter of  $3.25 \pm .01$  cm and a height of  $8.0 \pm 0.2$  cm.
2. Briefly outline the Procedure in your notebook.

## Introduction

The purpose of this experiment is to give you some experience with a simple thermodynamic system — a container of air. You will be studying the relationship of pressure and volume for a gas sample, and plotting the  $P - V$  diagram of a simple heat engine.

## Apparatus

1. Pasco interface and PC with *Data Studio* program;
2. Gas law apparatus: piston, air chamber, tubing;  
piston diameter:  $3.25 \pm .01$  cm; tubing diameter:  $\sim 0.3$  cm
3. Pressure Sensor;
4. Volume sensing pulley system;
5. Temperature baths: hot and cold water;
6. A 100 g mass with a hook; meter stick
7. Thermometer, barometer

## Experiment

The thermodynamic system and various sensors are illustrated in Fig. 9.1. There is a graduated cylinder with a graphite piston, an aluminum air chamber on the end of a plastic tube, and some temperature baths for immersing the air chamber. Sensors attached to the computer allow you to monitor the pressure and volume of the gas.

This experiment is to (1) verify Boyle's law and (2) to acquaint you with a simple heat engine in which the gas is used to convert heat energy to mechanical work.

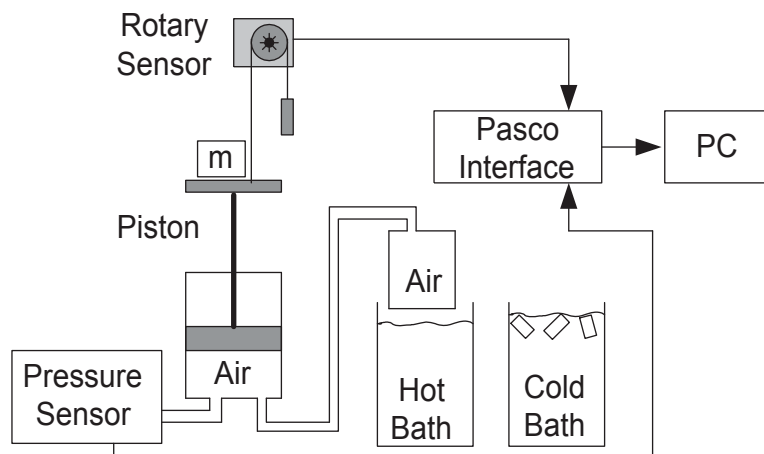


Figure 9.1: Gas behavior apparatus

Boyle’s Law relates the pressure,  $P$ , and volume,  $V$ , of a gas at constant temperature; with your help the computer will record the  $PV$  measurements. The  $P$  sensor measures *gauge pressure* (in kPa), so you will have to add atmospheric pressure to its readings to get the absolute pressure of the gas. The large pulley on the rotary sensor is used to track *changes* in the piston height, and you will tell the computer how to convert this change-in-height data into a measurement of  $V$  using the dimensions of the air chamber (given as the pre-lab problem) and the tubing length.

The engine operates in a cycle consisting of four stages: (1) a 100-g mass is placed on the piston, (2) the gas is warmed to lift the mass, (3) the mass is removed, and (4) the gas is cooled, completing the cycle.  $P$  and  $V$  will be monitored around the cycle.

## Theory

The ideal gas law relates the gas pressure  $P$ , volume  $V$ , and absolute temperature  $T$ :

$$P = nRT/V \quad (9.1)$$

where  $n$  is the number of moles of the gas,  $T$  is in Kelvins, and  $R$  is the universal gas constant, 8.31 J/mol·K. Air is not always very “ideal” but under the conditions of this experiment (temperatures and pressures not too low or high) Eq. (9.1) should be an excellent approximation.

Boyle’s Law says that if  $n$  and  $T$  are constant, then  $P$  is proportional to  $1/V$ ; that is,

$$P = (nRT) V^{-1}. \quad (9.2)$$

Gases are useful for converting thermal energy into mechanical energy because they undergo much larger volume changes than solids or liquids. For a gas confined at low

temperature in a container with a piston, warming the gas will raise the piston and a mass placed on top of it, thereby converting thermal energy to mechanical energy. An *engine* operates in a *cycle*; the mass must be removed and the gas cooled to return it to its starting point for a complete cycle. Figure 9.2 illustrates this simple engine cycle and a corresponding  $P - V$  diagram. We can assume that the  $a \rightarrow b$  segment, adding

