5: Langmuir's Probe

Purpose

The purpose of this lab is to measure some basic properties of plasmas: electron temperature, number density and plasma potential.

Introduction

When you think of electrical conductors, you probably think first of metals. In metals the valence electrons are not bound to particular nuclei, rather they are free to move through the solid and thus conduct an electrical current. However, by far the most common electrical conductors in the Universe are *plasmas*: a term first applied to hot ionized gases by Irving Langmuir (see below). In conditions rare on the surface of the Earth but common in the Universe as a whole, "high" temperatures[1] provide enough energy to eject electrons from atoms. Thus a plasma consists of a gas of freely flying electrons, ions, and yet unionized atoms. It should come as no surprise that during the extraordinary conditions of the Big Bang, all the matter in the Universe was ionized. About 380,000 years after the Big Bang, the Universe cooled enough for the first atoms to form. Surprisingly about 400 million years after that the Universe was re-ionized, and the vast majority of the matter in the universe remains ionized today (13.7 billion years after the Big Bang). Some of this ionized matter is at high density (hydrogen gas more dense than lead) and extremely high temperature at the center of stars, but most of it is believed to be at extremely low density in the vast spaces between the galaxies.

Perhaps the most obvious characteristic of conductors (metals and plasmas) is that they are shiny; that is, they reflect light. A moment's thought should convince you that this is

[1] What does "high temperature" mean? When you are attempting to make a Bose condensation at less than a millionth of a degree, liquid helium at 4 K would be called hot. When you are calculating conditions a few minutes after the Big Bang, a temperature of a billion degrees Kelvin would be called cool. An important insight: Nothing that has units can be said to be big or small! Things that have units need to be compared to a "normal state" before they can be declared big or small. Here the normal state refers to conditions allowing normal solids and liquids to exist. Tungsten, which is commonly used in the filaments of light bulbs, melts at about 3700 K; Carbon does a bit better: 3800 K. The "surface" of the Sun at 6000 K has no solid bits. At temperatures of about 5000 K most molecules have decomposed into elements which in turn have partially "ionized": ejecting one or more electrons to leave a positively charged core (an ion) and free electrons. I'll pick 5000 K as my comparison point for "hot", but of course some elements (e.g., sodium) begin to ionize a lower temperatures, and others (e.g., helium) ionize at higher temperatures. The key factor determining the ionized fraction in the Saha equation is the "first ionization energy".

not an "all-or-nothing" property. While metals reflect radio waves (see satellite TV dishes), microwaves (see the inside of your microwave) and visible light, they do not reflect higher frequency light like X-rays (lead, not aluminum, for X-ray protection). The free electron number density (units: number of free electrons/m^3), n_e, determines the behavior of light in a plasma. (Almost always plasmas are electrically neutral; i.e., the net electric charge density ρ is near zero. If the atoms are all singly ionized, we can conclude that the ion number density, n_i, equals n_e. In this lab we will be dealing with partially ionized argon with a neutral atom density $n_n \gg n_e \approx n_i$.) The free electron number density determines the *plasma frequency*, ω_p:

$$\omega_p = 2\pi f_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m}} \tag{5.1}$$

where $-e$ is the charge on an electron and m is the mass of an electron. If light with frequency f is aimed at a plasma: the light is reflected, if $f < f_p$; the light is transmitted, if $f > f_p$. Thus conductors are shiny only to light at frequencies below the plasma frequency. In order to reflect visible light, the plasma must be quite dense. Thus metals ($n_e \sim 10^{28}$ m^{-3}) look shiny, whereas semiconductors ($n_e \sim 10^{24}$ m^{-3}) do not. The plasma at "surface" of the Sun (with ionized fraction less than 0.1% and $n_e \sim 10^{20}$ m^{-3}) would also not look shiny. You will find the plasma used in this lab has even lower n_e; it will look transparent.

The defining characteristic of conductors (metals and plasmas) is that they can conduct an electric current. Since conductors conduct, they are usually at a nearly constant potential (voltage). (If potential differences exist, the resulting electric field will automatically direct current flow to erase the charge imbalance giving rise to the potential difference.) At first thought this combination (big current flow with nearly zero potential difference) may sound odd, but it is exactly what small resistance suggests. In fact the detection of big currents (through the magnetic field produced) first lead to the suggestion[2] of a conductor surrounding the Earth—an ionosphere. Edward Appleton (1924) confirmed the existence and location of this plasma layer by bouncing radio waves (supplied by a B.B.C. transmitter) off of it. In effect the ionosphere was the first object detected by radar. Appleton's early work with radar is credited with allowing development of radar in England just in time for the 1941 Battle of Britain. Appleton received the Nobel prize for his discovery of the ionosphere in 1947. (Much the same work was completed in this country just a bit later by Breit & Tuve.)

The plasma frequency in the ionosphere is around 3–10 MHz (corresponding to $n_e \sim 10^{11}$–10^{12} m^{-3}). Thus AM radio (at 1 MHz) can bounce to great distances, whereas CB radio (at 27 MHz) and FM (at 100 MHz) are limited to line-of-sight. (And of course when you look straight up you don't see yourself reflected in the ionospheric mirror, as $f_{\text{light}} \sim 5 \times 10^{14}$ Hz. On the other hand, extra terrestrials might listen to FM and TV, but we don't have to worry about them listening to AM radio.) The actual situation is a bit more complex. In the lowest layer of the ionosphere (D region), the fractional ionization is so low that AM radio is more absorbed than reflected. Sunlight powers the creation of new ions in the ionosphere, so when the Sun does down, ionization stops but recombination continues. In neutral-oxygen-rich plasmas like the D region, the plasma disappears without sunlight. Higher up in the ionosphere (the F region, where n_e is higher and n_n lower) total

[2]Faraday (1832), Gauss (1839), Kelvin (1860) all had ideas along this line, but the hypothesis is usually identified with the Scot Balfour Stewart (1882).

recombination takes much more than 12 hours, so the plasma persists through the night. Thus AM radio gets big bounces only at night.

I have located a plasma (a *hot* ionized gas) high in the Earth's atmosphere, yet folks climbing Mt. Everest think it's cold high up in the Earth's atmosphere. First, the ionosphere starts roughly 10 times higher than Mt. Everest, and in the F Region (about 200 km up) "temperature" is approaching 1000 K, warm by human standards if not by plasma standards. But the electrons are hotter still... up to three times hotter (still not quite hot by plasma standards). This is an odd thought: in an intimate mixture of electrons, ions, and neutral atoms, each has a different temperature. As you know, in a gas at equilibrium the particles (of mass M) have a particular distribution of speeds (derived by Maxwell and Boltzmann) in which the average translational kinetic energy, $\langle E_K \rangle$ is determined by the absolute temperature T:

$$\langle E_K \rangle = \frac{1}{2} M \langle v^2 \rangle = \frac{3}{2} kT \tag{5.2}$$

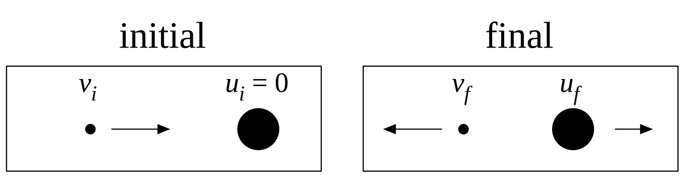
where k is the Boltzmann constant and $\langle\rangle$ denotes the average value. Thus, in a mixture of electrons (mass m) and ions (mass M_i) at different temperatures (say, $T_e > T_i$), you would typically find the electrons with more kinetic energy than the ions. (Note that even if $T_e = T_i$, the electrons would typically be moving much faster than the ions, as:

$$\frac{1}{2} m \langle v_e^2 \rangle = \frac{1}{2} M_i \langle v_i^2 \rangle \tag{5.3}$$

$$v_e\Big|_{rms} = \sqrt{\langle v_e^2 \rangle} = \sqrt{\frac{M_i}{m}}\; v_i\Big|_{rms} \tag{5.4}$$

that is the root-mean-square (rms) speed of the electrons will be $\sqrt{M_i/m} \approx \sqrt{40 \cdot 1827} \approx 270$ times the rms speed of the Argon ions, in the case of an ^{40}Ar plasma).

How can it be that the collisions between electrons and ions fail to average out the kinetic energy and hence the temperature? Consider a hard (in-line) elastic (energy conserving) collision between a slow moving (we'll assume stopped, $u_i = 0$) ion and a speeding electron (v_i).



We can find the final velocities (v_f & u_f) by applying conservation of momentum and energy. The quadratic equation that is energy conservation is equivalent to the linear equation of reversal of relative velocity:

$$v_i = u_f - v_f \tag{5.5}$$

$$mv_i = mv_f + Mu_f \tag{5.6}$$

with solution:

$$u_f = \frac{2m}{m+M}\, v_i \tag{5.7}$$

You can think of the ion velocity as being built up from a sequence of these random blows. Usually these collisions would be glancing, but as a maximum case, we can think of each blow as adding a velocity of $\Delta u = 2mv_i/(m+M)$ in a random direction. Consider the ion velocity vector before and after one of these successive blows:

$$\mathbf{u}_f = \mathbf{u}_i + \mathbf{\Delta u} \tag{5.8}$$
$$u_f^2 = u_i^2 + (\Delta u)^2 + 2\,\mathbf{u}_i \cdot \mathbf{\Delta u} \tag{5.9}$$

Now on average the dot product term will be zero (i.e., no special direction for $\mathbf{\Delta u}$), so on average the speed-squared increases by $(\Delta u)^2$ at each collision. Starting from rest, after N collisions we have:

$$u_f^2 = N\,(\Delta u)^2 \tag{5.10}$$
$$\frac{1}{2}\,M\,u_f^2 = N\,\frac{1}{2}\,M\,(\Delta u)^2 \tag{5.11}$$
$$= N\,\frac{1}{2}\,M\,\left[\frac{2m}{m+M}\,v_i\right]^2 \tag{5.12}$$
$$= N\,\frac{4m}{m+M}\,\frac{M}{m+M}\,\frac{1}{2}\,mv_i^2 \tag{5.13}$$

Thus for argon, $N \approx 18,000$ hard collisions are required for the ion kinetic energy to build up to the electron kinetic energy. Note that in nearly equal mass collisions (e.g., between an argon ion and an argon atom), nearly 100% of the kinetic energy may be transferred in one collision. Thus ions and neutral atoms are in close thermal contact; and electrons are in close contact with each other. But there is only a slow energy transfer between electrons and ions/atoms. In photoionization, electrons receive most of the photon's extra energy as kinetic energy. Slow energy transfer from the fast moving electrons heats the ions/atoms. When the Sun goes down, the electrons cool to nearly the ion temperature.

Note that the hottest thing near you now is the glow-discharge plasma inside a fluorescent bulb: $T_e > 3 \times 10^4$ K…hotter than the surface of the Sun, much hotter than the tungsten filament of an incandescent light bulb. The cool surface of the bulb gives testimony to the low plasma density ($n_e \sim 10^{16}$–10^{17} m^{-3}) inside the bulb. Containing this hot but rarefied electron gas heats the tube hardly at all — when the plasma's heat gets distributed over hugely more particles in the glass envelope, you have a hugely reduced average energy, and hence temperature.

