A new functional form to study the solar wind control of the magnetopause size and shape

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Abstract. In this study a new functional form, \( r = r_0 (2/(1 + \cos \theta))^{\alpha} \), is used to fit the size and shape of the magnetopause using crossings from ISEE 1 and 2, Active Magnetospheric Particle Tracer Explorers/Ion Release Module (AMPERE/IRM), and IMP 8 satellites. This functional form has two parameters, \( r_0 \) and \( \alpha \), representing the standoff distance and the level of tail flaring. The value \( r \) is the radial distance at an angle (\( \theta \)) between the Sun-Earth line and the direction of \( r \). It is found that \( r_0 \) varies with the interplanetary magnetic field (IMF) \( B_z \) component and has a break in the slope at \( B_z = 0 \) nT. The best-fit value of \( r_0 \) decreases with increasing southward IMF \( B_z \). For northward IMF \( B_z \), the best-fit value of \( r_0 \) increases slightly with increasing \( B_z \). The best-fit value of \( \alpha \) increases monotonically with decreasing IMF \( B_z \). The dynamic pressure \( (D_p) \) also changes \( r_0 \) and \( \alpha \). The parameters \( D_p \) and \( r_0 \) are related by a power law of \( -1/(6.6 \pm 0.8) \). The best-fit value of \( \alpha \) is slightly larger for larger dynamic pressure, which implies that \( D_p \) also has a role in flux transfer from the dayside to the nightside, but the size of this effect is small. An explicit function for the size and shape of the magnetopause, in terms of \( D_p \) and \( B_z \), is obtained by using multiple parameter fitting in a form that is useful for operational space applications such as predicting when satellites at geosynchronous orbit will be found in the magnetosheath.

1. Introduction

The solar wind interacts with the Earth’s dipole magnetic field, confining it in a magnetic cavity or magnetosphere with an outer boundary called the magnetopause. The size and shape of the magnetopause can be determined by the dynamic and static pressure of the solar wind and the magnetic pressure of the magnetosphere in the absence of solar wind coupling to the magnetosphere. In this study we use in situ magnetopause crossings and obtain a quantitative relation between the size and shape of the magnetopause and the solar wind parameters, dynamic pressure \( (D_p) \), and interplanetary magnetic field (IMF) \( B_z \). With this relation, we can predict the size and shape of the magnetopause for given values of IMF \( B_z \) and \( D_p \). This relation is useful for space weather operations and can be used for comparisons with numerical simulations or theoretical models.

Various models for the size and shape of the magnetopause have been studied in the past [Fairfield, 1971; Howe and Binsack, 1972; Formisano et al., 1979; Sibeck et al., 1991; Petrinec et al., 1991; Petrinec and Russell, 1993a, 1996; Roelof and Sibeck, 1993]. The Howe and Binsack [1972] model and the Petrinec and Russell [1996] model of the nightside magnetopause used inverse trigonometric functions to describe the magnetopause size and shape. The other models used either the general equation of an ellipsoid with two parameters (eccentricity and standoff distance) or the general quadratic equation. Fairfield [1995] discussed the limitations of using an elliptic equation. The ellipse must close at some point on the nightside and hence cannot represent a magnetopause that continues to flare. A new function should have the flexibility to produce a magnetopause which is open or closed. In this paper we present such a new function to fit the size and shape of the magnetopause:

\[
    r = r_0 \left( \frac{2}{1 + \cos \theta} \right)^\alpha
\]

where \( r_0 \) and \( \alpha \) are the standoff distance and the level of tail flaring, respectively. The parameter \( r_0 \) is the
distance at which balance is achieved between the solar wind dynamic pressure and Earth’s dipole magnetic field at the subsolar region. The value $r$ is the radial distance at an angle $(\theta)$ between the Earth Sun line and the direction of $r$. Unlike the equation of an ellipsoid, this functional form has the flexibility to produce a magnetopause which is closed ($\alpha < 0.5$), asymptotes to a finite tail radius ($\alpha = 0.5$), or expands with increasing distance from the Earth ($\alpha > 0.5$). The behavior of the new function in terms of $r_0$ and $\alpha$ is shown in Figure 1. The top panel of Figure 1 shows the function for different $r_0$ values and $\alpha = 0.5$. It can be seen that the curves for different $r_0$ are self-similar. The bottom panel of Figure 1 shows the function for different values of $\alpha$ and $r_0 = 10\; R_e$, where $R_e$ is Earth radii. It is found that the larger the value of $\alpha$, the more the tail flares.

In the study by Petrinec et al. [1991], an ellipsoid function was used to fit the position of crossings from ISEE 1 and 2 satellites. Since the apogee of ISEE 1 and 2 is 22.5 $R_e$, their results are limited principally to the dayside. Also, their data were only separated into two bins, southward and northward IMF $B_z$. Petrinec and Russell [1993a, 1996] extended their model to the magnetotail using a different functional form. This empirical model has a greater parametric extent than the earlier model [Petrinec et al., 1991]. In their study the value of $D_p$ ranges from 0.5 nPa to 8.0 nPa and the value of IMF $H_z$ is $\geq -10$ nT. They inferred the position of the magnetopause based on the total pressure balance at the magnetopause, and attached it in a piecewise continuous manner to the dayside model.

