The Electron Cyclotron Drift Instability

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1 Acronyms
1. Electron Cyclotron Drift Instability ≡ (ECDI)
2. Electron Cyclotron Harmonic ≡ (ECH)
3. Ion-Acoustic Waves ≡ (IAWs)
4. Electron-Acoustic Waves ≡ (EAWs)
5. Lower Hybrid Waves ≡ (LHWs)
6. Electrostatic ≡ (ES)
7. Electromagnetic ≡ (EM)
8. Modified Two-Stream Instability (MTSI)
9. Let ECHWs include: Bernstein, totem-pole, and (n + 1/2) waves

2 Other Names
1. Electron Cyclotron Drift Instability (ECDI)
2. Beam Cyclotron Instability [Lampe et al., 1971a,b, 1972]

3 Ashour-Abdalla and Kennel et. al., [1978a]
Ashour-Abdalla and Kennel [1978] examined nonconvective and convective ECHIs (f ≃ (n + 1/2)fce) finding:
1. nce controls which harmonic band can be excited through the upper hybrid frequency
2. Tce controls the spatial amplification → when 0 < Tce/The ≲ 10^{-2} ⇒ instability is nonconvective while larger values will eventually cause the instability to become convective
3. if Tce/The ≲ 5 \times 10^{-2}, quasi-linear diffusion increases Tce faster than resonant diffusion can heat/scatter hot electrons into the loss-cone
4. if nce/nee ≃ 3-5, the instability does not occur

4 Ashour-Abdalla and Kennel et. al., [1980]
Ashour-Abdalla et al. [1980] examined ECHIs finding that the waves heated the cold electrons perpendicular to the field faster than parallel.
5 Forslund et al., [1970]

Forslund et al. [1970] examined the electron cyclotron drift instability (ECDI), which is an instability that occurs when ions drift relative to electrons, $V_d$, across a magnetic field. The dispersion relation he used was:

$$(K\lambda_{De})^2 = -1 + e^{-2}I_0(\lambda) + 2\omega^2 \sum_{n=1}^{\infty} \frac{e^{-2}I_n(\lambda)}{\omega^2 - (n\Omega_{ce})^2} + \frac{T_e}{2T_i} Z'(\omega - kV_d - \frac{kV_T}{\Omega_{ce}})$$

where $\Omega_{ce} = eB/(m_e c)$, $\lambda + (k r_i)^2/2$, $\lambda = 1/2 \sqrt{k_\perp V_{Te}/\zeta_{Te}}$, $r_i = V_{Te}/\Omega_{ce}$, $V_{Te}^2 = 2T_e/m_e$, and $Z'(x)$ is the derivative of the plasma dispersion function given by $Z'(x) = -2[1 + x Z(x)]$. The plasma dispersion function can be written as:

$$Z(x) = i \frac{k}{|k|} \sqrt{\pi} e^{-x^2} - \left[ \frac{1}{x} + \frac{1}{2x^3} + \frac{3}{4x^5} + \cdots \right] \quad \text{(for } x \gg 1) \tag{2a}$$

$$Z(x) = i \frac{k}{|k|} \sqrt{\pi} e^{-x^2} - \left[ 2x - \frac{4}{3} x^3 + \frac{8}{15} x^5 + \cdots \right] \quad \text{(for } x \ll 1) \tag{2b}$$

$$Z'(x) = -2i \frac{k}{|k|} \sqrt{\pi} x e^{-x^2} + \left[ \frac{1}{x^2} + \frac{3}{2x^4} + \frac{15}{4x^6} + \cdots \right] \quad \text{(for } x \gg 1) \tag{2c}$$

$$Z'(x) = -2i \frac{k}{|k|} \sqrt{\pi} x e^{-x^2} - \left[ 2 - 4x^2 - \frac{8}{3} x^4 + \cdots \right] \quad \text{(for } x \ll 1) \tag{2d}$$