Plasma People

Irving Langmuir (1881–1957)

Born in Brooklyn, New York, Langmuir earned a B.S. (1903) in metallurgical engineering from Columbia University. As was common in those days, he went to Europe for advanced training and completed a Ph.D. (1906) in physical chemistry under Nobel laureate Walther Nernst at University of Göttingen in Germany. Langmuir returned to this country, taking the job of a college chemistry teacher at Stevens Institute in Hoboken, New Jersey. Dissatisfied with teaching, he tried industrial research at the recently opened General Electric Research Laboratory[3] in Schenectady, New York. Langmuir's work for G.E. involved the

[3] G.E. calls this lab, which opened in 1900, the "first U.S. industrial laboratory devoted to research, innovation and technology", but Edison's Menlo Park "invention factory" (1876) would often claim that honor. Bell Labs (founded 1925), with six Nobel prizes in physics, would probably claim to be the world's preeminent industrial research lab, but the break up of the "Ma Bell" monoploy has also reduced Bell Labs.

then fledgling[4] electric power industry. He begin with improving the performance of incandescent electric light bulb. (Langmuir is in the inventors hall of fame for patent number 1,180,159: the modern gas-filled tungsten-filament incandescent electric light.) His work with hot filaments naturally led to thermionic emission and improvements in the vacuum triode tube that had been invented by Lee de Forest[5] in 1906. Working with glow discharge tubes (think of a neon sign), he invented diagnostic tools like the Langmuir probe to investigate the resulting "plasma" (a word he coined). "Langmuir waves" were discovered in the plasma. Along the way he invented the mercury diffusion pump. In high vacuum, thin films can be adsorbed and studied. As he said in his 1932 Nobel prize lecture:

> When I first began to work in 1909 in an industrial research laboratory, I found that the high-vacuum technique which had been developed in incandescent lamp factories, especially after the introduction of the tungsten filament lamp, was far more advanced than that which had been used in university laboratories. This new technique seemed to open up wonderful opportunities for the study of chemical reactions on surfaces and of the physical properties of surfaces under well-defined conditions.

In 1946, Langmuir developed the idea of cloud seeding, which brought him into contact with meteorologist Bernard Vonnegut, brother of my favorite author Kurt Vonnegut. That's how Langmuir became the model for Dr. Felix Hoenikker, creator of "ice-nine" in the novel *Cat's Cradle*. In fact Langmuir created the ice-nine idea (a super-stable form of solid water, with resulting high melting point, never seen in nature for want of a seed crystal) for H.G. Wells who was visiting the G.E. lab.

Lyman Spitzer, Jr (1914–1997)

Lyman Spitzer was born in Toledo, Ohio, and completed his B.A. in physics from Yale in 1935. For a year he studied with Sir Arthur Eddington at Cambridge, but that did not work out so he returned to this country and entered Princeton. He completed his Ph.D. in 1938 under Henry Norris Russell, the dean of American astrophysics. Following war work on sonar, he returned to astrophysics. His 1946 proposal for a large space telescope earned him the title "Father of the Hubble Space Telescope".

Because of the bend in *The Curve of Blinding Energy*[6], the lowest energy nuclei are of middle mass (e.g., ^{56}Fe). Thus nuclear energy can be released by breaking apart big nuclei (like ^{235}Ur and ^{239}Pu): fission as in the mis-named atomic bomb or by combining small nuclei (like ^{2}H deuterium and ^{3}H tritium): fusion as in the hydrogen bomb. In 1943 Edward Teller and Stanislaw Ulam started theoretical work on bombs based on thermonuclear fusion then called "super" bombs. The end of WWII slowed all bomb work, but the explosion of the first Russian atomic bomb, "Joe 1", in 1949, rekindled U.S. bomb work. This history of renewed interest in H-bomb work is mixed up with Russian espionage—real and imagined,

[4]Although not germane to my topic, I can't resist mentioning the famous AC (with Tesla and Westinghouse) vs. DC (with Edison, J.P. Morgan, and G.E.) power wars just before the turn of the century. The battle had a variety of bizarre twists, e.g., each side inventing an electric chair based on the opposite power source aiming to prove that the opponent's source was more dangerous than theirs. Easy voltage transformation with AC guaranteed the victory we see today in every electrical outlet worldwide. Unfortunately the AC frequency did not get standardized so its 60 Hz here and 50 Hz in Europe.

[5]1873–1961; "Father of Radio", born Council Bluffs, Iowa, B.S & Ph.D. in engineering from Yale

[6]Title of an interesting book by John McPhee; ISBN: 0374515980; UF767 .M215 1974

"McCarthyism", and the removal of Robert Oppenheimer's[7] security clearance in 1954.[8]

Our piece of the fusion story starts with Spitzer's 1951 visit with John Wheeler[9] in Los Alamos at just the right moment. The building of the Super, in response to Joe 1, had been failing: difficult calculations showed model designs would not ignite. Energy lost by thermal radiation and expansion cooled the "bomb" faster than nuclear energy was released.[10] But just before Spitzer arrived, Ulam and Teller had come up with a new method (radiation coupling for fuel compression) that Oppenheimer called "technically sweet". Meanwhile, in a story Hollywood would love, Argentine president Juan Perón announced that his protegé, Ronald Richter an Austrian-German chemist, working in a secret island laboratory had achieved controlled fusion. The story (of course) fell apart in less than a year, but it got both Spitzer and funding agencies interested. Spitzer's idea (the "Stellarator") was a magnetically confined plasma that would slowly fuse hydrogen to helium, releasing nuclear energy to heat steam and turn electrical generators. Spitzer and Wheeler hatched a plan to return to Princeton with a bit of both projects (of course funded by the government[11]): Project Matterhorn B would work on bombs and Project Matterhorn S would work on stellarators. Matterhorn B made calculations for the thermonuclear stage of the test shot Mike (1 Nov 1952)—the first H-bomb. The device worked even better than they had calculated.

From 1951 until 1958 stellarator research was classified. Optimistic projections[12] for fusion reactors were believed by all—after all physicists had completed key projects (atomic bomb, radar, H-bomb) like clockwork. Why should controlled fusion be much different from the carefully calculated fusion of an H-bomb? Early hints of fusion success (neutron emission) turned out to be signs of failure: "instabilities" or disturbances that grew uncontrollably in the plasma. Turbulence—the intractable problem in hydrodynamics[13] from the 19^{th} century—came back to bite physics in the 1950s. Instead of the hoped-for "quiescent" plasma, experiment found large amplitude waves: a thrashing plasma that easily escaped the magnetic field configurations physicists had thought should confine it. In 1961 Spitzer turned directorship of the Princeton Plasma Physics Laboratory (PPPL) over to Melvin Gottlieb, and largely returned to astrophysical plasmas.

[7] J. Robert Oppenheimer (1904–1967): born New York, NY, Ph.D. (1927) Göttingen. Directed atomic bomb work at Los Alamos during WWII; 'father of the atomic bomb'.

[8] See: *Dark Sun*, by Richard Rhodes, UG1282.A8 R46 1995

[9] John Archibald Wheeler (1911-2008): born Jacksonville, FL, B.S.+Ph.D. (1933) Johns Hopkins, Feynman's major professor at Princeton in 1942. Famous book: *Gravitation*. Coined the word "black hole".

[10] *Curve of Binding Energy* p. 64: One day, at a meeting of people who were working on the problem of the fusion bomb, George Gamow placed a ball of cotton next to a piece of wood. He soaked the cotton with lighter fuel. He struck a match and ignited the cotton. It flashed and burned, a little fireball. The flame failed completely to ignite the wood which looked just as it had before—unscorched, unaffected. Gamow passed it around. It was petrified wood. He said, "That is where we are just now in the development of the hydrogen bomb."

[11] The U.S. Atomic Energy Commission (AEC) initiated the program for magnetic fusion research under the name Project Sherwood. In 1974 the AEC was disbanded and replaced by the Energy Research and Development Administration (ERDA). In 1977, ERDA in turn was disbanded and its responsibilities transferred to the new Department of Energy (DOE). Since 1977, DOE has managed the magnetic fusion research program.

[12] An August 1954 report on a theoretical Model D Stellarator (only Model B, with a 2" tube, had actually been built), using assumptions that proved false, projected a power output approximately four times that of Hoover Dam. The usual joke is that controlled fusion will always be just ten years away.

[13] Turbulence in hydrodynamics is one of the Clay Millennium Prize Problems (essentially Nobel + Hilbert for mathematics in this century): 1 million dollars for a solution!

The current focus for magnetically confined plasma research is the "tokamak": a particular doughnut-shape (torus) configuration that confines the plasma in part using a large current flowing through the plasma itself. Designed by Russians Igor Tamm and Andrei Sakharov, the T-3 Tokamak surprised the plasma physics world when results were made public in 1968. In 1969, PPPL quickly converted the C-Stellarator into the ST tokamak.

Returning to astrophysics, Spitzer's influential books demonstrate his connection to plasma physics: *Physics of Fully Ionized Gases* (1956 & 1962), *Diffuse Matter in Space* (1968), *Physical Processes in the Interstellar Medium* (1978), and *Dynamical Evolution of Globular Clusters* (1988).

Summary

Almost everywhere in time and space, plasmas predominate. While present in some natural phenomena at the surface of the Earth (e.g., lightning), plasmas were "discovered" in glow discharges. In the first half of the 1900s, plasma physics was honored with two Nobels[14] Langmuir worked to understand "industrial" plasmas in things that are now considered mundane like fluorescent lights. Appleton located the ionosphere: an astronomical plasma surrounding the Earth. Both Nobels were connected to larger historical events (the rise of radio and radar in time to stop Hitler at the English channel).

In the second half of the 1900s, plasma physics was connected unpleasant problems: politics (McCarthyism), espionage, and turbulence. While H-bombs worked as calculated, controlled fusion proved difficult and only slow progress has been achieved. Astrophysical plasmas (for example, around the Sun) have also proved difficult to understand.

In this century, "industrial" plasmas are again newsworthy with plasma etching for computer chip manufacture and plasma display screens for HDTV.

Since the calculation of plasma properties has proved so difficult, measurements of plasma properties ("plasma diagnostics") are critical. In all sorts of plasmas (astrophysical, thermonuclear, industrial), the primary plasma diagnostic has been that first developed by Langmuir. The purpose of this lab is to measure basic plasma properties (T_e, n_e) using a Langmuir probe.

Glow Discharge Tube

In a glow (or gas) discharge tube, a large voltage ($\sim$100 V) accelerates free electrons to speeds sufficient to cause ionization on collision with neutral atoms. The gas in the tube is usually at low pressure ($\sim$1 torr), so collisions are not so frequent that the electrons fail to reach the speed required for ionization.

Making an electrical connection to the plasma is a more complicated process than it might seem:

A. The creation of ions requires energetic collisions (say, energy transfer $\sim$10 eV). Kinetic

[14] An additional Nobel for plasma physics: Hannes Alfvén (1970). Note that Tamm and Sakharov (named in the context of the Tokamak) also received Nobels, but not for plasma physics work.