Strock et al. [1991] assembled a data set of magnetopause crossings from different studies that may have used different selection criteria. They separated the data by IMF $B_z$ without considering the variation of $D_p$ and by $D_p$ without considering the variation of IMF $B_z$. Roelof and Strock [1993] improved upon the fit with a method for determining the size and shape of the magnetopause as a bivariate function of hourly averages of IMF $B_z$ and $D_p$. The coefficients for their functions are rather complicated, so it is not easy to reproduce the size and shape of magnetopause for given IMF $B_z$ and $D_p$.

In this study different data sets, different assumptions, and different function than the previous models are used to fit the size and shape of the magnetopause. We also use in situ magnetopause crossings and a single function to determine the size and shape of the magnetopause at both dayside and nightside locations. Also, 5-min average IMF $B_z$ and $D_p$ are used to reflect more precisely the solar wind conditions corresponding to each individual crossing. Moreover, we fit the data simultaneously as a function of $D_p$ and IMF $H_z$ by using a multiple parameter fitting. An explicit function, with simple coefficients, is obtained at the last stage of our analysis.

2. Data Preparation

For this investigation we have used data from ISEE 1 and 2; AMPTE/IRM; GOES 2, 5, and 6; IMP 8; and ISEE 3. The orbits of ISEE 1 and 2, AMPTE/IRM, and IMP 8 traverse the magnetopause and can be used to identify the magnetopause crossings. The two satellites, IMP 8 and ISEE 3, are principally in the solar wind and were used to obtain solar wind conditions corresponding to each individual magnetopause crossing. The inclination of ISEE 1 and 2 satellites were initially 30°, with a perigee of 1.5 $R_e$ and an apogee of 22.5 $R_e$. The orbital period is 57 hours [Russell, 1975]. We used 4-s resolution data. The AMPTE/IRM satellite was launched on August 16, 1984, with a perigee of 557 km and an apogee of 18.83 $R_e$. The inclination is 28.5° and the orbital period is 44.3 hours. The data from AMPTE/IRM were obtained from the three-dimensional (3-D) plasma instrument and the fluxgate magnetometer. We used 4-s resolution data. Further details of these instruments are described by Paschmann et al. [1985] and Lühr et al. [1985]. The GOES series of geosynchronous meteorological satellites have two-axis magnetometers which measured the vector magnetic field at a 6.6 $R_e$ geosynchronous orbit by taking advantage of spacecraft spin.
We used data from the period January 1978 to December 1986 [Rufenach et al., 1989]. The IMP 8 satellite was launched on October 26, 1973, in an orbit covering a region from 25 \( R_e \) to 45 \( R_e \). Its initial orbit was more elliptical than intended, and its eccentricity decreased after launch. Its orbital inclination varied between 0° and about 55° with a periodicity of several years. The data from IMP 8, with a 5-min resolution, were averaged from higher resolution data by the National Space Science Data Center (NSSDC). The ISEE 3 satellite provided nearly continuous solar wind measurements in an orbit covering a region from 200 \( R_e \) to 260 \( R_e \) upstream of the Earth until August 1983. Note that the IMP 8 and ISEE 3 data we used in the present study were normalized to a uniform density calibration independent of ion temperature and velocity [Petrincc and Russell, 1993b].

The criterion that the magnetic field undergoes a sudden change in strength or direction, such as when crossing the magnetopause current layer, has been used to identify magnetopause crossings [Berchem and Russell, 1982; Paschmann et al., 1986; Song et al., 1988]. This criterion is good for identifying crossings which have a large magnetic shear at the magnetopause. Large shear at the dayside magnetopause usually accompanies southward IMF conditions. However, in this study in some situations, for example, for northward IMF, the field rotation was less than 20° and the field magnitude changed very little. It was hard to identify this kind of low-shear crossing using magnetometer data alone. Therefore, the plasma data were also used for identification. Low-shear crossings have been studied by Russell and Kiplic [1978] and Paschmann et al. [1978, 1993].

A total of 860 crossings were obtained using these identification criteria. Note that all of these crossings are from separate passes. If multiple crossings occur within an hour, the innermost crossing was chosen. Multiple magnetopause crossings are usually caused by the magnetopause oscillation corresponding to upstream variations. Many of the crossings are detected when the magnetopause moves from one equilibrium position to another and therefore do not represent the equilibrium position. Under these circumstances, we have chosen to use the innermost crossing to represent the pass. However, we note the possibility that the magnetopause may oscillate around its equilibrium position due to some instabilities such as Kelvin-Helmholtz instability. The percentage of multiple-crossing passes is only 13%, so there is little difference between using the innermost crossing and using the median crossing.