The instability results from a resonance between the otherwise purely electron cyclotron waves (ECWs) and ions where $\partial F_e/\partial V > 0$, or where $V < V_d$. Due to the resonance with the ions, $\gamma_{max}$ for each harmonic occurs near $\text{MAX}[(\partial F_e/\partial V)]$, which is about at $\omega/k \sim V_d - V_{Te}$. Note that for all $\omega \ll k V_i$, $\gamma_{max} > 0 \Rightarrow$ unstable growth. From this, one can see:

1. when $(k \lambda_{De}) > 1$, there is an attenuation of $\gamma \propto (k \lambda_{De})^{-4}$, which acts as a cutoff for large $(k \lambda_{De})$

2. for $(k r_i)$ large, but $(k \lambda_{De}) < 1$, all harmonics have roughly equal $\gamma$’s because the resonance condition is $(k r_i) \simeq n_v V_{Te}/(V_d - V_{Te})$

3. when $V_d > V_{Te}$ and $(k \lambda_{De}) < 1$, (noting that $\Im(Z') \sim 1$) $\gamma \propto \Omega_{ce} V_d/V_{Te}$

4. $\gamma \propto \Omega_{ce}/M_i$

5. for larger values of $\omega_{pe}/\Omega_{ce}$, the Debye length cutoff occurs at smaller values of $\omega_{pe}/\Omega_{ce}$, making more harmonics grow for larger $\omega_{pe}/\Omega_{ce}$.

Due to $\gamma$’s dependence on $\Re(Z')$, there is a strong interaction between the ECWs and ion modes. Note also that $T_i/T_e$ has little effect on $\gamma$. Since the Debye length cutoff implies the instability is most effective when $\omega_{pe} \sim \Omega_{ce}$ for $V_d > V_{Te}$, we assume that $\omega/k > V_{Te}$ and $\gg V_{Te}$. Using these assumptions, we can reduce Equation 1 to the following:

$$1 = \frac{\omega_{pe}^2}{\omega^2 - \Omega_{ce}^2} + \frac{\omega_{pe}^2 m_i/M_e}{(\omega - kV_d)^2} \tag{3}$$

which nicely reduces to the usual two-stream instability for $\omega_{pe}/\Omega_{ce} \gg 1$. Equation 3 represents an interaction between the upper hybrid and a Doppler-shifted lower hybrid mode.

There are three conditions which can quench the instability:

1. when $V_d \rightarrow 0$ (which could result from field diffusion due to the instability itself)

2. when the instability heats the electrons to the Debye-length cutoff

3. when the ions are resonantly heated until the MAX($\Im(Z')$) for the fundamental is very small and/or lies beyond the Debye-length cutoff

When $V_{Te} \gg V_d$ and $\omega_{pe}/\Omega_{ce} \gg 1$, the instability will preferentially heat the ions instead of the electrons. Since the resonance occurs near the MAX($\partial F_e/\partial V$), the ECDI can be an effective ion heating mechanism. In addition to heating the plasma, the ECDI produces an anomalous resistivity which causes a drift and diffusion across the magnetic field.

The ECDI is mostly a longitudinal instability until the particles become relativistic and it does not appear to be affected by finite $\beta_e$. 

\[2\]
6 Forslund et al., [1971]

Forslund et al. [1971] examined the nonlinear ECDI, finding significant electron heating due to adiabatic and non-adiabatic trapping. The ions were heated as well due to trapping. So the electrons are dragged across the magnetic field by the drifting ions, which produces an effective drag on the ions causing them to break the frozen-in condition and gain some thermal energy while losing bulk kinetic. Also, ion trapping does not reduce the heating rate.