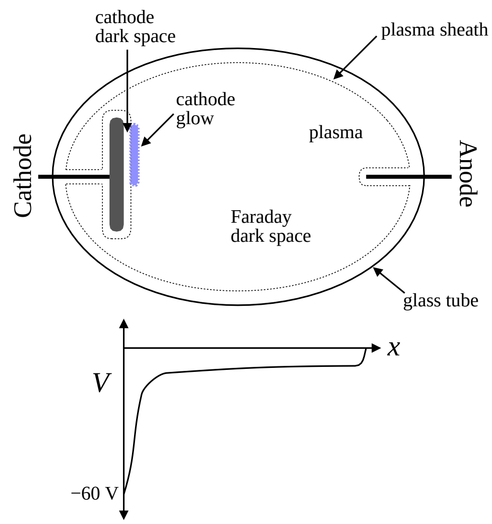


Figure 5.1: When a current flows between the anode (+) and cathode (–), the gas in the tube partially ionizes, providing electrons and ions to carry the current. The resulting plasma is at a nearly constant potential. Electric fields (from potential differences) exist mostly at the edge of the plasma, in the plasma sheath. The largest potential drop is near the cathode. The resulting cathode glow is the region of plasma creation. Increased discharge current I_c results in expanded coverage of the cathode by the cathode glow, but not much change in the cathode voltage V_c. Note that if the anode/cathode separation were larger, a *positive column* of excited gas would be created between the Faraday dark space and the anode.

energy for the collision must in turn come from potential differences of $\gtrsim$ 10 V. However, we've said that conductors (like the plasma) are at approximately constant potential. Thus ion creation must occur at the edge of the plasma.

B. It turns out that attempts to impose a potential difference on a plasma fail. Typically potential differences propagate only a short distance, called the Debye length λ_D, into the plasma:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k T_e}{e^2 n_e}} \tag{5.14}$$

Thus we expect the "edge" of the plasma to be just a few λ_D thick.

C. The small electron mass (compared to ions), guarantees high electron mobility in the plasma. Thus we expect electrons to carry the bulk of the current in the plasma. But this cannot be true in the immediate vicinity of the cathode. The electrons inside the cold cathode (unlike the heated cathode in thermionic emission) are strongly held—they will not spontaneously leave the cathode. Thus near the cathode the current must be carried by ions which are attracted to the negatively charged cathode. Once in contact with the cathode an ion can pick up an electron and float away as a neutral atom. Note particularly that there is no such problem with conduction at the anode: the plasma electrons are attracted to the anode and may directly enter the metal to continue the current. Thus we expect the active part of the discharge to be directly adjacent to the cathode.

D. If you stick a wire into a plasma, the surface of the wire will be bombarded with electrons, ions, and neutrals. Absent any electric forces, the impact rate per m^2 is given by

$$J = \frac{1}{4}\, n\, \langle v \rangle = \frac{1}{4}\, n\, \sqrt{\frac{8kT}{\pi M}} \tag{5.15}$$

where n is the number density of the particles and $\langle v \rangle$ is their average speed. If the particles follow the Maxwell-Boltzmann speed distribution, the average speed is determined by the temperature T and mass M of the particles. (Recall: $v_{rms} = \sqrt{3\pi/8}\ \langle v \rangle \approx 1.085\ \langle v \rangle$ for a Maxwell-Boltzmann distribution.) Since the electron mass is much less than the ion mass ($m \ll M_i$) and in this experiment the temperatures are also different ($T_e \gg T_i$), the average electron speed is much greater than the average ion speed. Thus an item placed in a plasma will collect many more electrons than ions, and hence will end up with a negative charge. The over-collection of electrons will stop only when the growing negative charge (repulsive to electrons, attractive to ions) reduces the electron current and increases the ion current so that a balance is reached and zero net current flows to the wire. The resulting potential is called the *floating potential*, V_f.

The upshot of these considerations is that objects immersed in a plasma do not actually contact the plasma. Instead the plasma produces a "sheath", a few Debye lengths thick, that prevents direct contact with the plasma. We begin by demonstrating the above equations.

The starting point for both equations is the Boltzmann factor:

$$\text{probability} = \mathcal{N} e^{-E/kT} \tag{5.16}$$

which reports the probability of finding a state of energy E in a system with temperature T, where $\mathcal{N}$ is a normalizing factor that is determined by the requirement that the total probability adds up to 1. The energy of an electron, the sum of kinetic and potential energy, is

$$E = \frac{1}{2}\, mv^2 - eV \tag{5.17}$$

where V is the voltage at the electron's position. (See that for an electron, a low energy region corresponds to a high voltage region. Thus Boltzmann's equation reports that electrons are most likely found in *high* voltage regions.) To find $\mathcal{N}$ add up the probability for all possible velocities and positions:

$$\begin{aligned}
1 &= \mathcal{N} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{-\infty}^{+\infty} dv_z \quad \int d\mathcal{V}\ \exp\left(\frac{-\frac{1}{2}\, mv^2 + eV(\mathbf{r})}{kT}\right) \\
&= \mathcal{N} \int_{-\infty}^{+\infty} e^{-mv_x^2/2kT} dv_x \int_{-\infty}^{+\infty} e^{-mv_y^2/2kT} dv_y \int_{-\infty}^{+\infty} e^{-mv_z^2/2kT} dv_z \quad \int d\mathcal{V}\ e^{eV(\mathbf{r})/kT} \\
&= \mathcal{N} \left[\frac{2\pi kT}{m}\right]^{3/2} \mathcal{V}\ e^{eV(\mathbf{r}_0)/kT}
\end{aligned} \tag{5.18}$$

where we have used the Gaussian integral:

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \tag{5.19}$$

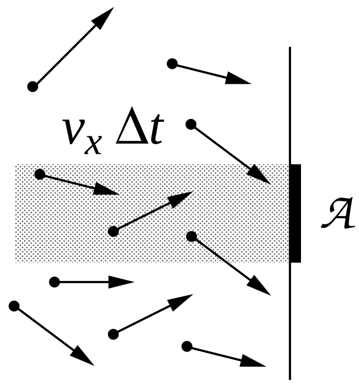


Figure 5.2: Seeking the number of electrons to hit the area $\mathcal{A}$, focus on just those electrons with some particular x-velocity, v_x. Electrons within $v_x\Delta t$ of the wall will hit it sometime during upcoming time interval Δt. The number of such electrons that will hit an area $\mathcal{A}$ will be equal to the number of such electrons in the shaded volume. Note that many electrons in that volume will *not* hit $\mathcal{A}$ because of large perpendicular velocities, but there will be matching electrons in neighboring volumes which will hit $\mathcal{A}$. To find the total number of hits, integrate over all possible v_x.

and the mean value theorem to replace the integral over the volume $\mathcal{V}$ of the electron gas with $\mathcal{V}$ times some value of the integrand in that domain. Thus:

$$\text{probability} = \frac{e^{-E/kT}}{\left[\frac{2\pi kT}{m}\right]^{3/2} \; \mathcal{V} e^{eV(\mathbf{r}_0)/kT}} = \frac{1}{\mathcal{V}} \left[\frac{m}{2\pi kT}\right]^{3/2} \exp\left(-\frac{\frac{1}{2}mv^2 - e(V - V_0)}{kT}\right) \tag{5.20}$$

So, if we have N (non-interacting) electrons in $\mathcal{V}$, the expected distribution of electrons (w.r.t. position and velocity) can be expressed as:

$$f = n_0 \left[\frac{m}{2\pi kT}\right]^{3/2} \exp\left(-\frac{\frac{1}{2}mv^2 - e(V - V_0)}{kT}\right) \tag{5.21}$$

where $n_0 = N/\mathcal{V}$ is the bulk electron density which is also the electron density at potential V_0. If we don't care about the distribution of electron speed, we can add up (integrate) over all possible speeds (which just reproduces the Gaussian factors from $\mathcal{N}$), resulting in the electron number density, n:

$$n = n_0 \exp\left(\frac{e(V - V_0)}{kT}\right) \tag{5.22}$$

D. Collision Rate: If we just care about the distribution of velocity in one direction (say, v_x), integrals over the other two directions (v_y, v_z) results in:

$$f_x = n_0 \left[\frac{m}{2\pi kT}\right]^{1/2} \exp\left(-\frac{\frac{1}{2}mv_x^2 - e(V - V_0)}{kT}\right) \tag{5.23}$$

We simplify further by considering the case where the potential is a constant (i.e., $V = V_0$). In order to calculate the number of electrons that hit an area $\mathcal{A}$ during the coming interval Δt, focus on a subset of electrons: those that happen to have a particular velocity v_x. (We will then add up all possible v_x, from v_{min} to ∞. For this immediate problem $v_{min} = 0$, i.e., any electron moving to the right can hit the wall; however, we'll need the more general case in a few pages.) An electron must be sufficiently close to the wall to hit it: within $v_x \Delta t$. The number of electron hits will be equal to the number of such electrons in the shaded volume in Figure 5.2. To find the total number of hits, integrate over all possible v_x:

$$\begin{aligned}
\text{number of hits} &= \int_{v_{min}}^{\infty} dv_x \; f_x \; \mathcal{A} v_x \Delta t && (5.24)\\
&= \mathcal{A}\Delta t \; n_0 \left[\frac{m}{2\pi kT}\right]^{1/2} \int_{v_{min}}^{\infty} \exp\left(-\frac{\frac{1}{2}mv_x^2}{kT}\right) v_x \; dv_x && (5.25)\\
&= \mathcal{A}\Delta t \; n_0 \left[\frac{m}{2\pi kT}\right]^{1/2} \frac{kT}{m} \int_{mv_{min}^2/2kT}^{\infty} e^{-y} dy && (5.26)\\
&= \mathcal{A}\Delta t \; n_0 \left[\frac{kT}{2\pi m}\right]^{1/2} \exp\left(-\frac{\frac{1}{2}mv_{min}^2}{kT}\right) && (5.27)\\
&\longrightarrow \mathcal{A}\Delta t \; n_0 \left[\frac{kT}{2\pi m}\right]^{1/2} \quad \text{for} \quad v_{min} \to 0 && (5.28)
\end{aligned}$$

Thus the particle current density, J_e, (units: hits per second per m^2) is

$$J_e = \frac{\text{hits}}{\mathcal{A}\Delta t} = \frac{1}{4}\, n_e \left[\frac{8kT}{\pi m}\right]^{1/2} = \frac{1}{4}\, n_e \, \langle v \rangle \tag{5.29}$$

B. Debye Length (λ_D): To introduce the Debye length, we consider a very simplified case: the potential variation due to varying electron density in a uniform (and nearly canceling, i.e., $n_i = n_{e0}$) ion density. By Poisson's equation:

$$\epsilon_0 \, \nabla^2 V = -e\left(n_i - n_{e0} e^{eV/kT}\right) = -en_{e0}\left(1 - e^{eV/kT}\right) \approx en_{e0}\left(\frac{eV}{kT}\right) \tag{5.30}$$

where we have assumed $eV/kT \ll 1$ so we can Taylor expand. If we consider variation in just one direction (x), we have:

$$\frac{d^2V}{dx^2} = \left(\frac{e^2 n_{e0}}{\epsilon_0 kT}\right) V = \frac{1}{\lambda_D^2} \, V \tag{5.31}$$

with exponentially growing/decaying solutions of the form: $V \propto \exp\left(\pm x/\lambda_D\right)$. We learn from this that deviations from charge neutrality take place on a length scale of λ_D, and are self-reinforcing so that very large changes in the potential can be accomplished in a distance of just a few λ_D.

In the cathode glow, high speed electrons (accelerated in the large electric field in the neighboring cathode dark space) collision-ionize the gas. Many of the newly created ions are attracted to the cathode. Accelerated by cathode dark space electric field, the ions crash into the cathode. The collision results in so-called secondary electron emission[15] from the cathode. These secondary electrons are then repelled from the cathode and cause further ionization in the cathode glow region. The observed voltage drop near the cathode (the "cathode fall"), is exactly that required so that, on average, a secondary electron is able to

[15] Secondary electron emission is the key to photomultiplier tubes used to detect single photons.