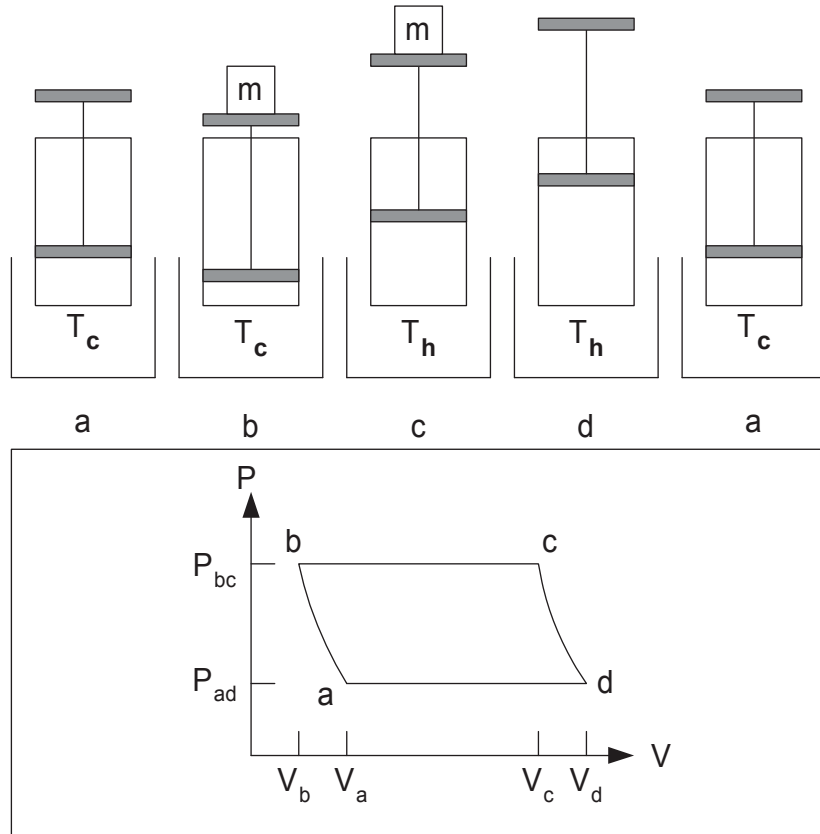


Figure 9.2: Engine cycle and  $P - V$  diagram.

the mass, and the  $c \rightarrow d$  segment, removing the mass, happen quickly enough to be *adiabatic*, so no heat is transferred. The other segments,  $b \rightarrow c$  and  $d \rightarrow a$ , are assumed to be *isobaric*, or constant pressure processes.

According to the first law of thermodynamics, the heat  $Q$  added to the gas appears in the form of work done by the gas,  $W$ , and a change in the internal energy of the gas,  $U$ :

$$Q = \Delta U + W. \quad (9.3)$$

The area of a  $P - V$  cycle gives the work done.

## Procedure

### Boyle's Law

1. Record atmospheric pressure.
2. Measure the length of the tubing that connects the gas law apparatus to the  $P$  sensor. Calculate the volume of air this tube contains using an inner diameter of 0.3 cm.
3. Tightly close the pinch clamp on the tubing leading to the second port on the gas law apparatus (which leads to the aluminum air chamber).
4. If it hasn't been done already, plug the  $P$  sensor into **ANALOG CHANNEL A** of the interface box. The rotary motion sensor allows you to monitor volume changes for the gas. Plug the sensor wires into **DIGITAL CHANNELS 1** and **2** of the interface box. Turn on the interface switch and then the PC.
5. Get the *Data Studio* program running and look for a file with the name "gas1.ds." It will have windows for volume and pressure tabular data, a calculator, and a window for the  $PV$  plot.
  - (a) The calculator will already be displayed. You should see a Definition box for defining formulas to calculate absolute pressure and volume from the  $P$  and  $V$  sensors. An icon ▼ to the right of the Definition box allows display of the Absolute Pressure or Volume formula. You must now modify these formulas for today's measurements.
  - (b) In the line defining absolute pressure replace the '101' with the actual atmospheric pressure. Click Accept.
  - (c) Add the tubing volume calculated in #2 to the cylinder volume (from the pre-lab problem: assuming an initial piston height of 8.0 cm). Add an additional 2 mL for the air inside the tubing within the cylinder base. Now, for example, if the resulting total volume comes out to 72.4 mL, replace the '70' in the volume formula with '72.4'. Result: **Volume = x\*100\*8.3 + 72.4**. (Note: In case you're curious, the 100 in this formula is there because  $x$  is measured in meters by the computer.)
6. The initial height (8 cm) of the piston can be adjusted by disconnecting the pressure sensor (twist & pull) to allow air flow in or out of the cylinder. The piston can be temporarily locked in place with the thumbscrew located on the device as you reconnect the tubing.
7. Practice your measurements by clicking on the START icon (don't forget to loosen the thumbscrew first). Push on the piston platform to see changes in  $V$  and  $P$ . Click on STOP to stop the measurements. Notice how the volume and absolute pressure are displayed. As you push down the pressure should rise as the volume

falls. A table and graph show all of the collected data. When the pressure plateaus at about 110 kPa, click STOP. NOTE: you are trying to get about a dozen data points; don't push too fast or too slow.