Since the magnetopause size and shape changes with variations in solar wind conditions, we need to know the solar wind condition for each individual crossing. The solar wind data from IMP 8 and ISEE 3 were used to specify the solar wind conditions during magnetopause crossings observed by ISEE 1 and 2 and AMPTE/IRM. We note that for much of IMP 8’s lifetime, there was no solar wind measurement available when IMP 8 crossed the magnetopause. Since we only use data from IMP 8 for \( X > 0 \), where \( X = X_{GSM} = X_{GSE} \) is the position of IMP 8 in a Sun-Earth direction, IMP 8 is always sunward of the Earth for these crossings. The time lag for the solar wind flowing from IMP 8 or ISEE 3 to the magnetopause has been taken to be 10 min for IMP 8 and 55 min for ISEE 3. Using 5-min-average data, the possible time lags for the IMP 8 data are 0, 5, 10, 15, and 20 min. Therefore, we chose 10 minutes as an average time lag. The data from IMP 8 were first used to examine solar wind conditions. If a data gap occurred or if IMP 8 was not in the solar wind, the data from ISEE 3 were used. Since the data from ISEE 1 and 2, AMPTE/IRM, and IMP 8 may have biases caused by the apogee of their orbits, we only used the crossings from positions \( X \geq -6 R_e \) for ISEE 1 and 2 and AMPTE/IRM, and \( X \geq -32 R_e \) for IMP 8. Moreover, we only used the crossings which had both corresponding solar wind plasma and magnetic data. Under these restrictions, there were 553 crossings available for our statistical study. There are 282 crossings from ISEE 1, 235 crossings from ISEE 2, 15 crossings from AMPTE/IRM, and 21 crossings from IMP 8.

The interplanetary control variables, \( D_p \) and \( B_z \), for these 553 crossings are plotted in Figure 2. The average value of \( D_p \) is 1.915 nPa and the average value of \( B_z \) is -0.595 nT. The solid elliptic curve in Figure 2 contains 92% of the entire data set. Six percent of the entire data set is in the area between the solid and dotted curves. The uncertainty is smaller in the area inside the solid curve. The uncertainty is larger in the area between the solid and dotted curves. Outside the dotted curve, the number of data points is not sufficient to obtain \( r_0 \) and \( \alpha \).

Figure 2. The interplanetary control variables, \( D_p \) and \( B_z \), for each individual crossing. The solid elliptic curve encloses 92% of the entire data set. The area between the solid and dotted curves contains 6% of the entire data set.
The motion of the Earth around the Sun causes an aberration or apparent rotation of the direction of flow and hence of the positions of magnetopause crossings. The solar wind aberration has been corrected by a rotation of angle \( \theta_{\text{aberr}} = \tan^{-1} \frac{V_{\text{sw}}}{V_{\text{es}}} \), where \( V_{\text{es}} \) is the velocity of Earth around Sun (30 km/s), and \( V_{\text{sw}} \) is the velocity of the solar wind. Axial symmetry has been assumed in this study. The magnetopause size and shape have been expressed in the aberrated distance perpendicular to the Earth-Sun line \( R = \sqrt{Y_{\text{GSE}}^2 + Z_{\text{GSE}}^2} = \sqrt{Y_{\text{GSM}}^2 + Z_{\text{GSM}}^2} \), where \( R \) is independent of GSE and GSM coordinates. If the magnetopause is not cylindrically symmetric, the cross section we obtain will be most appropriate near the equatorial region where the data were principally acquired.

3. Determination of \( r_0(B_z, D_p) \) and \( \alpha(B_z, D_p) \)

From (1), the radial distance, \( r \), is characterized by \( r_0 \) and \( \alpha \). We have examined the dependence of \( r_0 \) and \( \alpha \) on various solar wind parameters, specifically, IMF \( B_z \), \( B_y \), and \( B_z \); clock angle; cone angle; and \( D_p \). We found that \( r_0 \) and \( \alpha \) are significantly affected by only \( B_z \) and \( D_p \). In this section we will discuss how to obtain the functional forms of \( r_0 \) and \( \alpha \) in terms of \( B_z \) and \( D_p \).

Our objective is to obtain a function to best describe these measurements. The scheme of linear least squares fitting has been used to accomplish this objective. However, since (1) is a nonlinear function, we cannot use it directly. By taking the natural logarithm of both sides of the equation, (1) can be rewritten in a linear form for later use. We have

\[
\ln(r) = -\alpha \ln(1 + \cos \theta) + A
\]

where

\[
A = \ln(r_0) + \alpha \ln(2).
\]

The terms \( \ln(r) \) and \( \ln(1 + \cos \theta) \) are the variables to be fit, and the coefficients \( \alpha \) and \( A \) are the slope and the intercept of the fitting line. Equation (3) can be rewritten as

\[
r_0 = \exp^{A - \alpha \ln(2)}
\]

The position for each individual crossing has been transformed to the forms \( \ln(r) \) and \( \ln(1 + \cos \theta) \), and then plotted in the top panel of Figure 3. The data points show a linear tendency. We applied a linear least squares fit to these points, and obtained \( \alpha = -0.612 \) and \( A = 2.743 \). Using (4), we calculated \( r_0 = 10.161 \) \( R_e \), as noted in the bottom panel of Figure 3. Knowing \( r_0 \) and \( \alpha \), we can derive the size and shape of the magnetopause from (1) and plot it as the solid line in the bottom panel of Figure 3. The standard deviation is defined by

\[
\sqrt{\sum (r_i - r'_i)^2 / N}, \quad \text{where} \quad r_i \text{ is the observation,} \quad r'_i \text{ is the calculation, and} \quad N \text{ is the total number of data points.}
\]

The probable error of the best-fit value is estimated by the standard deviation over the square root of the total number of data points, that is, \( \sqrt{\sum (r_i - r'_i)^2 / N} \). The probable error of the best-fit value in this case is 0.044 \( R_e \). Note that there is an important physical difference between the probable error of best-fit value and the standard deviation. The more times we sample, the better we know the best-fit value. However, the standard deviation of the samples about the best-fit value will remain the same no matter how often we sample.

