Physics 338 LAB 1 Spring 2008

AC Circuits
Impedance & Filters

0. (See H&H p.29–42) Impedance ($\mathbf{Z}$) is the generalization of resistance to alternating current (“ac”) circuits, particularly circuits in which the electrical variables change sinusoidally ($\omega = 2\pi f$):

$$V = V_0 \cos(\omega t + \phi) = \mathrm{Re}\left(V_0 e^{j\phi}\, e^{j\omega t}\right) = \mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right)$$

Note that $\mathbf{V}$ is a complex number (i.e., $\mathbf{V} = a + bj$ for some real a and b) with magnitude V_0 and phase ϕ which can be related to the real and imaginary parts of $\mathbf{V}$:

$$\begin{aligned} V_0 &= \sqrt{a^2+b^2} = |\mathbf{V}| \qquad & a &= V_0 \cos\phi = \mathrm{Re}\,(\mathbf{V}) \\ \tan\phi &= b/a & b &= V_0 \sin\phi = \mathrm{Im}\,(\mathbf{V}) \end{aligned}$$

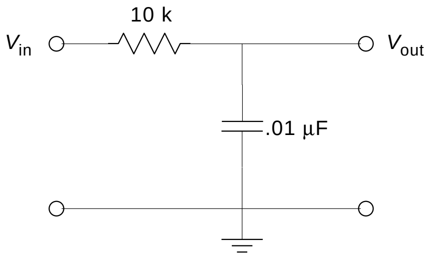
$V = IR$ is now generalized to $\mathbf{V} = \mathbf{IZ}$, which immediately implies that $|\mathbf{V}| = |\mathbf{I}| \cdot |\mathbf{Z}|$ and that the phase difference between $\mathbf{V}$ and $\mathbf{I}$ is the phase of $\mathbf{Z}$.

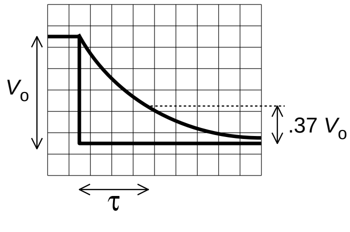
We note the following formulae for impedance:

component	impedance	phase relationship	frequency dependence of $\mathbf{Z}$
resistor	$\mathbf{Z}_R = R$	voltage & current in phase	constant
capacitor	$\mathbf{Z}_C = -j/\omega C$	voltage lags current	small at high frequency
inductor	$\mathbf{Z}_L = j\omega L$	voltage leads current	large at high frequency

1. *RC* Time Constant

Construct the circuit shown and drive it with a 500 Hz square wave. (Note: for this class always use the WAVETEK function generator as $V_{\rm in}$.) Sketch the input together with the output as seen on the scope. Measure the time constant τ for discharging (or charging) and compare it with the expected value: RC. For example, while discharging, the voltage initially V_0 above the square-wave bottom will asymptotically approach that bottom value. It falls to $V_0 e^{-1} \approx .37 V_0$ above the bottom value during a time interval of τ. Locate the spot on the scope where the trace is $.37 V_0$ above the bottom value and measure the interval of decay which is τ. Now vary the frequency of $V_{\rm in}$ up to about 100 kHz. Sketch/describe how the output's magnitude and waveshape change as the frequency is increased. (Of course, you will change the scope's scales for $V_{\rm out}$ to make the signal visible.)





2. *RC* Integrator

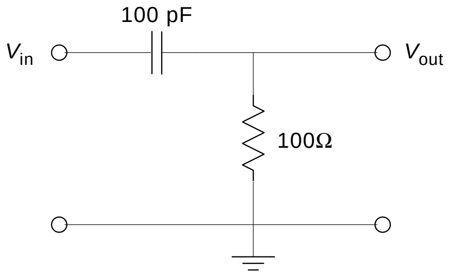
Set the frequency to about 100 kHz. At these frequencies ($f \gg f_{-3\rm dB} = 1/(2\pi RC)$), $V_{\rm out}$ is the integral of $V_{\rm in}$, hence the name “*RC* integrator.” Demonstrate that this circuit does in fact integrate by driving it with all the different types of waveforms available on your function generator; sketch corresponding $V_{\rm in}$ and $V_{\rm out}$ pairs. Convince me that $V_{\rm out} \propto \int V_{\rm in}\, dt$. What is the circuit's input impedance at dc and at infinite frequency?