8. When you are ready to record your measurements, make sure the cylinder starts out at an initial height of 8.0 cm (re-adjustment may be necessary). Proceed as above.
9. If the data appears suitable, select and copy the appropriate data points from the graph. Open *WAPP*<sup>+</sup> to analyze the data for comparison with Boyle's Law.
  - (a) In the *WAPP*<sup>+</sup> starting screen, mark "Enter a formula for  $y$  errors" and "Enter a formula for  $x$  errors." Then hit the Do Bulk button.
  - (b) Paste the data into the Block Copy & Paste area of the *WAPP*<sup>+</sup> screen.
  - (c) Identify the columns for *WAPP*<sup>+</sup>:  $P$  is the  $y$  variable and  $V$  is the  $x$  variable.
  - (d) Estimate the uncertainty in your volume ( $x$ ) based on your pre-lab calculations. The pressure ( $y$ ) uncertainty should have been given along with the value of atmospheric pressure.
  - (e) Initially, select a power function as your fitting function:  $y = Ax^B$ , based on Eq. (9.2). According to Boyle's Law, the  $B$  parameter should be  $-1.0$ , but don't require this yet — let *WAPP*<sup>+</sup> find out what it is. Hit the Submit Data button. If  $B$  isn't near  $-1$  contact your instructor.
  - (f) Go back to the fitting function screen and now hold the  $B$  parameter constant at  $-1$ . Have *WAPP*<sup>+</sup> recalculate the fit with a new value for  $A$ .
  - (g) Print a nicely labeled graph and a fit report, and attach them in your lab book with your comments.
10. According to Boyle's Law,  $A = nRT$ , so  $n$  may be calculated from the *WAPP*<sup>+</sup>  $A$  parameter as long as  $R$  and  $T$  are known. The value of  $R$  is 8313 mL·kPa/mol K. Use the room thermometer to determine the absolute temperature  $T$  in kelvins. Is  $n$  a reasonable number? (As a rough guess, you may recall that a mole of a gas at standard temperature and pressure fills a volume of about 22.4 liters, so a volume of 70 mL ought to contain about  $0.07/22.4 = 0.003$  moles of gas.)
11. Use the reported uncertainty in the  $A$  parameter to determine the uncertainty in  $n$ :
$$\delta n = n \frac{\delta A}{A} \tag{9.4}$$
12. Make any relevant comments on the validity of Boyle's Law based on your analysis.
13. When writing your report, give some thought as to how a small amount of air leakage and how a slight temperature increase, caused by pressing down on the piston, might affect your measurements. Would these errors be considered random or systematic?

## Heat Engine

1. Get two temperature baths ready in pails: a cold one with some ice water, and a hot one with hot water from the tap. The precise temperature of neither is critical; obviously the ice bath will be close to  $0^{\circ}\text{C}$ .
2. Make sure the pressure sensor and the aluminum air chamber are connected to the gas apparatus ports with both pinch clamps open as shown in Figure 9.1.
3. Open the Data Studio file “gas2.ds.” It will not be necessary to enter the calculator constants for total volume and absolute pressure since we are only interested in  $\Delta P$  and  $\Delta V$ .
4. Try a test run to assure that the  $P$  and  $V$  sensors work, and to practice the routine for making a nice cycle. Here’s the routine:
  - (a) Start with the air chamber in the cold water bath.
  - (b) Click on the **START** button of the *Data Studio* program.
  - (c) Gently place the 100 g mass on the piston. Watch the  $P - V$  graph.  $P$  should increase, while  $V$  decreases, like adiabat  $a \rightarrow b$  of Fig. 9.2.
  - (d) Immediately, move the air chamber into the hot water bath.  $V$  should rise, but  $P$  should remain constant, like the  $b \rightarrow c$  isobar of Fig. 9.2; constant pressure, with increasing temperature gives a volume increase. Leave the air chamber in the hot bath until the volume increase stalls.
  - (e) Gently remove the 100 g mass from the piston.  $P$  should decrease, while  $V$  increases, like the  $c \rightarrow d$  adiabat of Fig. 9.2.
  - (f) Immediately move the air chamber into the cold bath, and leave it there for now.  $P$  should remain constant and  $V$  should decrease, like the  $d \rightarrow a$  isobar of Fig. 9.2. Everything working OK? If not, get help. Click on the **STOP** button to end the trial run.
  - (g) Your  $P - V$  graph should be approximately a closed cycle. To improve it, try to do it quicker, especially moving the air chamber from one temperature bath to the other immediately after moving the 100 g mass, to reduce heat transfers between the air chamber and the temperature baths at the  $b$  and  $d$  corners of the cycle.
5. Now that you’ve practiced the engine cycle, do it for real. Start with the air chamber in the ice water bath. When the air chamber is cold, and everything is steady, start taking data as above. That’s one engine cycle. (Be watching for evidence of gas leaks. Does the  $P - V$  curve return to its starting place? Does  $P$  drop when it is expected to remain constant?)
6. If you had problems, try the cycle again. Get as good a  $P - V$  diagram as you can, and print it for in your notebook.
7. Here are some things to determine from your data:

- (a) Are the  $b \rightarrow c$  and  $d \rightarrow a$  segments nearly horizontal?
- (b) Is the pressure difference  $\Delta P = P_{bc} - P_{ad}$  between the upper and lower isobars consistent with the amount expected from adding a 100-g mass to the piston? Recall that  $P = \text{Force}/\text{Area}$ . (Convert your basic measurements to MKS units to make this comparison.)
- (c) The area inside a  $P - V$  cycle should correspond to the net work done by the gas in lifting the mass.
- Estimate the area inside your cycle in Joules ( $\text{Pa}\cdot\text{m}^3=\text{J}$ ). The cycle is roughly a parallelogram where the area is equal to the product of the base and the height:

$$W = \Delta V(\text{either base}) \times \Delta P. \quad (9.5)$$

To assist you in making measurements of  $\Delta V$  and  $\Delta P$ , there is a “Smart Tool” icon  you can click to get vertical and horizontal lines and the  $xy$  coordinates of their intersection.