In order to determine how the magnetopause size and shape varies with different solar wind parameters, we need to allow each parameter to vary independently. Since these conditions rarely exist in nature, there is much scatter in the data. To reduce the scatter, the data should be normalized to the same conditions. As mentioned previously, the size and shape of the magnetopause depends mainly on \( B_z \) and \( D_p \). We will discuss the \( B_z \) effect on magnetopause size and shape by normalizing the data to the average \( D_p \) and discuss the \( D_p \)
effect on magnetopause size and shape by normalizing the data to the average $B_z$.

The location of the magnetopause is determined by pressure balance between the solar wind dynamic pressure, $D_p$, and the pressure of the geomagnetic field, $B^2/2\mu_0$, where $\mu_0$ is the permeability of free space. It is expected that the subsolar magnetopause location, $r_0$, varies as the one-sixth root of the $D_p$ [Schield, 1969]:

$$r_0 = k(D_p)^{-\frac{1}{6}}$$  \hspace{1cm} (5)

where $k$ is a constant of proportionality. To normalize the data to the average value of $D_p$, we calculate the adjustment amount $\delta r_0$ of the standoff point determined from the average $D_p$ (= 1.915 nPa) value and from the corresponding $D_p$ value for each individual crossing using (5). If the $D_p$ value is greater than the average $D_p$ value, $\delta r_0$ is positive, and vice versa. The amount of $\delta r_0$ is traced back along $\alpha = 0.612$ (assuming the magnetopause is self-similar, and $\alpha$ is calculated from a linear fit in Figure 3) to calculate the adjustment amount $\delta r$ at the location of a crossing. We add $\delta r$ to the observed $r$ and obtain a normalized $r$ which corresponds to the average $D_p$. The same procedure used for producing

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**Figure 4.** The same format as Figure 3, but for all the data with normalization by $D_p$.

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**Figure 5.** The same format as Figure 3, but for northward IMF data with normalization by $D_p$.

Figure 3 was applied to the normalized data and plotted in Figure 4. The scatter of the data is reduced, and the probable error of the best-fit value decreases from 0.044 $R_e$ to 0.036 $R_e$.

The normalized data were separated into three bins on the basis of the orientation of the IMF; northward IMF ($B_z \geq 1.5$ nT), horizontal IMF ($-1.5$ nT $\leq B_z < 1.5$ nT), and southward IMF ($B_z < -1.5$ nT). The data in each bin have been processed and plotted in the same format as Figure 3. Figure 5 shows the northward IMF case. In this case the probable error of the best-fit value is 0.064 $R_e$, $\alpha$ is 0.563, and $r_0$ is 10.270 $R_e$. When the IMF is in the horizontal direction, as shown in Figure 6, the standoff distance and $\alpha$ do not change much. This suggests that there is no reconnection process occurring, and no magnetic flux removed from the dayside to the nightside for the cases of northward and horizontal IMF. However, both $r_0$ and $\alpha$ show significant changes when the IMF is southward, as shown in Figure 7. The best-fit value of $r_0$ changes from 10.270 $R_e$ to 9.451 $R_e$ and $\alpha$ changes from 0.563 to 0.651. As mentioned previously, a larger $\alpha$ means larger flaring in the tail. This implies that some magnetic flux has been transferred.
Figure 6. The same format as Figure 3, but for horizontal IMF data with normalization by \(D_p\). This feature is consistent with the results of Petrinec et al. [1991], Petrinec and Russell [1993a, 1996], and Roelof and Sibeck [1993].

Petrinec and Russell [1993a, 1996] limit their discussion to the range from \(-10\) to \(10\) nT, and Roelof and Sibeck [1993] limit their results from \(-7\) nT to \(7\) nT (IMF \(B_z\)). In this study we provide a formula for a greater range of \(B_z\), as indicated in Figure 2. The data have been separated into 34 overlapping bins, from \(-18\) nT to \(15\) nT, which are \(6\) nT wide and are shifted by \(1\) nT. The data in each bin have been processed by the same procedure used for producing Figure 3. We determine \(r_0\) and \(\alpha\) for each bin representing a specific value of IMF \(B_z\). For bins in the ranges \(-18\) nT \(< B_z < -12\) nT and \(8\) nT \(< B_z < 15\) nT, there are few data points. However, these data points are distributed evenly over local time, and we still can obtain a reasonable \(r_0\) and \(\alpha\). Outside the range \(-18\) nT \(< B_z < 15\) nT, the parameter \(\alpha\) becomes unreasonable. Figure 8 shows how \(r_0\) and \(\alpha\) vary with IMF \(B_z\). The diamond symbol with an error bar represents the best-fit value and probable error of the best-fit value for each bin. The number indicated above or below each error bar shows the number of data points in each bin. The variation of \(r_0\), as shown in the top panel of Figure 8, can be represented as two linear portions with a break at \(B_z = 0\) nT. The linear least squares fits have been applied separately to the points on either side of \(B_z = 0\) nT. The slopes of the right and left lines are 0.013 and 0.131, respectively. The best-fit value of \(r_0\) decreases when the value of southward IMF increases. This implies that the more the southward IMF, the more magnetic flux is removed from the subsolar region. Therefore, tail flaring increases proportionally to the flux lost at the subsolar region. As shown in the bottom panel of Figure 8, the best-fit value of \(\alpha\), which has been fit to one line, decreases with a slope of 0.011 when the IMF \(B_z\) increases. We can write the fitting results in the following form:

\[
r_0 = \begin{cases} 
10.150 + 0.013 B_z, & \text{for } B_z \geq 0 \\
10.146 + 0.131 B_z, & \text{for } B_z < 0
\end{cases}
\]

(6)

\[
\alpha = 0.590 - 0.011 B_z. 
\]

(7)

Note that (6) and (7) only apply for \(D_p = 1.915\) nPa, the average \(D_p\) for all crossings. The associated errors for (6) are 0.160 \(R_e\) for \(B_z \geq 0\) and 0.155 \(R_e\) for \(B_z < 0\). The associated error for (7) is 0.039.

Figure 7. The same format as Figure 3, but for southward IMF data with normalization by \(D_p\).
magnetosheath as a magnetopause crossing. This error would cause the radial distance of the magnetopause to be abnormally large. Moreover, $r_0$ is also different in Figures 8 and 9 for $B_z < 5$ nT. The time resolution of the solar wind measurements used to normalize the data sets may contribute to this discrepancy. Roelof and Sibeck [1993] used a 1-hour average of solar wind data sets, whereas we used 5-min resolution. Thus, several 1-hour intervals which average to a small positive value of IMF $B_z$ may actually have been southward during the period of time that the magnetopause was crossed. The same argument can be applied to intervals which average to small southward values of IMF $B_z$. Therefore, hourly averages of solar wind parameters will blur the trends in the magnetopause standoff distance for northward and southward IMF conditions. In our study the overlapping bins may also blur the response of the magnetopause position to weak northward and southward IMF. However, we represent all the data points as two

![Graph showing the variation of $r_0$ and $\alpha$ with $B_z$. The relation is for $D_p = 1.915$ nPa. The diamond symbols represent the best-fit values of $r_0$ and $\alpha$. The error bar shows the probable error of the best-fit value. The solid lines show the fits. The number indicated above or below each error bar shows the number of data points for each bin.](image)

**Figure 8.** The variation of $r_0$ and $\alpha$ with $B_z$. This relation is for $D_p = 1.915$ nPa. The diamond symbols represent the best-fit values of $r_0$ and $\alpha$. The error bar shows the probable error of the best-fit value. The solid lines show the fits. The number indicated above or below each error bar shows the number of data points for each bin.

We now apply the same procedure to the entire Roelof and Sibeck [1993] data set. The results are shown in Figure 9. These results are strikingly different from our results. The best-fit value of $r_0$ increases linearly until $B_z = 8$ nT and then increases sharply for larger values of northward IMF (Note that Roelof and Sibeck [1993] did not discuss the range $B_z > 7$ nT). We believe their data set contains several difficulties. First, their data were compiled by different investigators using different criteria. Thus it is not a homogeneous data set. Second, the magnetic fields on both sides of the magnetopause were in the same direction when the IMF was northward. If the investigations did not use plasma data to aid in identifying the crossing, their procedure risks identifying a discontinuity convected through the

![Graph showing the same format as Figure 8, but using Roelof and Sibeck [1993] data.](image)

**Figure 9.** The same format as Figure 8, but using Roelof and Sibeck [1993] data.
linear portions with a break in the slope at $B_z = 0$ nT. The slopes of the two portions have been determined by using a multiple parameter fit at the last stage of this fitting procedure. Also, the data set from Sibeck et al. [1991] and Roelof and Sibeck [1993] used crossings from spacecraft both in and outside of the equatorial plane. Sibeck et al. [1991] show a figure which illustrates different magnetopause locations in the equatorial plane and in the noon midnight meridian plane. Asymmetries due to the cusp could also contribute to the discrepancies.