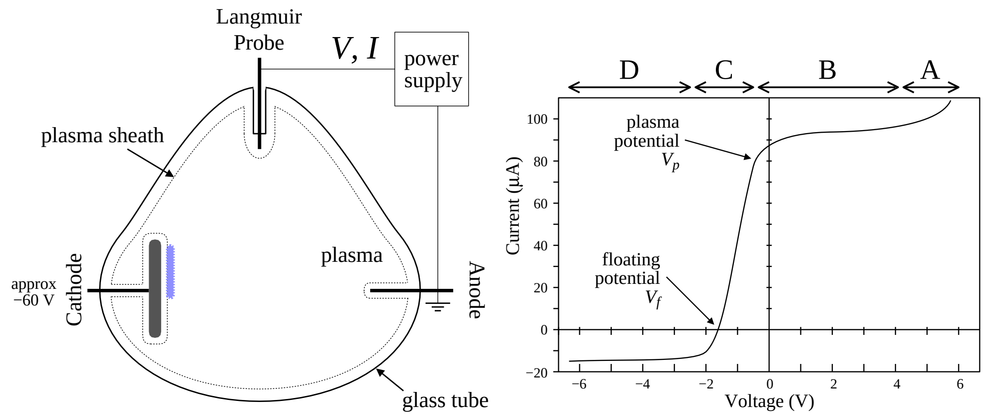


Figure 5.3: At its simplest, a Langmuir probe is just a wire stuck into the plasma. Of course, electrical contact with the plasma is limited by the plasma sheath. An external power supply allows the probe's voltage to be adjusted and the resulting current measured. The characteristics of the plasma can be determined by careful analysis of the resulting IV relationship.

reproduce itself through the above mechanism. Clearly the cathode fall depends both on the gas (how easy it is to ionize) and the cathode material (how easy is it to eject secondary electrons). Luckily we will not need to understand in detail the processes maintaining the discharge near the cathode. A properly operating Langmuir probe does not utilize secondary electron emission, that is we limit voltage drop near the probe to much less than the cathode fall.

Langmuir Probe Theory

As stated above, if a wire is stuck into a plasma, instead of connecting to the plasma potential (V_p) it instead charges to a negative potential (the floating potential: $V_f < V_p$) so as to retard the electron current enough so that it matches the ion current. An equal flow of electrons and ions—zero net current— is achieved and charging stops. If the wire is held at potentials above or below V_f, a net current will flow into the plasma (positive I: plasma electrons attracted to the probe), or out from the plasma (negative I: plasma ions attracted to the probe). Figure 5.3 displays a Langmuir probe IV curve dividing it up into four regions (A–D):

A. When the Langmuir probe is well above the plasma potential it begins to collect some of the discharge current, essentially replacing the anode.

B. When the probe is at the plasma potential (left side of region B), there is no plasma sheath, and the surface of the probe collects ions and electrons that hit it. The electron current is much larger than the ion current, so at V_p the current is approximately:

$$I_p = e\mathcal{A}\frac{1}{4}n_e\left[\frac{8kT_e}{\pi m}\right]^{1/2} \tag{5.32}$$

where $\mathcal{A}$ is the area of the probe. For $V > V_p$, the sheath forms, effectively expanding slightly the collecting area. Thus the probe current increases slightly and then levels off in this region.

C. When the Langmuir probe's potential is below V_p, it begins to repel electrons. Only electrons with sufficient kinetic energy can hit the probe. The minimum approach velocity v_{min} allowing a probe hit can be determined using conservation of energy:

$$\frac{1}{2}\,mv_{min}^2 - eV_p = -eV \tag{5.33}$$

$$\frac{1}{2}\,mv_{min}^2 = e(V_p - V) \tag{5.34}$$

where the probe is at potential V. Equation 5.27 then reports that the resulting electron current is

$$I = e\mathcal{A}\,n_e \left[\frac{kT_e}{2\pi m}\right]^{1/2} \exp\left(-\frac{e(V_p - V)}{kT_e}\right) \tag{5.35}$$

For $V \ll V_p$, very few of the electrons have the required velocity. At $V = V_f$ the electron current has been suppressed so much that it equals the ion current. (The ion current is always present in region C, but typically for $V > V_f$ it is "negligible" in comparison to the electron current.)

D. In this region the probe is surrounded by a well developed sheath repelling all electrons. Ions that random-walk past the sheath boundary will be collected by the probe. As the sheath area is little affected by the probe voltage, the collected ion current is approximately constant. (At very negative voltages, $V \sim -60$ V, secondary electron emission following ion hits leads to large currents, and a glow discharge.) The equation for this ion current is a bit surprising (that is not analogous to the seeming equivalent electron current in region B):

$$I_i \approx -\frac{1}{2}e\mathcal{A}\,n_i\,u_B \tag{5.36}$$

where u_B is the Bohm[16] velocity—a surprising combination of electron temperature and ion mass:

$$u_B = \sqrt{kT_e/M_i} \tag{5.37}$$

Note that Equation 5.15 would have suggested a similar result but with the thermal velocity rather than the Bohm velocity.

Region D: Ion Current

As shown in Figure 5.4, consider bulk plasma ($n_i = n_e = n_0$, located at $x = 0$ with a plasma potential we take to be zero $V_p = 0$) near a planar Langmuir probe (located at $x = b$ biased with a negative voltage). In the bulk plasma, the ions are approaching the probe with a

[16] David Bohm (1917–1992) Born: Wiles-Barre, PA, B.S. (1939) PSU, joined Communist Party (1942), Ph.D. (1943) Berkeley — Oppenheimer's last student before he became director at Los Alamos. Cited for contempt by McCarthy's House Un-American Activities Committee, Bohm was arrested in 1950. Although acquitted at trial, he was nevertheless blacklisted, and ended up in London.

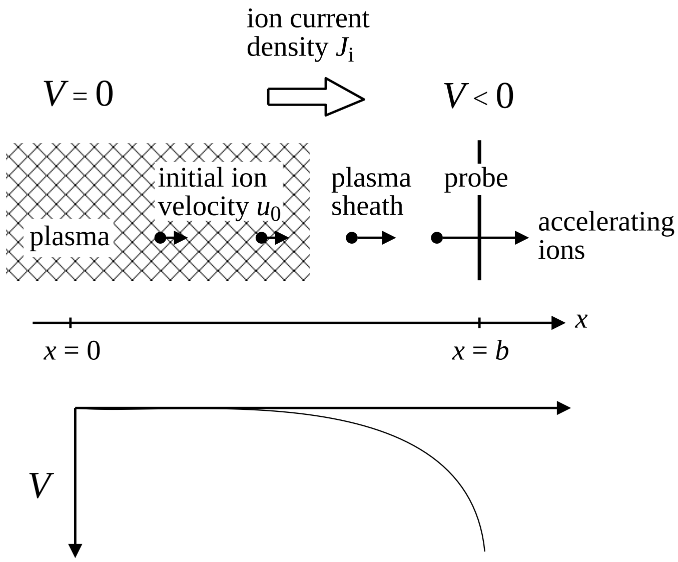


Figure 5.4: When the Langmuir probe is negatively biased relative to the plasma, it attracts ions from the plasma (and repels electrons). Of course, the the accelerating electric field (from voltage difference) is largely confined to the edge of the plasma, the plasma sheath. The resulting ion current density J_i should be continuous: the same for every x.

velocity u_0. The ions are accelerated towards the negatively biased probe; we can determine their velocity, $u(x)$ at any position by applying conservation of energy:

$$\frac{1}{2}M_i u^2 + eV(x) = \frac{1}{2}M_i u_0^2 \tag{5.38}$$

$$u(x) = u_0\sqrt{1 - \frac{2eV(x)}{M_i u_0^2}} \tag{5.39}$$

The moving ions constitute a steady electric current density:

$$J_i = en(x)u(x) = en_0u_0 \tag{5.40}$$

(i.e., J_i doesn't depend on position), so $n(x)$ must decrease as the ions speed toward the probe.

$$n(x) = n_0u_0/u(x) = \frac{n_0}{\sqrt{1 - \frac{2eV(x)}{M_i u_0^2}}} \tag{5.41}$$

The varying charge density affects the electric potential which in turn affects the electron density through the Boltzmann equation (Equation 5.22). Poisson's equation reads:

$$\frac{d^2V}{dx^2} = -\frac{\rho(x)}{\epsilon_0} = \frac{e}{\epsilon_0}\, n_0 \left[\exp\left(\frac{eV(x)}{kT_e}\right) - \frac{1}{\sqrt{1 - \frac{2eV(x)}{M_i u_0^2}}}\right] \tag{5.42}$$

The first step in solving most any differential equation, is to convert it to dimensionless form. We adopt dimensionless versions of voltage, position, and velocity:

$$\tilde{V} = \frac{eV}{kT_e} \tag{5.43}$$

$$\tilde{x} = \frac{x}{\lambda_D} = x\left[\frac{n_0 e^2}{\epsilon_0 k T_e}\right]^{1/2} \tag{5.44}$$

$$\tilde{u}_0 = \frac{u_0}{u_B} = \frac{u_0}{\sqrt{kT_e/M_i}} \tag{5.45}$$

If we multiply Equation 5.42, by e/kT_e, we find:

$$\frac{d^2\tilde{V}}{dx^2} = \frac{1}{\lambda_D^2}\left[\exp(\tilde{V}) - \frac{1}{\sqrt{1-\frac{2\tilde{V}}{\tilde{u}_0^2}}}\right] \tag{5.46}$$

or

$$\frac{d^2\tilde{V}}{d\tilde{x}^2} = \exp(\tilde{V}) - \frac{1}{\sqrt{1-\frac{2\tilde{V}}{\tilde{u}_0^2}}} \tag{5.47}$$

First let's simplify notation, by dropping all those tildes.

$$\frac{d^2V}{dx^2} = \exp(V) - \frac{1}{\sqrt{1-\frac{2V}{u_0^2}}} \tag{5.48}$$

The physical solution should show the neutral plasma picking up a positive charge density as we approach the probe (as electrons are repelled and ions attracted to the probe). The r.h.s. of Equation 5.48 is proportional to the charge density $n_e - n_i$ which is nearly zero at $x = 0$ (where $V = 0$) and monotonically declines as x approaches the probe (where $V < 0$). If we Taylor expand the r.h.s., we find:

$$\begin{aligned} n_e - n_i \quad &\propto \quad \exp(V) - \frac{1}{\sqrt{1-\frac{2V}{u_0^2}}} \\ &\approx \left(1 + V + \frac{1}{2}V^2 + \cdots\right) - \left(1 + \frac{1}{u_0^2}\,V + \frac{3}{2u_0^4}\,V^2 + \cdots\right) \\ &= \left(1 - \frac{1}{u_0^2}\right)\,V + \frac{1}{2}\left(1 - \frac{3}{u_0^4}\right)\,V^2 + \cdots \end{aligned} \tag{5.49}$$

Thus $u_0 \geq 1$ is required for $n_i > n_e$ when $V < 0$. Maximizing the extent of charge neutrality requires $u_0 = 1$. As shown in Figure 5.5, the choice $u_0 = 1$ works throughout the $V < 0$ range (i.e., beyond the range of convergence of the Taylor's series).

Note that we can numerically solve this differential equation using *Mathematica*:

```
NDSolve[{v''[x]==Exp[v[x]]-1/Sqrt[1-2 v[x]/u0^2],v[0]==0,v'[0]==0},v,{x,0,20}]
```

but, it's not quite that simple. The unique solution with these boundary conditions, is $V = 0$ everywhere. There must be some small electric field ($V'(0) < 0 \Longleftrightarrow E > 0$) at $x = 0$ due to probe. The smaller the choice of E, the more distant the probe. See Figure 5.6.