- Calculate the work done in lifting the mass:  $mg\Delta y$ . ( $\Delta y$  can be determined by dividing  $\Delta V$  by piston area  $A$ .)
  - Compare the results of (i) and (ii).
8. For a “Quick Report” card for your lab instructor, list (with correct units, significant figures, and uncertainty) your measured  $n$  and the amount of work done in your  $P - V$  engine cycle.
9. Add any other comments you may have on the quality and meaning of your engine cycle graphs, your other data, or the experiment. Finish your lab report and hand it in.

## CHECKLIST

Pre-Lab Exercise	
Brief outline of Procedure	
Report atmospheric pressure	
Measurement of tubing volume	
WAPP <sup>+</sup> fit report for $P$ vs. $V$	
properly label $P$ and $V$ columns in fit report	
WAPP <sup>+</sup> graph of $P$ vs. $V$	
WAPP <sup>+</sup> $A$ and $B$ parameters and the calculation of $n$	
$P - V$ cycle graph	
comparison: $\Delta P$ and Force/Area	
comparison: $P - V$ cycle area and $mg\Delta y$	
Conclusions, Discussion	





# 10. LAB PRACTICAL EXAM

The ability to collect useful data (and estimate the accuracy of that data) and the ability to analyze data (interpreting the results of computer programs; making proper plots) are two goals of this course (see page 5). This lab practical exam will test only the most easily-tested skills, namely: interacting with and interpreting the results of computer programs.

**Because you need one of the 8 lab computers, and because there can be as many as 16 students in a lab section, your time will be limited to 1 hour. This should be plenty of time to accomplish the required tasks. You will need to sign up for the hour in which you will take the exam.**

In the exam you will be *given* a dataset, told the appropriate functional form, and, working individually, you will:

- Enter this data into a spreadsheet (including numbers in scientific notation)
- Transfer the data to *WAPP*<sup>+</sup>.
  - Tell *WAPP*<sup>+</sup> about the errors in the data. All the datasets assume no *x*-error and generally (but not always) there will be a simple formula for *y*-error.
  - Tell *WAPP*<sup>+</sup> the proper functional form.
  - Tell *WAPP*<sup>+</sup> to find the parameters of the best possible fit.
  - Print out the resulting fit report.
- Report the scientific validity of the resulting fit. (Think reduced  $\chi^2$ .)
- Report the resulting best-fit parameters (including proper units, significant figures, and uncertainty).
- Calculate some quantity based on those parameters and, most importantly, find the error in that calculated quantity. (That is: error propagation using either the high-low method of page 17 or the formula-based methods of page 85 in Appendix A.) You must show these calculations by self-documenting your spreadsheet.
- Produce proper (axes labels, title, etc.) hardcopy plots of the data with fitted curve. In most cases, two hardcopy plots (one with normal scales, and one in which you have selected scales to linearize the curve) are required. The quadratic functional form cannot be linearized and the linear functional form is already linear using normal scales, so in those two cases you need only turn in one hardcopy plot.

You can find practice exam datasets at:

<http://www.physics.csbsju.edu/lab/practical> Your exam will consist of one of these on-line problems (but probably a different data set). Therefore, if, before lab, you assure you can do all the practice problems, you should be able to get 100% on your actual practical exam!

Your results for this exam include:

1. a quick report card (50 points) reporting:
  - (a) units,
  - (b) significant figures,
  - (c) uncertainties,
  - (d) the functional parameters,
  - (e) a calculated quantity, and
  - (f) assessment of scientific validity;
2. hardcopy plots (20 points);
3. hardcopy of your self-documented spreadsheet showing the dataset and specified calculations (20 points),
4. hardcopy of the  $WAPP^+$  fit report with all these other items nicely stapled together and handed to your instructor (10 points).

Your lab notebook is not needed for this exam. You may use this lab manual and the textbook as references during this exam.

# APPENDIX A — UNCERTAINTIES

## Introduction

All *measured* quantities have some uncertainty associated with them. It follows that anything you *calculate* using this measurement will also have some uncertainty associated with it. Any time you report a quantity, for example a quantity called  $Y$ , you should also report the amount of uncertainty  $\delta Y$  it has, in the form  $Y \pm \delta Y$ . This appendix will assist you in determining the uncertainties for measured and calculated quantities.