7 Forslund et al., [1972]

Forslund et al. [1972] examined the perpendicular anomalous resistivity it causes at collisionless shocks and in lab plasmas. They consider a relative drift, \( V_d \), between electrons and ions that is perpendicular to both the magnetic field and the shock normal. They treat the ion trajectories as straight because the \( \gamma_{ECDI} \gg \Omega_i \). If one assumes that \( \lambda \gg 1 \), then one can approximate:

\[
e^{-\lambda I_n(\lambda)} \simeq \sqrt{\frac{1}{2\pi \lambda}}
\]

They also assume that the plasma dispersion functions are of the form:

\[
Z'_{i} = \frac{\omega_{i} - k \cdot V_d}{k V_{Ti}}
\]

\[
Z'_{e} = \frac{\omega_{e} - n\Omega_{ce}}{k V_{Te}}
\]

As one might expect, the largest values of \( \gamma \) occur when \( k_{\parallel} \rightarrow 0 \), which results in \( \Re[Z'] = \Im[Z'] = 0 \). This simplifies the growth rate calculation and real frequency result to:

\[
\frac{\gamma}{\Omega_{ce}} \simeq \frac{n_{e}}{\sqrt{\pi kr_{e}}} \left[ \frac{T_{e}/(2T_{i})}{1 + (k_{\lambda_{De}})^{2} - (T_{e}/2T_{i})\Re[Z']^{2} + [(T_{e}/2T_{i})\Im[Z']^{2}]^{2}} \right]
\]

\[
\frac{\omega_{e} - n\Omega_{ce}}{\Omega_{ce}} \simeq \frac{\gamma}{\Omega_{ce}} \left[ 1 + (k_{\lambda_{De}})^{2} - (T_{e}/2T_{i})\Re[Z']^{2} \right]
\]

If we drop the term associated with \( \Re[Z'] \) in the denominator of Equation 6a (typically okay when \( T_{e} \approx T_{i} \)), and noting that \( \max(\Im[Z']) \approx 1.5 \) (when its argument is \( \sim -0.7 \)), then Equations 6a and 6b reduce to:

\[
\frac{\gamma}{\Omega_{ce}} \simeq \frac{n_{e}}{\sqrt{\pi kr_{e}}} \left( \frac{T_{e}}{2T_{i}} \right)^{3/2} \left[ 1 + (k_{\lambda_{De}})^{2} \right]^{2} + [(3T_{e}/4T_{i})^{2}]^{2}
\]

\[
\frac{\omega_{e} - k \cdot V_d}{k V_{Ti}} \simeq -0.7
\]

assuming \( \omega_{e} \approx n\Omega_{ce} \) and \( \cos \theta = k \cdot V_{d}/(k V_{d}) \), then we can reduce Equations 7a and 7b down to:

\[
k \simeq \frac{n\Omega_{ce}}{V_{d} - 0.7V_{Ti}}
\]

\[
\frac{\gamma}{\Omega_{ce}} \simeq \frac{\cos \theta}{\sqrt{\pi}} \left( \frac{V_{d}}{V_{Te}} \right) \left( \frac{T_{e}}{2T_{i}} \right) \left[ 1 + (n\Omega_{ce}/\sqrt{\pi})^{2} \right]^{2}
\]

\[
\frac{\omega_{e} - k \cdot V_d}{k V_{Te}} \simeq \frac{\cos \theta}{\sqrt{\pi}} \left( \frac{V_{d}}{V_{Te}} \right) \left( \frac{T_{e}}{2T_{i}} \right) \left[ 1 + \left( (n\cos \theta)(V_{d}/V_{Te})(\Omega_{ce}/\sqrt{\pi}) \right)^{2} \right]
\]

where I have ignored any electron-electron or electron-ion collisions.

Not that for all harmonics with \( (k \lambda_{De}) < 1 \), \( \gamma_{\text{max}} \neq \gamma_{\text{max}(n, m, M)} \) and occurs at the same \( k_{\perp} = k \cos \theta \) for all \( k_{\perp} \) in the plane perpendicular to \( B_{e} \). In this case, the last term in the brackets of Equation 8c reduces to unity leaving the growth rate to be:

\[
\frac{\gamma}{\Omega_{ce}} \simeq \frac{\cos \theta}{\sqrt{\pi}} \left( \frac{V_{d}}{V_{Te}} \right) \left( \frac{T_{e}}{2T_{i}} \right)
\]
The expected turbulence fills a relatively wide range of angles in the plane perpendicular to \( \mathbf{B}_o \) but parallel to \( \mathbf{V}_d \). Anisotropic heating due to cyclotron interactions can change this fan-like resonance into a cone that extends into the plane containing \( \mathbf{B}_o \). From Equation 8c when \( (k \lambda_D) > 1 \), there is a strong Debye-length cutoff \( \propto (k \lambda_D)^{-4} \) which allows us to estimate an approximate instability criterion:

\[
\frac{V_d}{V_{Te}} \gtrsim \frac{n}{\Omega_e} \frac{\Omega_e}{\sqrt{2} \omega_{pe}}
\]