We have concluded, $u_0 = 1$, or returning to dimensioned variables: $u_0 = u_B$. However, we must now determine how the ions arrived at this velocity. It must be that in the much

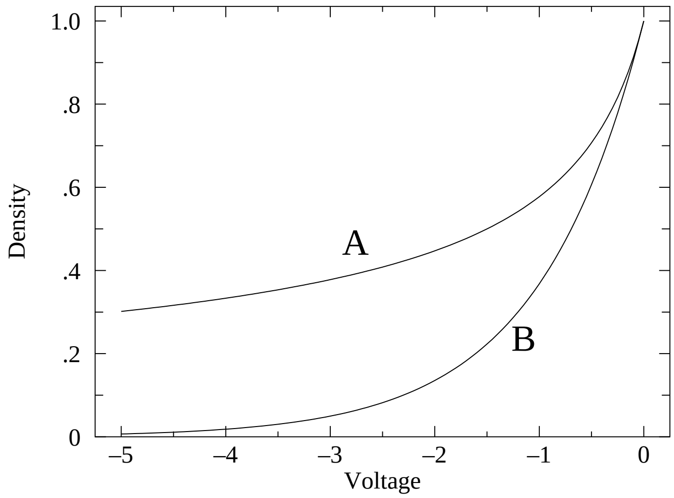


Figure 5.5: A plot of the ion density (A) $n_i \propto \left[1 - 2V(x)/u_0^2\right]^{-1/2}$ and the electron density (B) $n_e \propto e^{V(x)}$ for $V \leq 0$ in the vicinity of the probe. (With this axis choice the bulk plasma, at $V = 0$, is to the right and the probe, at $V < 0$, is on the left.) The case $u_0 = 1$ is displayed. Notice that $n_i > n_e$ throughout the plasma sheath, as is required. If $u_0 < 1$, there would be a region near $V = 0$ where $n_e > n_i$. If $u_0 > 1$, the densities in region near $V = 0$ would not be tangent. There would be a first-order mismatch between n_e and n_i: $n_i > n_e$ for $V < 0$ and $n_i < n_e$ for $V > 0$.

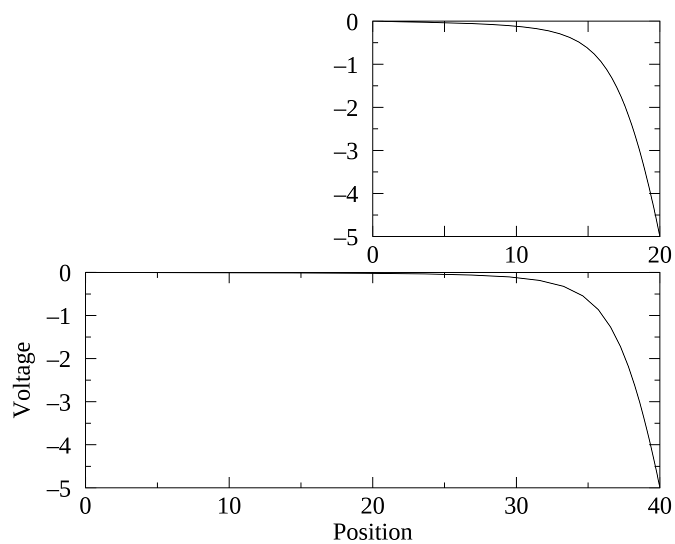


Figure 5.6: *Mathematica* solutions of our differential equation (5.48) with $V'(0) = -.0068$ (top) and $V'(0) = -.00067$ (bottom). Smaller E simply places the probe at a greater distance without much changing the voltage in the vicinity of the probe (i.e., in the sheath).

larger region ("pre-sheath", $x \ll 0$), under conditions of near charge neutrality ($n_e \approx n_i$), the ions traveled "downhill" a potential difference of:

$$V(-\infty) - V(0) = \frac{\text{K.E.}}{e} = \frac{\frac{1}{2}M_i u_B^2}{e} = \frac{kT_e}{2e} \tag{5.50}$$

Thus, at $x = 0$, the plasma density has been depleted compared to that at $x = -\infty$:

$$n_i = n_e = n_0 = n_\infty e^{\Delta V/kT_e} = n_\infty e^{-1/2} \tag{5.51}$$

The corresponding current density is:

$$J = en_0 u_0 = en_\infty e^{-1/2} u_B \approx .61\ en_\infty \sqrt{\frac{kT_e}{M_i}} \tag{5.52}$$

Clearly this is an approximate argument. Other arguments can give lower (e.g., .40) constants. We adopt as our final equation (if only approximate: $\pm 20\%$)

$$J_i \approx \frac{1}{2}\ en_\infty \sqrt{\frac{kT_e}{M_i}} \tag{5.53}$$

Region C

Plasma Potential, V_p

We have said that for $V < V_p$, the electron current to the probe is exponentially growing essentially because of the exponential form of the Maxwell-Boltzmann distribution. For $V > V_p$, the electron current to the probe continues to grow, but only because of expanding collecting area due to an expanding plasma sheath. The boundary between two cases is defined by the point of maximum slope. The point where the slope is a maximum, of course, has the second derivative zero—an inflection point. Thus V_p is defined by $I''(V_p) = 0$. (This is called the Druyvesteyn criteria.)

In calculus you've learned how to apply a precise definition of derivative to find the derivatives of various functions, but how can you determine the second derivative from a set of data?

We begin with a qualitative treatment. If you have a set of equally spaced data points: $x_i = ih$ where $i \in \mathbb{Z}$ (i is an integer, h might have been called Δx), then $(f(x_{i+1}) - f(x_i))/h$ (the slope of a line through the points $(x_i, f(x_i))$ & $(x_{i+1}, f(x_{i+1}))$ ought to be something like $f'(x_i + h/2)$ (the derivative half way between x_i & x_{i+1}). Similarly, $f'(x_i - h/2) \approx (f(x_i) - f(x_{i-1}))/h$. Thus:

$$f'' \approx \frac{f'(x_i + h/2) - f'(x_i - h/2)}{h} = \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2} \tag{5.54}$$

where we imagine f'' is evaluated half way between $x_i + h/2$ & $x_i - h/2$, that is, at x_i.

More formally, theoretically f has a Taylor expansion:

$$f(x_i + y) = f(x_i) + f'(x_i)y + \frac{1}{2}\ f''(x_i)y^2 + \frac{1}{6}\ f'''(x_i)y^3 + \frac{1}{24}\ f''''(x_i)y^4 + \cdots \tag{5.55}$$

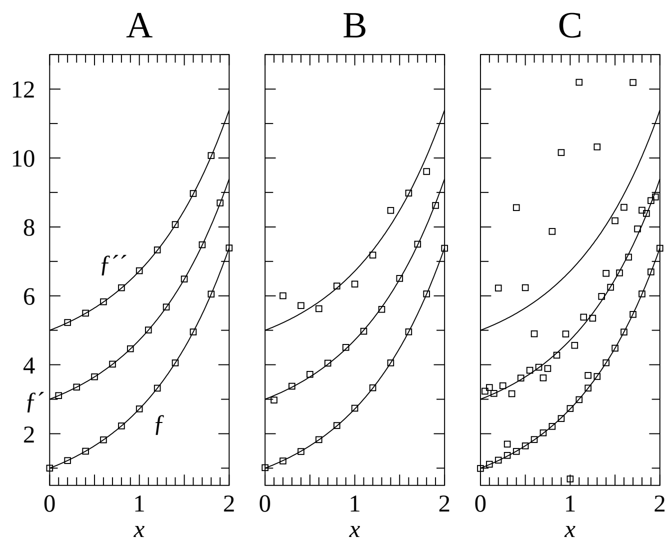


Figure 5.7: Numerical differentiation applied to the function $f(x) = e^x$. f' and f'' have been vertically offset by 2 and 4 respectively so that they do not coincide with f. (A) Here with $h = .2$, the numerically calculated f' and f'' (based on the shown data points) accurately track the actual derivatives (shown as curves). (B) With random noise of magnitude $\pm.02$ added to the y values, the data points seem to accurately follow the function, but numerically calculated f' and f'' increasingly diverge. See that numerical differentiation exacerbates noise. (C) Doubling the number of data points (with the same level of random noise as in B), makes the situation much worse. While the data points seem to follow f adequately, f' shows large deviations and f'' does not at all track the actual derivative.

So:

$$
\begin{aligned}
f(x_i + h) = f(x_{i+1}) &= f(x_i) + f'(x_i)h + \frac{1}{2}\, f''(x_i)h^2 + \frac{1}{6}\, f'''(x_i)h^3 + \frac{1}{24}\, f''''(x_i)h^4 + \cdots \\
f(x_i - h) = f(x_{i-1}) &= f(x_i) - f'(x_i)h + \frac{1}{2}\, f''(x_i)h^2 - \frac{1}{6}\, f'''(x_i)h^3 + \frac{1}{24}\, f''''(x_i)h^4 + \cdots \\
f(x_i + h) + f(x_i - h) &= 2f(x_i) + f''(x_i)h^2 + \frac{1}{12}\, f''''(x_i)h^4 + \cdots \qquad (5.56)
\end{aligned}
$$

with the result:

$$\frac{f(x_i + h) + f(x_i - h) - 2f(x_i)}{h^2} = f''(x_i) + \frac{1}{12}\, f''''(x_i)h^2 + \cdots \qquad (5.57)$$

Thus if $f''''(x_i)h^2/f''(x_i) \ll 1$ (which should follow for sufficiently small h), Equation 5.54 should provide a good approximation for $f''(x_i)$. While ever smaller h looks good mathematically, Figure 5.7 shows that too small h plus noise, is a problem.

Root Finding—Floating Potential, V_f & Plasma Potential, V_p

The floating potential is defined by $I(V_f) = 0$. Of course, it is unlikely that any collected data point exactly has $I = 0$. Instead a sequence of points with $I < 0$ will be followed by points with $I > 0$, with V_f lying somewhere between two measured points. We seek to interpolate to find the best estimate for V_f.

We begin with the generic case, where we seek a root, i.e., the x value such that $y(x) = 0$. We start with two data points (x_1, y_1) and (x_2, y_2) with $y_1 < 0$ and $y_2 > 0$. The line connecting these two points has equation:

$$y = \frac{\Delta y}{\Delta x}\ (x - x_1) + y_1 = \frac{y_2 - y_1}{x_2 - x_1}\ (x - x_1) + y_1 \tag{5.58}$$

We seek the x value for which the corresponding y value is zero:

$$\frac{\Delta y}{\Delta x}\ (x - x_1) + y_1 \quad = \quad y = 0 \tag{5.59}$$

$$x - x_1 \quad = \quad -y_1\ \frac{\Delta x}{\Delta y} \tag{5.60}$$

$$x \quad = \quad x_1 - y_1\ \frac{\Delta x}{\Delta y} \tag{5.61}$$

In the case of the floating potential ($I(V_f) = 0$), our (x_i, y_i) are a sequence of voltages with measured currents: $x_i = V_i$, $y_i = I(V_i)$, and we can apply the above formula to find the voltage, V_f where the current is zero.

The same generic result can be used to estimate the plasma potential, where $I''(V_p) = 0$. Here $x_i = V_i$, $y_i = I''(V_i)$. Just as in the floating voltage case, it is unlikely that any collected data point exactly has $I'' = 0$. Instead a sequence of points with $I'' > 0$ will be followed by points with $I'' < 0$, with V_p lying somewhere between two measured points. You will apply the generic interpolate formula to find the best estimate for V_p.