3. Low-pass Filter

   This same circuit also operates as a low-pass filter. Compute its -3 dB frequency $f_{-3\text{dB}}$ (at which $V_{\text{out}} = V_{\text{in}}/\sqrt{2}$) and verify this value experimentally using a sine wave input. Vary the the frequency from 10 Hz to 1 MHz while viewing V_{out} on the scope; briefly report what you observe. Now make a table of attenuation ($V_{\text{out}}/V_{\text{in}}$) and phase shift as measured at frequencies of about 0.1, 0.2, 1, 2, 5, 10, 20, and 100 times $f_{-3\text{dB}}$. (Note if you set V_{in} to an convenient value—like 1 V rms— and use the MEASURE menu on the scope, the calculation of $V_{\text{out}}/V_{\text{in}}$ is trivial. Simultaneous display of V_{in} and V_{out} on your scope allows you to observe the lead/lag time and convert it to phase shift: $\phi = 360° \ \Delta t/T$, where Δt and T can be measured in divisions by eye. Alternatively you could use the CURSOR menu to accurately find the times in seconds, but that level of accuracy is not required here.) Using a straightedge and supplied graph paper, make an accurate *Bode plot*: a log-log graph of attenuation vs. f. Check that the plot's slope for $f \gg f_{-3\text{dB}}$ is -6 dB per octave (or, equivalently, -20 dB per decade). What is the expected phase shift at $f_{-3\text{dB}}$? Does V_{out} lead or lag V_{in}?

4. *RC* Differentiator

   Construct the circuit shown (note the new component values), drive it with a 100 kHz triangle wave, and check that V_{out} is the time derivative of V_{in}. Verify the circuit's differentiation behavior by sketching V_{in} and V_{out} for all the other types of waveforms available on your function generator. Convince me that $V_{\text{out}} \propto \frac{d}{dt}V_{\text{in}}$. Why is the output signal so small? (Hint: Calculate this circuit's $f_{-3\text{dB}}$.)

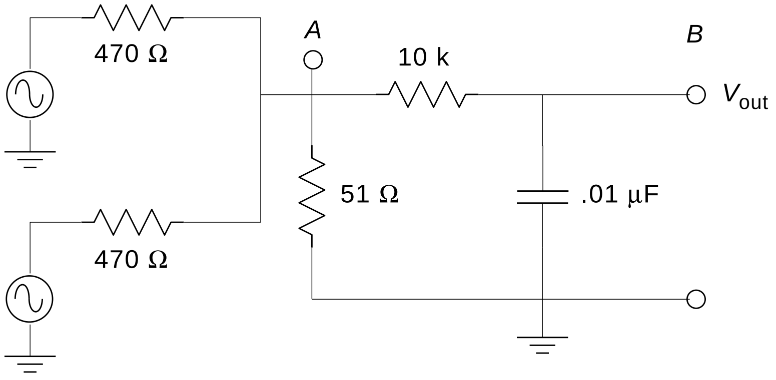


5. High-pass Filter

   Build a high-pass filter from the components of part 1 of this lab. Measure its $f_{-3\text{dB}}$: is it what you would compute it to be? Make a table of attenuation and phase shift as measured at frequencies of about 0.02, 0.05, 0.1, 0.2, .5, 1, 2, and 10 times $f_{-3\text{dB}}$, and again make a log-log plot of attenuation vs. f. Verify that the slope is $+6$ dB per octave for $f \ll f_{-3\text{dB}}$ What is the expected phase shift at $f_{-3\text{dB}}$? Does V_{out} lead or lag V_{in}?