Using the data set constructed for the study in this paper, we have shown how $r_0$ and $\alpha$ change with IMF $B_z$. Now we want to see how these two parameters vary with $D_p$. In a manner similar to our earlier analysis, the data need to be normalized to the same value of IMF $H_z$, $-0.395$ nT (the average $H_z$ for all crossings). We can obtain $r_0$ and $\alpha$ from (6) and (7) for a given $B_z$ and then the location of the magnetopause. Therefore, we can calculate $\delta r$ between the locations for the average $B_z$ and for each individual crossing (each crossing has a corresponding $H_z$) at a specific $\theta$. We add $\delta r$ to observed $r$ and obtain a normalized $r$ which corresponds to the average $B_z$. This procedure reduces the probable error of the best-fit value to 0.043 Re, as shown in Figure 10. These normalized data have been divided into three bins on the basis of the value of $D_p$: $D_p \geq 2.5$ nPa, $1.5 \leq D_p < 2.5$ nPa, and $D_p < 1.5$ nPa, and the results are plotted in Figures 11-13. Comparing the best-fit values of $r_0$ and $\alpha$ in these figures, we find that the best-fit value of $\alpha$ increases slightly with increasing $D_p$ and that the best-fit value of $r_0$ increases when $D_p$ decreases. These results imply that $D_p$ also has a role in flux transfer from the dayside to the nightside. To further understand this relation, the data have been separated into 17 overlapping bins, from 0.5 nPa to 8.5 nPa, which are 4 nPa wide and are shifted by 0.5 nPa. For bins in the range 6 nPa < $D_p < 8.5$ nPa, there are few data points. However, these data points distribute evenly over local time, and we still can obtain reasonable values for $r_0$ and $\alpha$. Outside the range of 0.5 nPa < $D_p < 8.5$ nPa, $\alpha$ is unreasonable. The best-fit values of $r_0$ and $\alpha$ for each bin are plotted in Figure 14. The best-fit value of $r_0$ has a power law of $-1/6.6$ with respect to $D_p$, which is slightly different from the $-1/6$ theoretical prediction. A linear trend for $\alpha$ has a 0.010 slope. The fitting results for $B_z = -0.395$ nT can be written as follows:

$$\ln(r) = -0.608 \ln(1+\cos(\theta)) + 2.619$$

$D_p > 2.5$ nPa

$\text{Probable Error} = 0.083$ Re

$\text{Data Number} = 108$

$\alpha = 0.608 \quad r_0 = 9.003$ Re

$\text{Probable Error} = 0.038$ Re

$\text{Data Number} = 108$

$\ln(r) = -0.608 \ln(1+\cos(\theta)) + 2.619$

$R(\text{Re})$

$X(\text{Re})$

$\alpha = 0.608 \quad r_0 = 9.003$ Re

$\text{Probable Error} = 0.038$ Re

$\text{Data Number} = 108$

$\ln(r) = -0.608 \ln(1+\cos(\theta)) + 2.619$

$R(\text{Re})$

$X(\text{Re})$
1.5 nPa < $D_p$ < 2.5 nPa
Probable Error = 0.065 Re  Data Number = 145

$$
\ln(r) = -0.601 \ln(1+\cos(\theta)) + 2.716
$$

Figure 12. The same format as Figure 3, but for intermediate $D_p$ with normalization by $B_z$.

$D_p$ simultaneously by using a multiple parameter fitting technique. We have used a gradient search nonlinear optimization technique [Blevington, 1969; Khurana and Kivelson, 1994] to obtain the least squares solution. The technique predicts the locations of the magnetopause crossings by following the gradient of the root mean square difference in the parameter space. Since the functional form for the relationship between the dependent variable (magnetopause location) and the independent variables ($B_z$ and $D_p$) is known, we can combine (6)-(9) to obtain

$$
r_0 = \begin{cases} 
(a_1 + a_2 B_z)(D_p)^{-\frac{1}{a_3}}, & \text{for } B_z \geq 0 \\
(a_1 + a_3 D_p)(1 + a_4 D_p), & \text{for } D_p < 0
\end{cases}
$$

where parameters $a_1$ through $a_4$ are to be optimized by using the gradient search technique. Note that the work of previous sections is necessary to obtain the form of (10) and (11). The initial seed values of these parameters are shown in the first column of Table 1. We have added a factor of 1.16 to the proton's dynamic pressure.

4. Multiple Parameter Fitting

In the previous section the effect of dynamic pressure on the magnetopause size and shape was obtained by normalizing the crossing data for the effect of $B_z$. Similarly, the effect of $B_z$ on the magnetopause location was obtained by normalizing the data for the dynamic pressure. In this section we will consider the effect of $B_z$ and $D_p$ simultaneously by using a multiple parameter fitting technique. We have used a gradient search nonlinear optimization technique [Blevington, 1969; Khurana and Kivelson, 1994] to obtain the least squares solution. The technique predicts the locations of the magnetopause crossings by following the gradient of the root mean square difference in the parameter space. Since the functional form for the relationship between the dependent variable (magnetopause location) and the independent variables ($B_z$ and $D_p$) is known, we can combine (6)-(9) to obtain

$$
r_0 = \begin{cases} 
(a_1 + a_2 B_z)(D_p)^{-\frac{1}{a_3}}, & \text{for } B_z \geq 0 \\
(a_1 + a_3 D_p)(1 + a_4 D_p), & \text{for } D_p < 0
\end{cases}
$$

where parameters $a_1$ through $a_4$ are to be optimized by using the gradient search technique. Note that the work of previous sections is necessary to obtain the form of (10) and (11). The initial seed values of these parameters are shown in the first column of Table 1. We have added a factor of 1.16 to the proton's dynamic pressure.
Table 1. The Coefficients of Equations (10) and (11) Before and After the Multiple Parameters Fit

