Physics 364 MIDTERM EXAM 1999 May 1

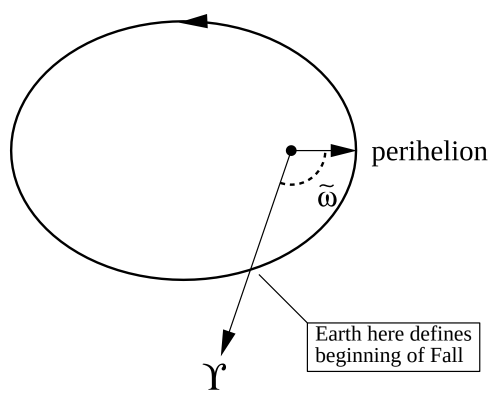
**Answer 10 of the following 14 questions**

1. It is often said that the end stars of the Big Dipper ($\alpha$ and $\beta$) "point" to the pole star Polaris and that the distance from $\alpha$ to Polaris is about 5 times the distance between $\alpha$ and $\beta$. How accurate are these statements? Consider the $\beta$, $\alpha$, Polaris spherical triangle. Find the size of the sides and the value of the "pointing" angle (that should be nearly 180°).

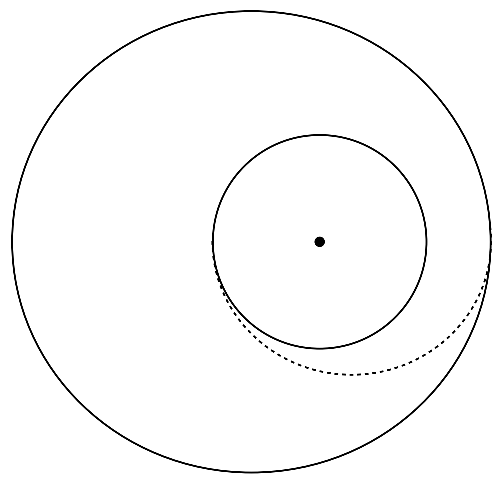
2. At the SJU observatory (latitude= 45° 34' 58", longitude=94° 23' 25") you observe the star Sirius when it is on the meridian, and then again 1 hour, 2 hours and 3 hours later producing photon counts of: 8273, 8102, 7727, and 7132. How many photons would you estimate you'd get without the Earth's atmosphere in the way. What is your estimated error in that count?

3. On 1999 May 2 the location of the Sun will be: RA=$2^h35^m$ dec=$15°11'$. At what altitude will the Sun culminate here? On this date what is the location of the mean Sun? At what time of day will the Sun culminate here? At what hour angle will the Sun have zero altitude (i.e., approximately sunrise)? What is the hour angle of sunrise if you include atmospheric refraction and the diameter of the Sun in your definition of sunrise?

4. Calculate the length of the Earth's seasons (Fall, Winter, Spring, Summer) from the Earth's orbit. Fall starts when the Earth is in the direction of the first point of Aries, and each season lasts 90° of true anomaly. The angle from the first point of Aries to perihelion $\varpi \approx 103.0°$, the eccentricity of the Earth's orbit is $e \approx 0.01674$. Find the mean anomaly for the start of each season and find the length of each season (accurate to 0.01 day).



5. Consider the problem of sending a satellite from Earth to Mars. The motion of the satellite is (of course) ballistic except for brief periods when the rocket engine is fired to change orbits. The transfer orbit must connect the orbits of Earth and Mars:



   Assume that the orbits of Mars and Earth are co-planar, that Earth's orbit is circular, but that otherwise the orbital elements ($e$s and $a$s) are realistic. Calculate the $\Delta v$ needed for transfer from the Earth's orbit to transfer orbit, and from transfer orbit to the orbit of Mars. How long will the trip take? Do the same if the trip is started half a year later than the above picture.

6. Recall that with spectroscopic binaries one can only determine $m \sin^3 \theta$, where $m$ is the mass of the star and $\theta$ is the (unknown) inclination of the orbit. (E.g., if we view the orbit face on $\theta = 0$ and we see no Doppler shift, whereas if we see the orbit edge on $\theta = 90°$ and we see maximum Doppler shifts.) Here is a list of all known measurements of $m \sin^3 \theta$ (in units of $M_{\mathrm{sun}}$) for A2V stars: $\{2.3, 2.3, 2.25, 2.0, 1.9, 1.7, 1.65, 1.6, 1.5, 1.4, 1.4, .44, .32, .035\}$. Note that these measurements are subject to an obvious bias: we don't see the spectroscopic binary if $\sin \theta$ is small. Your job is to come up with a statistical method of extracting an estimate of <u>the</u> mass of A2V stars from this statistical data. The problem is similar to that posed in the bubble chamber lab. You need to come up with an expected cumulative fraction function and then fit it to the data. However in this case the data set has been "censored": we are missing an unknown number of small $\theta$ binaries. Come up with a method of dealing with this problem.