## Measurements

### Quantity $Y$ measured once

The amount of uncertainty in a measurement usually depends on how the measurement was made. For instance, if you measure the temperature in a room using a short Celsius thermometer that is marked in  $1^\circ$  increments, it may be that the best you can do is report the temperature to the nearest degree. You might report that the temperature is  $T = 25^\circ\text{C}$ , and mean that the temperature is in the range  $24.5^\circ\text{C} \leq T \leq 25.5^\circ\text{C}$ ; if this is the case, you should report it as  $T = 25.0 \pm 0.5^\circ\text{C}$ . It is common when reporting data to take the uncertainty as  $1/2$  the smallest unit on the measuring device. Sometimes you can do better than this. If the thermometer is long enough, you might be able to realistically estimate where the mercury column ends between two degree markings, and report a temperature of, say,  $25.3^\circ\text{C}$ . If you mean that the temperature is certainly between  $25.25^\circ\text{C}$  and  $25.35^\circ\text{C}$ , then say so by reporting  $T = 25.30 \pm 0.05^\circ\text{C}$ . However, if you only mean that it's certainly between  $25.1$  and  $25.5$  degrees, you should report  $T = 25.3 \pm 0.2^\circ\text{C}$ . The idea is to report a measurement as accurately as possible, but to also state clearly its range of possible values.

It can be very useful to put the uncertainty into a fractional or percentage form. As we shall see, the percent uncertainty in a calculated quantity is often related in a simple way to the percent uncertainty of the measured quantity. In the example above, where  $T = 25 \pm 0.5^\circ\text{C}$ , we could have said  $T = 25^\circ\text{C} \pm 2\%$ , since  $0.5/25 \times 100\% = 2\%$ .

In summary, when only a single measurement of quantity  $Y$  is performed, express it as  $Y \pm \delta Y$  where  $\delta Y$  represents the range of possible values for  $Y$ .

$$\text{Fractional uncertainty in } Y = \frac{\delta Y}{Y}$$

$$\text{Percentage uncertainty in } Y = \frac{\delta Y}{Y} \times 100\%$$

## Quantity $Y$ measured $N$ times

Sometimes repeated measurements of a quantity give different results. This may be because the quantity actually varies; for example, your weight may vary from day to day. On the other hand, it may be an indication of the uncertainty of the measurement; for instance, you may be measuring the time for a pendulum to swing once, and get a different result each time, even though the pendulum always behaves the same. In either case, repetition can tell us something useful. We can take the average, or mean,  $\bar{Y}$  as the best overall value of the quantity  $Y$ . We can also find how much the measurement typically varies from the average. This typical variation is called the standard deviation,  $\sigma_Y$ . But the best estimate of the uncertainty in  $\bar{Y}$  is called the standard deviation of the mean,  $\sigma_{\bar{Y}} = \sigma_Y/\sqrt{N}$ . The idea is that the mean of many measurements is less uncertain than the measurements themselves. If we have  $N$  measurements of quantity  $Y$ , call them  $(Y_1, Y_2, \dots, Y_N)$ , and they are all taken to be equally trustworthy, then:

$$\text{Mean of } Y = \bar{Y} = \frac{Y_1 + Y_2 + \dots + Y_N}{N}$$

$$\text{Standard deviation of } Y = \sigma_Y = \sqrt{\frac{(Y_1 - \bar{Y})^2 + (Y_2 - \bar{Y})^2 + \dots + (Y_N - \bar{Y})^2}{N-1}}$$

$$\text{Standard deviation of } \bar{Y} = \sigma_{\bar{Y}} = \sigma_Y/\sqrt{N}$$

Standard deviations can be tedious to calculate if  $N$  is large; many hand calculators have a button for computing  $\sigma_Y$ . If  $N = 2$ , the above formulas take on simple and reasonable forms:

$$\bar{Y} = \frac{Y_1 + Y_2}{2}$$

$$\sigma_Y = \frac{|Y_1 - Y_2|}{\sqrt{2}}$$

$$\sigma_{\bar{Y}} = \frac{|Y_1 - Y_2|}{2}$$

## Quantity $Y$ measured $N$ times with different uncertainties

Suppose we somehow get several measurements of some quantity  $Y$ , and the measurements have different uncertainties associated with them. There is a way to get a “best” estimate of  $Y$  by taking all the measurements into consideration, but counting most heavily on the more precise ones; it’s called the *weighted average*. Furthermore, we can attach an uncertainty to it that is less than the smallest of the individual uncertainties. If the measurements of  $Y$  are  $(Y_1, Y_2, \dots, Y_N)$  and their uncertainties are  $(\delta_1, \delta_2, \dots, \delta_N)$ , we calculate weighting factors  $W_1, W_2, \dots, W_N$  given by:

$$W_i = \frac{1}{\delta_i^2}$$

and use them to compute

$$Y_{best} = \frac{W_1 Y_1 + W_2 Y_2 + \dots + W_N Y_N}{W_1 + W_2 + \dots + W_N}$$

Notice that this is like an average of the  $Y$  values, but with each one weighted by a factor that increases as its uncertainty decreases. So the ones that are most reliable influence the average the most. The weighted average has a “best” uncertainty associated with it, given by

$$\sigma_{Y_{best}} = \frac{1}{\sqrt{W_1 + W_2 + \dots + W_N}}$$

## Calculations

This important section gives formulas for finding uncertainties in calculations. Suppose quantities  $A, B, C, \dots$ , are measured and used to compute quantity  $Y$ . The uncertainty  $\delta Y$  in the value of  $Y$  depends on the uncertainties ( $\delta A, \delta B, \delta C, \dots$ ) in the measurements and the way the measurements are used in the calculation. A little calculus gives approximate formulas to use for a “worst-case” estimate. Some statistical analysis gives similar but slightly better formulas to use when the uncertainties are all independent and random.