<table>
<thead>
<tr>
<th></th>
<th>Before Fit</th>
<th>After Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>10.2</td>
<td>11.4 ± 0.2</td>
</tr>
<tr>
<td>(a_2)</td>
<td>0.013</td>
<td>0.013 ± 0.001</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.13</td>
<td>0.14 ± 0.01</td>
</tr>
<tr>
<td>(a_4)</td>
<td>6.6</td>
<td>6.6 ± 0.8</td>
</tr>
<tr>
<td>(a_5)</td>
<td>0.59</td>
<td>0.58 ± 0.02</td>
</tr>
<tr>
<td>(a_6)</td>
<td>-0.011</td>
<td>-0.010 ± 0.002</td>
</tr>
<tr>
<td>(a_7)</td>
<td>0.010</td>
<td>0.010 ± 0.001</td>
</tr>
</tbody>
</table>

The right column shows the coefficients before the fit; and the left column shows the coefficients with their uncertainties after the fit.

1.97 \(R_e\) to 1.24 \(R_e\) after we apply the multiple parameter fit. Equations (10) and (11) can now be rewritten as

\[
r_0 = \begin{cases} 
(11.4 + 0.013B_z)(D_p)^{-0.5}, & \text{for } B_z \geq 0 \\
(11.4 + 0.14B_z)(D_p)^{-0.5}, & \text{for } B_z < 0 
\end{cases} \tag{12}
\]

\[
\alpha = (0.58 - 0.010B_z)(1 + 0.010D_p), \tag{13}
\]

The dependence of \(r_0\) and \(\alpha\) on the solar wind parameters \(B_z\) and the logarithm of \(D_p\) can be illustrated graphically by using contour and surface plots. Figure 16 shows \(r_0\) and Figure 17 shows \(\alpha\). The asterisks in the bottom panel of Figure 16 show magnetopause crossings at geosynchronous orbit taken from Table 1 of Rufenach et al. [1989]. Five-minute resolution observa-

Figure 14. The variation of \(r_0\) and \(\alpha\) with \(D_p\). This relation is for \(B_z = -0.595\) nT. The diamond symbols represent the best-fit value of \(r_0\) and \(\alpha\). The error bar shows the probable error of the best-fit value. The solid lines show the fits. The number indicated above or below each error bar shows the number of data points for each bin.

to reflect the contribution from the solar wind helium content. Neglecting the helium content will cause the magnetopause location to be overestimated by 2.5%. The final optimized values and uncertainties are shown in the second column of the table. The uncertainty of each coefficient is obtained by the Monte Carlo simulation method. The multiple parameter fittings have been run 200 times using one third of the total points, which are sampled randomly each time. We obtain 200 sets of coefficients and take their standard deviations as their uncertainties. The comparison between the analytic and the observed radial locations of the magnetopause are shown in Figure 15. The solid line has a slope of 1. The data points are distributed evenly around the solid line. The standard deviation between the analytic and the observed values is improved from

Figure 15. The comparison between the observed and the analytic radial distances. The solid line represents where both the observed and the analytic values are the same.
netopause moves earthward and tail flaring increases when the IMF is southward. Also, the size and shape of the magnetopause is not self-similar for various solar wind dynamic pressure when the IMF is northward. However, as discussed below, there are some differences between our model and the others.

Figure 18 shows a comparison with the Petrinec et al. [1991] model. Petrinec et al. [1991] only divided the data which were normalized by $D_p = 2$ nPa, into southward and northward bins. In comparison to their results, we chose $D_p = 1.915$ nPa and $B_z = 2.856$ nT for northward IMF, and $B_z = -3.403$ nT for southward IMF. We obtained these values by averaging the corresponding $B_z$ values for the northward and southward IMF bins, respectively. The solid curves in Figure 18 represent the size and shape of the magnetopause from the present study, and the dotted lines show the size and shape from their model. It can be seen that our results are very consistent with their results only on the dayside. Since they fit the data to an elliptic form, this

5. Comparisons With Previous Models

In this section we compare our results with models developed by Petrinec et al. [1991], Petrinec and Russell [1993a, 1996], and Roelof and Sibeck [1993]. All these models and ours have confirmed that the dayside mag-

---

**Figure 16.** Surface and contour plots for $r_e$ as a function of $B_z$ and $\log(D_p)$. The contour interval is 0.5 $r_e$. The star signs show the GOES crossings at 6.6 $r_e$.

**Figure 17.** Surface and contour plots for $\alpha$ as a function of $B_z$ and $\log(D_p)$. The contour interval is 0.04.
for $D_z \geq 0$ nT, $B_z = -5$ nT, and $B_z = -10$ nT for three different values of $D_p$. They showed no dependence on northward $B_z$. We have plotted our results in the same format as theirs in the top panel of Figure 19. We find that our three curves intersect with each other (at $-12$ $R_e$ for $D_p = 0.5$ nPa, at $-10$ $R_e$ for $D_p = 2$ nPa, and at $-6$ $R_e$ for $D_p = 8$ nPa), and the position of the intersecting point depends on $D_p$. Our tail radii are typically smaller than theirs, and the difference between ours and theirs is the largest for the case when $D_p = 0.5$ nPa. Petrinec and Russell [1993a] has a similar convergence of solutions at the terminator as ours. Their dependence on $D_p$ is stronger than this model's. Note that they used data determined from the pressure balance between the solar wind and magnetosphere and not directly from in situ magnetopause measurements.