Electron Temperature, T_e

If we combine Equation 5.36 for the ion current with Equation 5.35 for the electron current, we have an estimate for the total current through out Region C:

$$I \quad = \quad -\frac{1}{2}\ e\mathcal{A}\ n_i\ u_B + \frac{1}{4} e\mathcal{A}\ n_e \left[\frac{8kT_e}{\pi m}\right]^{1/2} \exp\left(-\frac{e(V_p - V)}{kT_e}\right) \tag{5.62}$$

$$= \quad \frac{1}{2} e\mathcal{A}\ n u_B \left\{-1 + \left[\frac{2M_i}{\pi m}\right]^{1/2} \exp\left(-\frac{e(V_p - V)}{kT_e}\right)\right\} \tag{5.63}$$

$$= \quad k_1 + k_2 \exp\left((V - V_p)/k_3\right) \tag{5.64}$$

Use of this equation is based on a long list of assumptions (e.g., constant collection area $\mathcal{A}$, Maxwell-Boltzmann electron speed distribution, $n_i = n_e = n \ldots$). These assumptions are not exactly true, so we do not expect this equation to be exactly satisfied: we are seeking a simplified model of reality not reality itself. By fitting this model equation to measured IV data, we can estimate the parameters k_1, k_2, k_3. Since $k_3 = kT_e/e$ we can use it to estimate the electron temperature. Similarly, k_1 represents a measure of the ion current from which (once T_e is known) n_i can be calculated using Equation 5.36.

In order to start a non-linear fit as in Equation 5.64, we need initial estimates for the parameters k_i. Measurement of V_f (where $I = 0$) and Equation 5.63, provide an estimate for T_e:

$$
\begin{aligned}
0 = I &= \frac{1}{2}\, e\mathcal{A}\, n\, u_B \left\{ -1 + \left[\frac{2M_i}{\pi m}\right]^{1/2} \exp\left(\frac{e(V_f - V_p)}{kT_e}\right) \right\} && (5.65) \\
1 &= \left[\frac{2M_i}{\pi m}\right]^{1/2} \exp\left(\frac{e(V_f - V_p)}{kT_e}\right) && (5.66) \\
0 &= \frac{1}{2}\ln\left[\frac{2M_i}{\pi m}\right] - \left(\frac{e(V_p - V_f)}{kT_e}\right) && (5.67) \\
\frac{e}{kT_e}(V_p - V_f) &= \frac{1}{2}\ln\left[\frac{2M_i}{\pi m}\right] && (5.68) \\
\frac{kT_e}{e} &= \frac{2(V_p - V_f)}{\ln\left[\frac{2M_i}{\pi m}\right]} && (5.69)
\end{aligned}
$$

Notice that the unit for $k_3 = kT_e/e$ is volts, so $ek_3 = kT_e$ in units of J or, most simply, $k_3 = kT_e$ in units eV[17] (since eV=$e\times$Volt). It is common practice in plasma physics to report "the temperature" [meaning kT] in eV.

The ion current k_1 can be estimated from the "saturated current" in Region D, i.e., the nearly constant current a volt or so below V_f. (For future convenience, we name this "saturated current" in Region D, I_i.)

See that for $V = V_p$, $I = k_1 + k_2$. Since the ion current is "negligible" for most of region C, we can estimate k_2 from the measured current near V_p. (For future convenience, we name the current actually measured at the data point just below V_p, I_p.)

Plasma Number Density, n

Given kT_e, we have several of measuring n:

A. measured I_i and Equation 5.36, and

B. measured I_p and Equation 5.32,

Additionally we could use the fit values of I_i or I_p (`k1` or `k2`, in Equation 5.64). These methods will give answers that differ by a factor of 5 or more! When different ways of measuring the same thing give different results, "systematic error" is the name of the problem. The source of this problem is our imperfect model of current flow in Region C (all those inaccurate assumptions). In particular, both (A) and (B) are hindered by the assumption of a Maxwell-Boltzmann speed distribution. (In fact measurements in Region C are commonly used to *measure*[18] that speed distribution.) Often fit parameters are preferred to individual data points (essentially because the fit averages over several data points), but that is not the case here. Thus the method considered most accurate is (A), although it

[17]Recall: 1 eV = 1.6022×10^{-19} J is the energy an electron gains in going through a potential difference of 1 V.

[18]For example, Langmuir probes have been used to measure the electron speed distribution in the plasma that gives rise to aurora (northern lights). The results show a non-Maxwellian speed distribution: lots of high-speed "suprathermal" electrons.

Table 5.1: R.C.A. 0A4-G Gas Triode Specifications

probe length	0.34 cm
probe diameter	0.08 cm
tube volume (approx.)	40 cm^3
peak cathode current	100 mA
DC cathode current	25 mA
starter anode drop (approx.)	60 V
anode drop (approx.)	70 V
minimum anode to cathode breakdown voltage	
(starter anode potential 0 volts)	225 V

could be improved[19] to account for the slowly varying—rather than constant—ion current in Region D. (That is, how precisely is the ion saturation current determined?) However, even with that ambiguity resolved, Equation 5.36 itself has expected variations of the order of 20%. Mostly physicists just live with that level of accuracy, as improved measurement methods (like microwave phase shifts due to the index of refraction of the plasma) are often not worth the effort.

Apparatus: 0A4-G Gas Triode Tube

Figure 5.8 displays the anatomy of a 0A4-G gas triode[20]. As shown in Figure 5.9, a discharge through the argon gas is controlled by a Keithley 2400 current source. Various cathode currents ($I_c = -5, -10, -20, -40$ mA) will produce various plasma densities. During tube operation, you should see the cathode glow expand as larger discharge currents are produced. Note also that the cathode voltage ($V_c \sim -60$ V) varies just a bit, over this factor of 8 increase in I_c. A glow discharge does not act like a resistor! A Keithley 2420 is used to sweep the probe voltage and simultaneously measure the probe current. Figure 5.10 shows representative results. You should note that the cathode glow is not perfectly stable. It can jump in position for no obvious reason. If a jump occurs during a probe sweep, the resulting data will look noisy (I'' randomly jumping in sign rather than smoothly going from $I'' > 0$ to $I'' < 0$) and cannot be used. Figure 5.11 shows fits to the data between V_f and V_p. The resulting reduced χ^2 were of order 10^5: the measurement errors are much smaller than the deviations between the reality and the model. While the model is "wrong", it nevertheless supplies a reasonable representation of the data. A `fudge` of the errors allows parameter error estimates to be extracted from the covariance matrix, but it's hard to give meaning to the resulting error.

[19]This issue is addressed by Chen in report LTP-111 `Chen206R.pdf` listed as a web reference at the end of this document. Models of the Universe can usually be improved at a cost of greater complexity. Choosing an appropriate level of complexity is something of an art. Here we are using the simplest possible model.

[20]This tube is also called a cold cathode control tube. In its usual applications, what is here called the anode is called the starter anode and what is here called the Langmuir probe is called the anode.

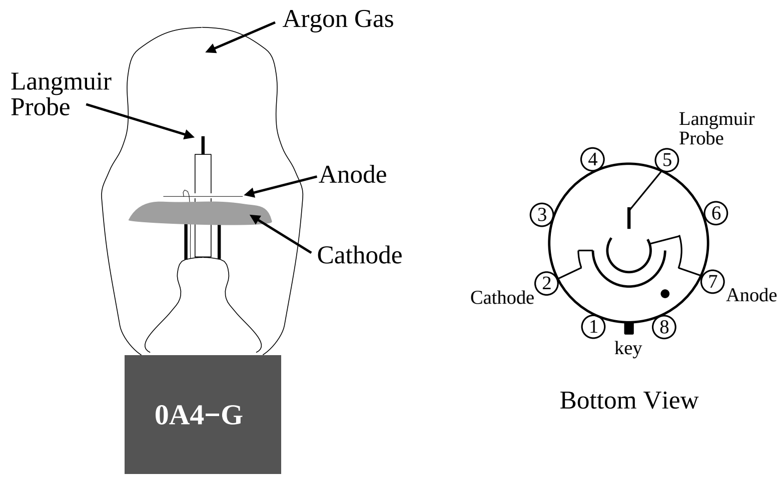


Figure 5.8: The 0A4-G triode has a large cold cathode and two "anodes" surrounded by low pressure argon gas. A glow discharge in the argon gas may be maintained by an approximate −60 V drop between the cathode (pin 2) and the "starter anode" (pin 7). The pin 5 anode may then be used as a Langmuir probe in the resulting plasma. The figure shows an R.C.A. 0A4G; A Sylvania 0A4G has the same components arranged differently.

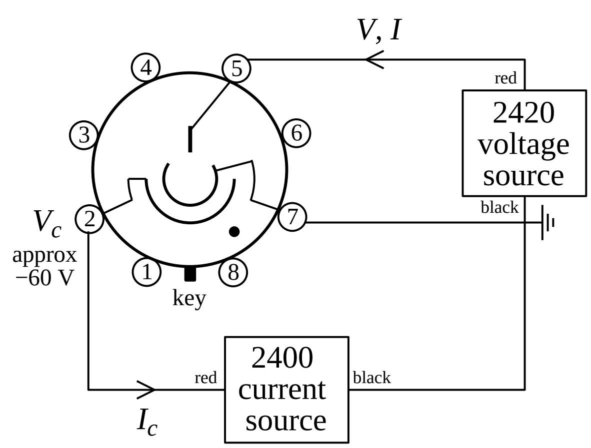


Figure 5.9: When the cathode is held at a voltage of $V_c \approx -60$ V relative to the anode, the argon gas in the tube partially ionizes and a discharge is set up between the anode and cathode. The discharge current is controlled by a Keithley 2400 in current-source mode, e.g., $I_c = -20$ mA. The voltage V on the Langmuir probe is swept by the Keithley 2420, and the current I is simultaneously measured. The resulting IV curve allows us to determine the characteristics of the plasma. Note that the pinout shows the tube as viewed from the bottom.

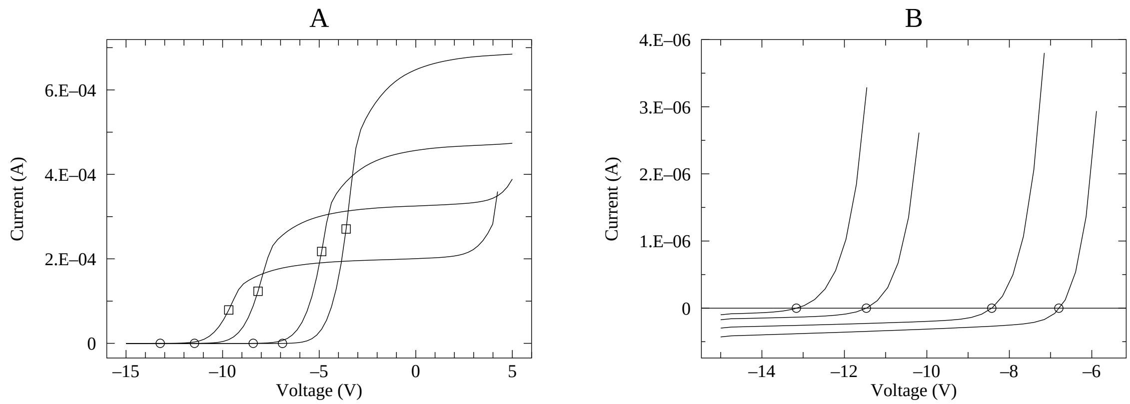


Figure 5.10: The IV curves for the Langmuir probe in a 0A4G tube for $I_c = -40, -20, -10, -5$ mA. The plasma potential V_p is marked with a square; The floating potential V_f is marked with a circle. Since the ion current is so much smaller than the electron current, blowing up the y scale by a factor of 200 is required to see it (see plot B). In this lab we are primarily concerned with Region C: between V_f and V_p.