When the measurements  $A, B, C, \dots$  have estimated uncertainties  $\delta A, \delta B, \delta C, \dots$ , the uncertainty  $\delta Y$  depends on the formula for  $Y$  in the following ways.

Addition and Subtraction: If  $Y = A \pm B \pm C \pm \dots$ ,

$$\text{then } \delta Y \leq \delta A + \delta B + \delta C + \dots$$

Multiplication and Division: If  $Y = \frac{AB}{D}$ ,

$$\text{then } \frac{\delta Y}{Y} \leq \frac{\delta A}{A} + \frac{\delta B}{B} + \frac{\delta D}{D}$$

Exponentiation: If  $Y = A^a B^b D^d$ ,

$$\text{then } \frac{\delta Y}{Y} \leq a \frac{\delta A}{A} + b \frac{\delta B}{B} + d \frac{\delta D}{D}$$

Logarithms: If  $Y = \ln A$ ,

$$\text{then } \delta Y = \frac{\delta A}{A}$$

## General remarks on calculated uncertainties

Note how fractional uncertainties like  $\delta A/A$  appear in formulas for fractional uncertainty in  $Y$ . When measured quantities are multiplied, divided, or raised to a power in calculating  $Y$ , the easiest way to find  $\delta Y$  is to use fractional or percentage uncertainties.

In exponent formulas, the exponents don't have to be integers. For example, the period  $T$  of a pendulum depends on the square root of its length  $L$ ; the formula is  $T = (2\pi/\sqrt{g})L^{1/2}$ , so measuring  $L \pm \delta L$  gives  $\delta T/T = \delta L/2L$ . The significance of a measured quantity in determining  $\delta Y$  obviously increases with higher exponents. As another example, the volume of a cube is  $V = L^3$ , where  $L$  is the length of a side. Measuring  $L \pm 10\%$ , we obtain  $V \pm 30\%$ . To get  $V$  within 10%, we must measure  $L$  within about 3%.

The formula for obtaining  $Y$  from measurements  $A, B, C, \dots$  may be more complicated than the simple operations listed above. Rather than learn the calculus and statistics to find the correct uncertainty relationships, you can just plug in the maximum and minimum values of  $A, B, C, \dots$  and find the resulting range of  $Y$  values by the "high-low" method.

## Significant Digits

The rules for significant digits are important in determining where to truncate values when using a computer or calculator. *As a rule, the last significant figure in any stated number should be of the same order of magnitude (in the same decimal position) as the uncertainty.* In the physics lab, you should use the following rules in addition to reporting uncertainties:

1. *Nonzero integers.* Nonzero integers always count as significant figures.
2. *Zeros.* There are three classes of zeros:
  - (a) **Leading zeros** are zeros that *precede* all the nonzero digits. These do not count as significant figures. In the number 0.0025, the three zeros simply indicate the position of the decimal point. This number has only two significant figures.
  - (b) **Captive zeros** are zeros *between* nonzero digits. These always count as significant figures. The number 1.008 has four significant figures.
  - (c) **Trailing zeros** are zeros at the *right end* of the number. They are significant only if the number contains a decimal point. The number 100 has only one significant figure, whereas the number  $1.00 \times 10^2$  has three significant figures. The number one hundred written as 100. also has three significant figures.

3. *Exact numbers.* Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 apples, 8 molecules. Such numbers are called *exact numbers*. They can be assumed to have an infinite number of significant figures. Other examples of exact numbers are the 2 in  $2\pi r$  (the circumference of a circle) and the 4 and 3 in  $4/3\pi r^3$  (the volume of a sphere). Exact numbers also can arise from definitions. For example, one inch is defined as *exactly* 2.54 centimeters. Thus in the statement  $1 \text{ in} = 2.54 \text{ cm}$ , neither the 2.54 nor the 1 limits the number of significant figures when used in a calculation.

For calculations in the lab it is a good idea to carry at least one extra significant figure until the final result is obtained, then round to the proper number of digits as the last step. The rule of thumb is to drop all but one extra significant figure when multiplying/dividing (do this first). Lastly, for addition and subtraction, line up decimal places and add. In your final result round to the left-most decimal place in the added (subtracted) numbers. For example, adding:  $1.05 + 3.797 + 6.71236$

Lining up decimals:

$$\begin{array}{r} 6.71236 \\ 3.797 \\ + 1.05 \\ \hline 11.55936 \end{array} = 11.56 \text{ because } 1.05 \text{ has the left-most decimal place}$$

At some point you will likely find a number reported *without* a value for its uncertainty. In this case the author has assumed some value of uncertainty that you can only guess. A reasonable guess would be  $\pm$  the last decimal place (e.g. 15.5 would be assumed to be  $15.5 \pm 0.1$ ).