7. The four differential equations of stellar structure are first order, non-linear, and *coupled*. They cannot be solved with "paper and pencil". However, if we simplify them, we can solve them. Consider the case of a star made of constant density material. Write down the differential equations for $P(r)$ and $M(r)$. Solve them! If the Sun had a constant density equal to its average density, find the central pressure.

8. Consider the case of a star made from an isothermal ideal gas, where $P \propto \rho$. Solve the pressure differential equation for $M(r)$ and plug that into the $M(r)$ differential equation. This result, when combined with the ideal gas law, should be a second order differential equation for just $\rho$. Show that $\rho \propto r^{-2}$ solves this differential equation. What is wrong with this solution?

9. Since hydrogen is the most common element seen in the universe, its absorption lines are quite important. Lyman $\alpha$ (L$\alpha$) is the transition: $n = 1 \to n = 2$. Calculate its wavelength and energy in eV. What type (X-ray, UV, visible, IR, ...) of light is it? What wavelength of light would just ionize an H-atom in its ground state? Calculate the wavelength and energy in eV of H$\alpha$ (the lowest energy Balmer series line: $n = 2 \to n = 3$) and of the Balmer limit: $n = 2 \to n = \infty$. Highly excited H-atoms can be seen with radio telescopes. Calculate the frequency and wavelength of $n = 200 \to n = 201$ (called 200$\alpha$).

10. Consider a binary star system: star A (with apparent magnitude $m_A = 1.5$) and star B (with $m_B = 3$) in orbit about each other. What magnitude do they have together? What magnitude would they have together if (in an eclipse) B covered up exactly half of the surface of A?

11. When we talk about the spectrum of a star we are often talking about a plot of $F_\lambda$ vs $\lambda$. What exactly is $F_\lambda$? Give a *careful* definition of $F_\lambda$ including its units and how it might be experimentally determined (*determined:* give me the details of what you would measure, what would you divide by what, etc., to get $F_\lambda$).

12. According to Zombeck's *Handbook of Space Astronomy and Astrophysics* a $V = 0$ star has $F_\nu = 3.67 \times 10^{-23}\mathrm{Wm^{-2}Hz^{-1}}$ at a wavelength of $\lambda = .55\mu$m. If the spectrum has a blackbody shape but with the above normalization, integrate to find the bolometric flux and then convert to a bolometric magnitude. The resulting bolometric magnitude is the bolometric correction (because we started at $V = 0$). Do the above calculation for a blackbody of temperature $T$, and plot the results for $T$ in the range 3,000 K to 30,000 K. Hint: convert the flux integral to dimensionless form and use:

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

13. Given material with the composition of the Sun and compressed to the density present at the Sun's core (see p.299 for data) at what temperature will the radiation pressure equal the gas pressure.

14. Sketch an HR diagram and approximately place the stars in the table on that diagram. Answer the questions using the data in the table.

| Star Name | Absolute Magnitude $M_V$ | Apparent Magnitude $m_V$ | Spectral Type | Luminosity Class |
|---|---|---|---|---|
| 1. Spica | $-3.5$ | $+1.0$ | B1 | III |
| 2. Antares | $-3.8$ | $+1.0$ | M2 | I |
| 3. Sirius | $+1.4$ | $-1.5$ | A1 | V |
| 4. Rigel Kent | $+4.4$ | $+0.0$ | G2 | V |
| 5. Fomalhaut | $+2.0$ | $+1.2$ | A3 | V |
| 6. Deneb | $-7.2$ | $+1.3$ | A2 | I |
| 7. Canopus | $-3.5$ | $-0.7$ | F0 | II |
| 8. Regulus | $-0.7$ | $+1.4$ | B7 | V |
| 9. Aldebaran | $-0.5$ | $+0.9$ | K5 | III |

(a) Which star has the greatest apparent brightness?

(b) Which star has the same spectral type as the Sun?

(c) Which star is intrinsically the brightest?

(d) Which star has the highest surface temperature?

(e) Which star has the lowest surface temperature?

(f) Which star is furthest away?

(g) Which star has the largest radius?