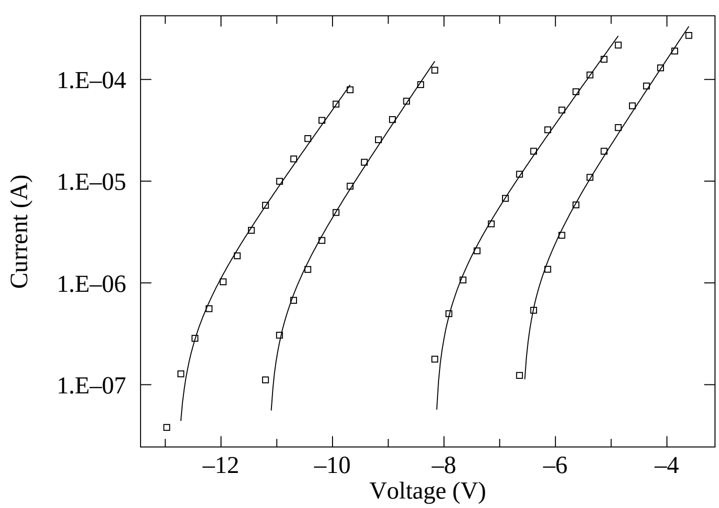


Figure 5.11: In Region C, between V_f and V_p, $I > 0$ so we can display it on a log scale. Assuming a constant ion current (k_1), constant sheath area, and a Maxwell-Boltzmann distribution of electron speed, we can fit: $I(V) = k_1 + k_2 * \exp((V - V_p)/k_3)$. The horrendous reduced χ^2 shows that these assumptions are not exact, nevertheless the fit does a reasonable job of representing the data (say, $\pm 10\%$)

Computer Data Collection

As part of this experiment you will write a computer program to control the experiment. *Plagiarism Warning*: like all lab work, this program is to be your own work! Programs strikingly similar to previous programs will alarm the grader. I understand that programming may be new (and difficult) experience for you. Please consult with me as you write the program, and test the program (with tube disconnected!) before attempting a final data-collecting run.

In the following I'm assuming the probe voltage V is stored in array `v(i)`, probe current I is stored in array `a(i)`, and the second derivative of prove current I'' is stored in array `app(i)`. Your program will control all aspects of data collection. In particular it will:

0. Declare and define all variables.
1. Open (i.e., create integer nicknames—i.e., `iunit`—for) the enets `gpib0` and `gpb1`.
2. Initialize the source-meters which must be told the maximum voltage and current to occur during the experiment. For the 2420, you can limit V, I to 25. V and .005 A; For the 2400, limit V, I to 100. V and 0.1 A;
3. Display the status of all devices before starting data collection.
4. Open files:
   (a) `VI.dump.dat` (intended for all V, I, I'' of probe, with comments (!) for cathode I_c, V_c)
   (b) `VI.fit.dat` (intended for Region C: $V, I, \delta I, I''$ of probe, with comments for cathode I_c, V_c, calculated V_f, V_p, estimated T_e, measured I_i, I_p, and the number of data points. The data points in this file are for fitting Equation 5.64:

   $$\mathtt{f(x) = k1 + k2 * exp((x - k4)/k3)} \tag{5.70}$$

   Note that $\mathtt{k4}=V_p$ is a constant, not an adjustable parameter.)
5. Tell the 2400 source-meter to source a cathode current, $I_c = -20$ mA
6. Let the system sleep for 60 seconds to approach thermal equilibrium.
7. Repeat the below data collection process <u>six</u> times. Since you will need just three repeats for each I_c, this will probably produce more data than is needed. However some data sets may be noisy because of unstable (moving, flickering) cathode glow. Noisy data will have multiple sign changes in calculated I''; Discard this data. If one run of this program fails to produce enough good data (three repeats), simply rename the data files (to preserve the data produced in the initial run), and re-run the program.

   Do the following for four different I_c: 5, 10, 20 40 mA (e.g., `acath=-.005*2**j` for `j=0,3`). Thus there will be 4×6 voltage sweeps.

   (a) Tell the 2400 source-meter to source a cathode current, I_c
   (b) Let the system sleep for 10 seconds to approach thermal equilibrium.

(c) Tell the 2420 to perform a linear probe voltage sweep from V_{min} to V_{max}, including N data points (say, $N = 100$). In Figure 5.10, you can see that the choice made was $V_{min} = -15.$ and $V_{max} = +5.$, but your choices will vary and must be determined by trial and error (see below). The aim is to find a range that includes from a few volts below Region C to a few volts above Region C for every cathode current I_c.

(d) Turn off the probe voltage.

(e) Repeat (a) thus obtaining up-to-date values for I_c, V_c

(f) Write a comment ('!') line to the file `VI.dump.dat` containing I_c, V_c from the 2400.

(g) Write a line to the file `VI.dump.dat` containing the first probe (V, I) data point: `v(1), a(1)`

(h) Do for `i=2,N-1` the following:

   i. Calculate I'' and store the value in the `i`th spot of an array i.e., `app(i)=a(i+1)+a(i-1)-2*a(i)`.

   ii. Write the probe data: V, I, I'', i.e., `v(i), a(i), app(i)`, to the file `VI.dump.dat`

(i) Write a line to the file: `VI.dump.dat` containing the last (`i=N`) data point V, I.

(j) Find the data point just before V_f with code[21] like:

```
          do i=2,N-1
                 if(a(i)>0.and.a(i+1)>0)goto 100
          enddo
100       ivf=i-1
          if(ivf.lt.15) STOP
```

The final line halts the program if `ivf`< 15, as we need Region D data to find the ion current, I_i. If the program `STOP`s, V_{min} will need to be reduced to capture this data, and the program re-run.

(k) Using Equation 5.61 and the data points at `ivf` and `ivf+1`, find V_f. Note: in the general case Equation 5.61, we were seeking x such that $y(x) = 0$; Here we are seeking V_f which is defined as the voltage such that $I(V_f) = 0$, so, for example, $x_1 \rightarrow$`v(ivf)`, $y_1 \rightarrow$`a(ivf)`, and $x \rightarrow V_f$.

(l) Find the data point just before V_p with code[22] like:

```
          do i=ivf+1,N-2
                 if(app(i)>0.and.app(i+1)>0)goto 200
          enddo
          STOP
200       ivp=i-1
```

Note that the program is halted if the plasma potential has not been found before we run out of data. In that case V_{max} will need to be increased to capture this data, and the program re-run.

[21] The check for two successive data points gone positive is done to avoid mistaking one point of noise for V_f.

[22] The check for two successive data points with $I'' < 0$ is done to avoid mistaking one point of noise for V_p

(m) Using Equation 5.61 and the data points at `ivp` and `ivp+1`, find V_p. Note V_p is defined as the voltage such that $I''(V_p) = 0$, so, for example, $x_1 \rightarrow$`v(ivp)`, $y_1 \rightarrow$`app(ivp)` , and $x \rightarrow V_p$.

(n) Write comment lines to the file: `VI.fit.dat` recording:

i. I_c, V_c, V_f, V_p (also `print` this to the screen, so you can monitor data collection)

ii. `'!set k1= ',a(ivf-15),' k2= ',a(ivp),' k3= ', 0.18615*`$(V_p - V_f)$
The aim here is to record basic plasma parameters (which are also needed as an initial guess in the fit to Equation 5.70) in a format that allows easy copy and paste into `plot` and `fit`. `k1` is the measured ion current I_i; `k2` is the measured current at V_p (denoted I_p); `k3` is an estimate for kT_e/e, the electron temperature in eV.
Notes: The estimate for `k3` is based on Equation 5.69 where $0.18615 = 2/\ln(2M_i/\pi m)$. We are assuming that `ivf-15` is far enough below V_f (i.e., in Region D) that $I_i = \mathtt{a}(\mathtt{ivf} - 15)$. You should check (after the fact) that this is the case, i.e., $V_f - \mathtt{v}(\mathtt{ivf} - 15) \gg$ `k3`. If this condition is not met, simply use a larger offset than `15` and re-run the program.

iii. `'!set k4= ',` V_p`, ' npoint= ', ivp-ivf`
`k4` is the (not adjusted) plasma potential, V_p, whereas `k1-k3` are varied from the above initial guesses to get the best fit to Equation 5.70 in Region C.

(o) For `i=ivf+1,ivp` write the probe data $V, I, \delta I, I''$ to the file `VI.fit.dat` with one $V, I, \delta I, I''$ data 'point' per line.

8. Turn off the output of the 2400.

9. Close all files.

Data Analysis

If all has gone well you have three good data sets for four different I_c, something like 1000 data points. We could spend the next semester analyzing this data! Instead I suggest below a simplified analysis scheme.

Start by making a composite plot similar to Figure 5.10A showing one IV curve for each of the four I_c. For these plots, choose V_{min} & V_{max} so that all behaviors (Regions A–D) are displayed. Make a similar plot showing three IV curves for one of the four I_c. The aim here is to test for reproducibility; because of time limitations we'll just check the reproducibility of this one selected I_c. (I'll call this selected I_c data set I_c^*.) These six data sets (one IV curve is on both plots) will be analyzed in greater detail. In order to properly `fit` the data, you will want to have more than 15 data points in the region between V_f and V_p (this may require reducing the range V_{min} through V_{max} for the probe voltage sweep so it focuses just on Region C: say from about 4 V negative of the lowest V_f to 2 V positive of the highest V_p and then re-taking the data). Additionally you should find that all six data sets have approximately the same number of data points.

Electron Temperature in eV: `k3`
Fit Equation 5.70 to the six data sets. Note that the file itself contains the required initial guesses for adjusted parameters `k1-k3` and the fixed value for `k4`. Expect to see large

reduced χ^2 which signals a too-simple model. *No definite meaning can be attached to error estimates from such poor fits*, nevertheless some sort of nonsense needs to be reported. On page 16 we explored options for dealing with such 'unusual' fits. Option #5: 'In dire circumstances' you can use `fudge` command in `fit` to change your errors so that a reduced χ^2 near one will be obtained. A re-fit with these enlarged errors then gives a new covariance matrice from which an estimate of parameter errors can be determined. Parameter values within the range allowed by these errors would produce as good (or bad) a fit to the data as the 'best' values. As the name suggests `fudge` is not exactly legitimate[23], nevertheless it is what Linfit has been silently doing all these years. Option #4: 'Bootstrap' the data: repeatedly fit subsets of the data and see how the resulting fit parameters vary with the multiple fits. This is essentially a way to repeat the experiment without taking new data. The `fit` command `boots` will report the results of `nboot` (default: 25) re-fits to subsets of your data along with the standard deviation of the fit parameters. Using either option, we are particularly interested in the some estimate of error in fit value of `k3`: use the square root of the appropriate diagonal element of the covariance matrice of a `fudge`d fit or the reported standard deviation of `k3` from a `boots`trap.

For I_c^*, compare the two errors for k_3: σ (standard deviation of 3 fit values of k_3) and that from the fudged covariance matrix. Are they in the same ballpark (say, within a factor of two)?

Calculate the electron temperature of the plasma both in eV and K. Comment on the relationship (if any) between T_e and I_c.