# APPENDIX B — DATA HANDLING

## Tables

Experimental data are usually recorded first in a tabular form, as shown below. The table should have labeled columns for the controlled quantity and the resultant quantities. Each column label should indicate the units for that quantity. A representative estimate of the uncertainty should be made and recorded for the first entry and any other entries where there is a significantly different uncertainty. Leave room for more columns if other quantities will be calculated from the direct measurements. A sample calculation should also be shown at the bottom of the table for those quantities which are calculated.

Point #	$L$ (cm)	$\pm\delta L$ (cm)	$T$ (sec)	$\pm\delta T$ (sec)	Other Quantities (units)
1	1.50	0.05	0.25	0.01	...
2	1.60	"	0.40	0.02	...
...	...	...	...	...	...

## Graphs

It is much easier to recognize a pattern from a graph than a table, so we will use graphs extensively in this laboratory. An additional appendix should be available in the lab to explain more about how functional relationships may be determined using graphs and curve-fitting programs such as *WAPP+*. An initial graph can be used to monitor an experiment as data is being collected. As each measurement is made, put it on the graph. After a few points are plotted, a pattern may become apparent which can serve to tell whether subsequent measurements fit the pattern. Although we may not know the exact pattern the data will follow, large deviations from any pattern can be seen easily and you can catch some mistakes in time to correct them. This initial graph will also serve as the starting point for data analysis. A refined and more complete final graph should be made for the lab report. A sample graph is shown in Fig. B.1.

Here are some guidelines for proper graphing:

- Put a title on each graph.
- Label each axis with the name and units of the quantity.
- Select the graph scale such that the data spans most of the full page.
- Indicate the uncertainty of each data point using error bar symbols, unless the error bars are smaller than the dots representing the data.

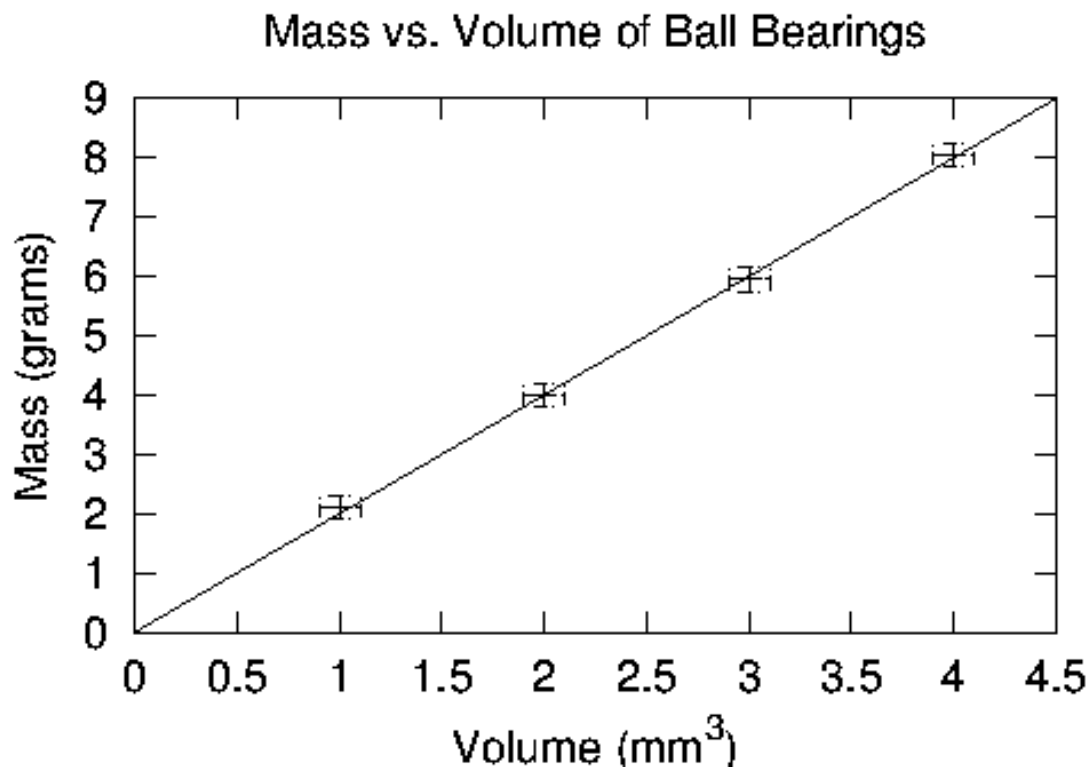


Figure B.1: A sample graph.

- Draw a smooth curve through the data; DO NOT simply connect the dots.

# APPENDIX C — WAPP<sup>+</sup>

## Using WAPP<sup>+</sup>

1. Find WAPP<sup>+</sup>: WAPP<sup>+</sup> is a web-based program that can be found at

<http://www.physics.csbsju.edu/stats/WAPP2.html>.

However it's often faster just to Google “wapp+”. You should bookmark this page for fast future reference. Also,

[http://www.physics.csbsju.edu/lab/105\\_WAPPonline.mp4](http://www.physics.csbsju.edu/lab/105_WAPPonline.mp4)

provides a video introduction to this program.