Roelof and Sibeck [1993] also studied the size and shape of the magnetopause covering both dayside and nightside and for various IMF $B_z$ and $D_p$ conditions. The equation they used is an ellipse. An ellipse must close at some point in the tail, thus it cannot represent a magnetopause that continues to flare. Their results were shown in Figure 9 of their paper. For comparison with their results, we chose two cases which are similar to theirs (rows 1 and 4 in Figure 20) and another two cases which are different from theirs (rows 2 and 3 in Figure 20). The left column is produced using this model and the right column is from theirs. The Roelof and Sibeck [1993] model shows that the magnetotail flaring for different $R_e$ values depends strongly on $D_p$. Also, the flaring varies with $D_p$ when the IMF is northward. In the present study the flaring changes slightly with $D_p$ for northward and southward IMF $B_z$.

6. Conclusions

This study provides another look at the fitting of the magnetopause size and shape which is independent of Petrinec et al. [1991], Petrinec and Russell [1993a, 1996], and Roelof and Sibeck [1993]. We have used crossings from ISEE 1 and 2, AMPTE/IRM, and IMP 8 satellites, which we identified using consistent criteria, and fit in a new functional form which is characterized by two parameters, $r_0$ and $\alpha$. The parameters $r_0$ and $\alpha$ represent the balance between the solar wind dynamic pressure and the magnetic pressure of the Earth's dipole field, and the level of flaring, respectively. The parameters $r_0$ and $\alpha$ are controlled by the IMF $B_z$ and $D_p$. To obtain the initial relations, we normalized the data by the average $D_p$ or $D_z$ and binned the data using a range of $B_z$ or $D_p$ conditions. Then we used a multiple parameter fit to determine the coefficients for $r_0$ and $\alpha$ as a function of $R_e$ and $D_p$. The final form of the fit with coefficients for the size and shape of the magnetopause is given in (1), (12), and (13). These equations have some general features; for example, the magnetopause moves inward when the IMF is southward, and the standoff distance increases very slightly when the northward IMF increases. For the tail region, the flaring becomes larger when the IMF is southward, which is
generally believed to be caused by reconnection at the dayside removing magnetic flux to the nightside. Also the flaring of the magnetopause increases slightly as $D_p$ increases.

Moreover, we obtain a power law of $-1/(6.6\pm0.8)$ for $r_n$ versus $D_p$. The theoretical prediction is $-1/6$ for a perfect dipole in a vacuum, which is within the error range. Some variation in this value may be due to the thermal pressure interior to the magnetosphere. Another possible cause for the variation is associated with neglecting of the static pressure (especially magnetic pressure) in the solar wind. When the solar wind has a small dynamic pressure but a large magnetic pressure, the values of the standoff distance may actually correspond to a higher total pressure of the solar wind, and the effect is to move the power law curve closer to $-1/6$. However, we have examined the real solar wind measurements in our data set, and the difference between the dynamic and magnetic pressures is about 2 orders of magnitude. We also have added the magnetic pressure into the total pressure and recalculated the power law. The power law remains unchanged.

Petricec et al. [1991] and Petrinec and Russell [1993a] used two separate functional forms to represent the size and shape of the magnetopause at both dayside and nightside locations, but the form derived in this paper is a single functional form which eliminates the flaring angle discontinuity across the terminator. Furthermore, our model is an explicit expression in a simple conic form. This formula is useful for operational space applications such as predicting when satellites at geosynchronous orbit will find themselves in the magnetosheath.

Determining the size and shape of the magnetopause in the tail is a very difficult problem. Several factors influence the magnetopause position, and these influences become more important at further distances downstream from the Earth. The most notable of these influences are the exact (rather than average) angle of the solar wind flow vector with respect to the Sun-Earth line;
the dipole tilt angle of the Earth (asymmetries in the tail cross section that may result); the static pressure of the solar wind; and dynamic processes internal to the magnetotail (i.e., substorm phase). The error in our magnetopause model may be larger when we extend it further downtail. In our study there are only 21 data points beyond $X = -10 \, R_e$ because of many data gaps in the IMP 8 data and the corresponding solar wind data from ISEE 3 is limited. In spite of these limitations, our model seems to fit the data very well, as illustrated in Figures 5-7 which show the fit for different IMF $B_z$ conditions normalized by dynamic pressure and in Figures 11-13 which show the fit for different dynamic pressure normalized by IMF $B_z$.

The comparisons with previous models serve to show how different data sets, with different assumptions, and different functional forms can lead to very different dependencies of the magnetopause size and shape on solar wind parameters. Also, the size and shape of the magnetopause is uncertain at extreme values of the IMF $B_z$ and solar wind dynamic pressure. The magnetopause size and shape cannot be determined with more certainty until additional data sets are available from future missions.
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