Plasma Number Density: n
We are interested in two sorts of 'error' in the plasma density n: reproducibility and an estimate of 'systematic' error. In the first case we're asking: "given the same I_c is applied, how much do conditions in the plasma vary?" In the second case we interested in what the actual value of n is. We can estimate this systematic error by measuring n by two different methods:[24] n calculated using I_i (Method A: let's call this n_i) and n calculated using I_p (Method B: let's call this n_p). Let's be clear here: n_i, n_p, n_e, n are all supposed to be the same thing: the number of electrons (or ions) per m^3 in the plasma. So

$$\frac{n_p}{n_i} = \frac{-I_p/I_i}{\sqrt{2M_i/\pi m}} \tag{5.71}$$

should be one; but it won't be: systematic error is present!

In your lab notebook record these results in tables similar that shown in Table 5.2[25]. (For this example, I selected $I_c^* = 20$ mA.) Note particularly to record exactly the proper number of significant figures in these tables! You should also copy & paste each full `fit` report into a long concatenated file and include the resulting file in your notebook. As previously noted, the reduced χ^2 for these fits is likely to be "horrendous". Pick out your highest reduced χ^2 fit, and plot the fitted function along with the data points on semi-log paper. (The results should look similar to Figure 5.11, but with just one data set.) Do the same for the best fit.

---

[23] for further discussion see: `http://www.physics.csbsju.edu/stats/WAPP2_fit.html`

[24] See page 124; we are discussing here only methods A and B; methods using the fit parameters `k1` and `k2` would be additional options.

[25] The spreadsheet `gnumeric` may be of use.

Table 5.2: Simplified data table for reporting Langmuir Probe results.

	k_3 (V)		I_c (mA)	k_3 (V)	fit error	σ
			40			
			20			
			10			
			5			

	I_i (A)		I_c (mA)	median I_i	σ	% error in I_i	n_i (m^{-3})	% error in n_i
			40					
			20					
			10					
			5					

	I_p (A)		I_c (mA)	median I_p	σ	% error in I_p	n_p/n_i
			40				
			20				
			10				
			5				

Calculate[26] n for each I_c based on the median I_i. Comment on the relationship (if any) between n and I_c. Make a power-law fit and log-log plot of the data[27]

Derive Equation 5.71. Calculate the n_p/n_i and see that n has systematic error, i.e., n calculated from I_p will be several times larger than the value of n calculated from I_i. This proves that there are problems with our simple theory.

Miscellaneous Calculations: 'Lawson Product' $n\tau$, λ_D, f_p
In 1957 J. D. Lawson determined that power generation from thermonuclear fusion required temperature, plasma density n and the plasma confinement time τ to meet certain criteria: a temperature of at least 10^4 eV with the product: $n\tau > 10^{21}\ \mathrm{m}^{-3}\cdot\mathrm{s}$. We can estimate τ because we know that our plasma is moving toward any surface at the Bohm velocity u_B. Given that typically the plasma is within about 1 cm of a wall, find how long it remains confined, and calculate the "Lawson product" $n\tau$ for the I_c^* plasma. (It is possible to directly measure the plasma confinement time by using a scope to time the decay of the plasma when the glow discharge is suddenly turned off.)

Calculate the Debye length (λ_D, Eq. 5.14) and the plasma frequency (f_p, Eq. 5.1) for I_c^*. Compare λ_D to the diameter of the Langmuir probe. Will the sheath substantially expand the collecting area $\mathcal{A}$? According to Koller (p. 140, reporting the results of Compton & Langmuir[28]) the mean free path of an electron in a 1 torr argon gas is 0.45 mm. Compare

[26] `http://www.physics.csbsju.edu/cgi-bin/twk/plasma.html` can do this in one click.
[27] I'd use WAPP$^+$: `http://www.physics.csbsju.edu/stats/WAPP2.html`.
[28] Rev. Mod. Phys. **2** (1930) 208

your λ_D to this electron mean free path.

Report Checklist

1. Write an introductory paragraph describing the basic physics behind this experiment. For example, why does the Langmuir probe current increase exponentially with probe voltage? Why is it that probe currents allow the calculation of plasma density? (This manual has many pages on these topics; your job is condense this into a few sentences and no equations.)

2. Computer program: Print out a copy of your program and tape it into your lab notebook.

3. Plots:

   (a) Similar to Figure 5.10A showing one IV curve for each of the four I_c.

   (b) Similar to Figure 5.10A but showing three IV curves for I_c^*.

   (c) Two plots similar to Figure 5.11, showing the Region C fit to the data. (These plots are to display the best and worst reduced χ^2; Record the reduced χ^2 on each plot.)

   (d) Power law fit and log-log plot of four (I_c, n) data points.

4. Tabulated results from six `fits` for k_3 (T_e) at four different cathode currents including `fudge`d or `boots`trapped results for δk_3 and the standard deviation of three k_3 for I_c^*. You should copy & paste each full `fit` report into a long concatenated file. Print that file and tape it into your notebook. Record the identifying letter on your tube.

5. Tabulated results for measured I_i and I_p at four different cathode currents including reproducibility errors estimated from the standard deviation.

6. Calculations (self-document spreadsheet or show sample calculations):

   (a) kT_e in units if eV for four I_c with estimates for errors.

   (b) T_e in units of K for four I_c.

   (c) n calculated from median I_i and `k3` for four I_c (with and estimate of reproducibility error)

   (d) n_p/n_i calculated from median I_i and I_p (which provides an estimate of systematic error)

   (e) λ_D

   (f) f_p

   (g) Lawson product at I_c^*

7. Derivation of n_p/n_i (Eq. 5.71).

8. Answers to the questions posed in the Data Analysis section:

   (a) Are they in the same ballpark?

   (b) Comment on the relationship (if any) between T_e and I_c.

(c) Comment on the relationship (if any) between n and I_c.

(d) Will the sheath substantially expand the collecting area $\mathcal{A}$?

(e) Compare your λ_D to this electron mean free path.

9. Discussion of errors:

   (a) Two methods were used to find "errors" in the electron temperature, T_e: (A) fudged/bootstrap error from a fit and (B) σ from lack of reproducibility. What is the meaning and significance of fudged/bootstrap error? What is the meaning and significance of reproducibility error? How would you respond to the question: "What is the error in T_e?" (Note a few sentences are required, not a number!)

   (b) Two methods were used to find the plasma number density, n: methods based on I_i and I_p. What is the meaning and significance of the fact that two different ways of measuring n produced different results. How would you respond to the question: "What is n?" (Note a few sentences are required, not a number!)

   (c) Consider any one of the basic plasma parameters (n, T_e, V_f, V_p) measured in this lab. Report any evidence that there is systematic error in the parameter. Report your best guess for the total error (systematic and random) in the parameter. Report how this error could be reduced.

Comment: Uncertainty

Area: Systemtic Error

Both methods of calculating plasma density (n_i and n_p) used the probe area $\mathcal{A}$, which entered as an overall factor. The probe area was calculated based on the probe geometry data listed in Table 5.1 which was supplied by reference #9 with no uncertainties. Clearly if the actual probe geometry differs from that in Table 5.1 there will be a systematic error in both methods of calculating n. If I guess the uncertainty in length and diameter measurements based on the number of supplied sigfigs, I find a 6% uncertainty in $\mathcal{A}$. In 2008 a Sylvania 0A4G tube was destroyed, and I took the opportunity to measure the the probe. The results[29] were quite different from those reported in Table 5.1: $\mathcal{A}$ was about 20% larger for this Sylvania tube then for the RCA tube of reference #9. (The Sylvania/GE 0A4G also looks different from the RCA 0A4G.)

There is an additional significant problem: because the plasma sheath extends several λ_D beyond the physical probe, there will be particle collection beyond the surface area of the probe—the effective area is larger than the geometric surface area; we have made a Spherical Cow by the simple assumption of $\mathcal{A} = \pi d\ell$. A glace at Fig. 5.6 on page 120, shows that the plasma sheath extends about $10 \times \lambda_D$ for $\Delta V \sim 5 \times kT_e/e$, which amounts to a large correction to probe diameter. Since the plasma sheath for ions[30] in Region D has no reason to be identical to the plasma sheath for electrons at the plasma potential, the effective $\mathcal{A}$ in the two methods is probably not the same, and hence $\mathcal{A}$ does not really cancel out in deriving Eq. 5.71.

[29] diameter= 0.025", length= .2", the surrounding metal can is about 2 cm×3 cm.

[30] For example, the mean free path for an electron is much larger than the mean free path for an ion (approximately $4\sqrt{2}\times$).

While we have lots of systematic and spherical cow error in our measurement of T_e and n, it should also be emphasized that we do have 'the right end of the stick'.

Interpreting Results

Inspection of Figure 5.11 shows the unmistakable signs of "large reduced χ^2 ": The fitted curve misses many error bars. The miss might be called "small", but the error bar is smaller still (in fact too small to show in this figure). The plasma density calculated from I_i disagrees with that calculated from I_p. What should be recorded as our uncertainty in T_e and n? The problem is the result of using simplified theory. What can we conclude from the fits using inexact[31] theory?

First the exponential IV relationship is clearly reflecting the Boltzmann factor at work. To the extent that there is <u>an</u> electron temperature, our k_3 estimate must be fairly accurate.

On the other hand our n values disagree by a factor of three, and there are reasons to suggest (e.g., uncertain $\mathcal{A}$) the systematic error may be even larger. The disagreement between n_i and n_p could be improved by better theory. (The assumed Maxwell-Boltzmann speed distribution, collisions, varying collecting areas due to sheath expansion, ... can be corrected —see particularly References 3 & 5.) However, for many practical purposes one is interested in reproducible control rather measurement. One might be told that: "silicon wafer etching is to proceed when the <u>indicated</u> n reaches 10^{14} m^{-3}" with no concern for what n actually is. Usually the reproducibility error found in the lab is only a few percent, which is often good enough for industrial control.

Of course, physics is most interested in reality, and what we have found is systematic error: two different ways of measuring n disagree. While one might argue that calculation of n based on I_i is more robust than that based on I_p, fundamentally what is needed is additional methods of determining n to resolve the problem (see Reference 8). This is also beyond the aims of this lab. The best physics you can do based on this data is to report our estimated errors with a clear warning that the systematic errors may be larger than the reported uncertainty.

References

1. Maxwell & Benedict *Theory of Gaseous Conduction and Electronics* 1941 Chapter X

2. Koller *Physics of Electron Tubes* 1937 Chapter X

3. ed. Huddlestone & Leonard *Plasma Diagnostic Techniques* 1965 Chapter 4 by F. F. Chen Dewey 537.16028 H87

4. Chen *Introduction to Plasma Physics and Controlled Fusion* 1984 p. 295

5. ed. Auciello & Flamm *Plasma Diagnostics* 1989 Volume 1: Discharge Parameters and Chemistry, Chapter 3 by N. Hershkowitz (ILL Madison)

6. Hutchinson *Principles of Plasma Diagnostics* 2001 Chapter 3 (ILL Bemidji)

[31] Note that <u>all</u> theory is inexact.

7. `http://www.physics.uiowa.edu/~fskiff/Physics_132/Lab%20Manual/B4Langmuir.PDF`

8. `http://www.ee.ucla.edu/~ffchen/Publs/Chen206R.pdf`

9. I. Alexeff, J. T. Pytlinski, and N. L. Oleson Am. J. Phys. **45** (1977) 860–866, also see Am. J. Phys. **45** (1977) 1196–1199, Am. J. Phys. **46** (1978) 1276–1278, Am. J. Phys. **52** (1984) 753–754