2. Start Excel: It is recommended to use Excel for all data analysis this semester.
3. Enter data into Excel: Enter your data values and formula-based calculations in spreadsheet columns. For example, generically calling a controlled variable  $x$ , with uncertainties  $\delta x$ , you may be measuring a related variable  $y$  with uncertainty  $\delta y$ , and then perhaps computing a third quantity  $z$  that depends on the variables somehow, and has an uncertainty  $\delta z$  that depends on the other measurements and their uncertainties. Use the spreadsheet columns to record data and perform calculations:

$x$	$\delta x$	$y$	$\delta y$	$z$	$\delta z$

4. WAPP<sup>+</sup>  $y$  and  $x$  error: The default setting for WAPP<sup>+</sup> is “Enter  $y$  error for each data point” and “Enter  $x$  error for each data point”.

Other options are available. For instance, if the uncertainties are uniform for  $\delta x$  and  $\delta y$ , or readily expressed with a formula, you could have avoided those three Excel columns and instead entered uniform or formula-based error entry in WAPP<sup>+</sup>. (The appropriate values/formulas would then be entered in WAPP<sup>+</sup>.) Make your desired selection and then click the **Do Bulk** button. A “Data Entry” page should appear.

5. Copy the numbers from Excel: Either by using:

- (a) Block Mode:

Highlight an array of up to six columns of numbers (no column headings, just numbers) in Excel. Copy those values using your favorite method (e.g., right click, control-C, or clicking the appropriate button on the top ribbon). Paste those numbers into the large WAPP<sup>+</sup> data entry box that has a background image of spreadsheet cells, using your favorite method (right click, control-V, or clicking Paste from the Edit menu). There is no reason for the columns

to line up with either the cell-image or the column headings. As far as the browser is concerned this is just a big lump of numbers. The numbers you paste in will almost certainly not align with anything. WAPP<sup>+</sup> is only concerned that there is some sort of separator between numbers (space, comma, tab, new-line, etc.); it is not concerned about the column alignment visible to you.

(b) Column Mode:

If appropriate, you can use the “Column Copy & Paste” region and, in separate copy & paste actions, move the appropriate columns of data into the appropriate WAPP<sup>+</sup> columns.

6. Tell WAPP<sup>+</sup> which column is which: In Block Mode, you must tell WAPP<sup>+</sup> the identity of each column you pasted in, so it can perform the data analysis you wish to do. In Column Mode, the headings were already in place when you entered the data.
7. Tell WAPP<sup>+</sup> which function should fit the data: Select from the choices:
  - Linear:  $y = A + Bx$  (which should be the default).
  - Power:  $y = Ax^B$
  - Exponential:  $y = A \exp(Bx)$
  - Natural Log:  $y = A + B \log(x)$
  - Inverse  $x$ :  $y = A + B/x$
  - Inverse  $y$ :  $1/y = A + Bx$
  - Inverse  $x$  &  $y$ :  $1/y = A + B/x$
  - Arrhenius:  $y = A \exp(B/x)$
  - Inverse-Natural Log:  $1/y = A + B \log(x)$

If you look at the bottom of the WAPP<sup>+</sup> data entry page you’ll find these words:

Occasionally the value of  $B$  is set by theory to a known value. If that situation applies to this data, enter here that fixed value of  $B$ . (Usually you will leave this blank.)

$B =$

8. Compute fit: click the Submit Data button. The resulting page includes the ‘fit report’ that gives the best values for parameters  $A$  and  $B$  and the uncertainties for each.

There is an important quantity called “reduced chi-squared” ( $\bar{\chi}^2$ ) which tells how well the computed curve fits the data:

- if  $\bar{\chi}^2$  is approximately 1 then the curve is a good fit within the error bars;

- if  $\bar{\chi}^2$  is much larger than 1, the curve misses the error bars by too much to be considered reliable;
- if  $\bar{\chi}^2$  is much less than 1, the curve misses the points by distances that are unexpectedly small compared to the error bars — your error bars are probably larger than necessary. WAPP<sup>+</sup> may give you a warning about this, but don't feel bad, just think about whether the error bars could have been smaller, and fix them if necessary; otherwise go on.

So if the  $\bar{\chi}^2$  value is far from 1 either direction, you ought to check either your data or error estimates. By looking at the plot of the curve you may see a problem you should correct.

9. Request a plot: For a linear (normal) plot select “X Scale Options” Linear and “Y Scale Options” Linear (which should be the default). Power law data should produce a straight line if plotted on Log-Log paper. For a Log-Log plot select “X Scale Options” Log and “Y Scale Options” Log. For the final version of a plot, enter appropriate  $x$  axis labels,  $y$  axis labels, and plot title. Click Make Plot button. On the following page click [PDF File](#). If everything looks OK, make a final pdf plot.
10. Print the fit report: Hit the Internet Explorer back arrow a couple of times to return to the fit report page. The bottom part of the fit report page involves plotting the data and need not be printed. Rather than print the entire page, you can save some trees by highlighting (left-click sweep) the report area, down to and including the line Data Reference, and then print just the selected area.