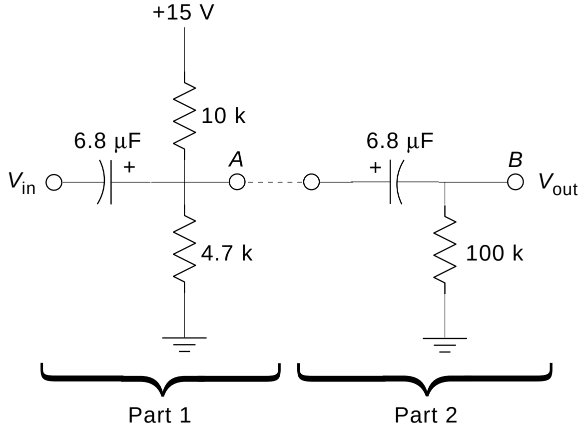
6. A Filter Example

   Construct this circuit using two function generators. What is the purpose of the two 470Ω resistors? Let the input signals have equal amplitudes, one at frequency $\sim \frac{1}{10} f_{-3\text{dB}}$ and the other at $\sim 10 \ f_{-3\text{dB}}$. Observe and sketch the signal at A and at the filter's output B. Play around with it until you can explain what the circuit is doing. Swap the locations of the capacitor and resistor (0.01 μF↔10 kΩ) making a low-pass filter into a high-pass filter. Explain the resulting signal at B.



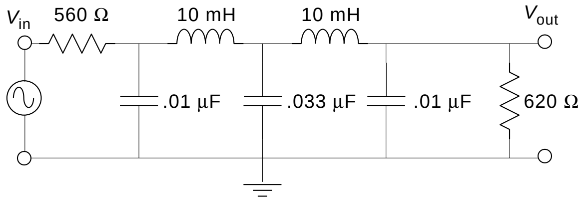
7. Blocking Capacitor

Capacitors are commonly used to block dc voltages while letting ac voltages pass. Construct part 1 of this circuit, which adds a dc voltage to the function generator voltage. Use the scope (on dc coupling, with CH1 and CH2 zero volts set to the same location) to observe (and sketch) the output at A along with V_{in}. Describe what's going on. Now add part 2 of the circuit, where the added capacitor blocks the dc voltage at A. Observe (and sketch as before) the output at B and describe it. Part 2 of the circuit is nothing more than another high-pass filter: what is its $f_{-3\text{dB}}$? How does the capacitor in part 1 of the circuit also function as a blocking capacitor?



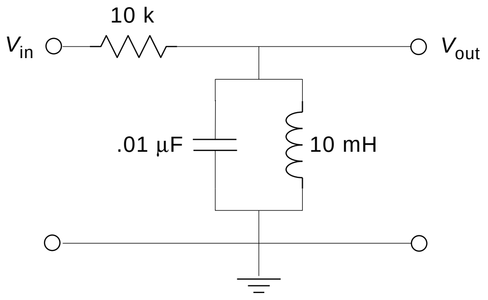
8. *LC* Filter

I've constructed the beastie shown. (For the curious, it's a 5-pole, low-pass Butterworth filter in the π-configuration: see Appendix H of Horowitz and Hill if you want more details.) At low frequencies the gain never exceeds about $\frac{1}{2}$, so $f_{-3\text{dB}}$ is defined[1] as the frequency where the gain is 3 dB lower than it is at low frequencies, i.e., a gain of about $\frac{1}{2\sqrt{2}} = .35$. Using a sine wave as input, take data of attenuation vs. frequency and plot them on log-log paper. Use frequencies of about 0.01, 0.1, 0.2, 0.5, 1, 2, and 4 times $f_{-3\text{dB}}$. How does the slope of this filter's cut-off compare with that of the simple *RC* filter? (I.e., compare the dB-per-octave falloff of this filter to the simple *RC* filter.)



9. *LC* Resonant Circuit

Construct the parallel *LC* resonant circuit shown. Using a constant-amplitude sine wave as input, observe its response over a range of frequencies centered on its resonant frequency, and sketch its resonance curve (i.e., the magnitude of V_{out} as a function of frequency). Measure its resonant frequency and compare to what you would compute from the component values. (*Measure* your component L and C with bridge and capacitance meter.)



---

[1] **Note:** The general definition of $f_{-3\text{dB}}$ is <u>not</u> that the gain is .707, rather that the gain has <u>changed</u> from some standard value by a factor of .707, i.e., the gain is 3 dB less than 'usual'.