From the full resonance condition (Equation 8a) one can see that if \( V_{Ti} \to > V_d \), then \( k \to > \) Debye-length cutoff for all harmonics. Thus one also must demand that \( V_d \gtrsim V_{Ti} \) for an instability to occur.

There are four possible ways to stabilize this instability:

1. \( V_d \) is reduced by resistive broadening of \( \mathbf{B}_o \)

2. \( V_{Ti} \) is increased by resistive heating which results in successively lower harmonics being stabilized until the \( n = 1 \) (fundamental) is stabilized at \( V_d/V_{Ti} \propto \Omega_e/\omega_{pe} \). The primary electron heating will be perpendicular to \( \mathbf{B}_o \), various longitudinal and transverse instabilities driven by the resulting anisotropy could rapidly convert some electron thermal energy into parallel momentum, thus producing effectively an increase in electron thermal conductivity.

3. \( V_{Ti} \) can be increased by heating until \( V_d \simeq V_{Ti} \), however this is less likely than (1) or (2) because electron heating is more effective

4. plasma compression \( \perp \mathbf{B}_o \) can reduce \( \Omega_e/\omega_{pe} \) by \( \sqrt{N_2/N_1} \Rightarrow \) increases the instability threshold.

When considering the full dispersion relation (not shown), there are effects to consider when \( k_{||} \neq 0 \) that enter through \( Z' (\equiv Z'[(\omega_k - n \Omega_e)/(k_1 V_{Ti})]) \) in the combinations \( /2 \Im[Z'] \) and \( (1 + \Re[Z']/2) \). Both of these terms go to zero (from above and below, respectively) for argument \( \to 0 \), and through their quotient \( (\Im[Z']/2)/(1 + \Re[Z']/2) \) [argument \( \to 0 \), from the \( k_{||} > 0 \) side]. However, when all terms are considered, \( \gamma/\Omega_e \) decreases monotonically \( \propto k_{||} \) increasing \( k_{||} \). The cutoff occurs when the argument of \( Z' \) \( \to 0 \), thus \( (\omega_k - n \Omega_e)/\Omega_e \sim n/(\sqrt{2} k r_e) \) and the resonance condition simplifies to \( \omega_e/k \simeq V_{Te} \cos \theta \). If we assume \( (k \lambda_D) < 1 \), then the spread in \( k_i \) can be shown to be:

\[
\frac{k_i}{k} \lesssim \frac{n}{\sqrt{\pi}} \frac{(k r_e)^2}{\sqrt{k r_e}} \left( \frac{1}{\sqrt{\pi n}} \right) \left( \frac{V_{Te} \cos \theta}{V_{Ti}} \right) ^2
\]

The spread in \( k_i \) is less for higher harmonics and in general, much narrower than for \( k_{||} \). The damping which limits \( k_i \) is cyclotron, NOT Landau, damping! Thus, as the Debye-length (or collisional) cutoff of a harmonic is approached and exceeded, its \( k_i \) spread \( \to 0 \). The important conclusions are:

1. an instability still exists if a weak magnetic field and \( T_e \sim T_i \) but does not exist if the magnetic field \( \to 0 \)

2. the instability is difficult to stabilize (linearly) without significant magnetic field diffusion or electron heating

3. electron collision frequency \( \propto \gamma \) is required to stabilize

4. instability occupies a broad cone of angles in the plane \( \perp \mathbf{B}_o \) but a very narrow cone of angles in the plane \( \parallel \mathbf{B}_o \)

### 7.1 Analytical Linear Theory

The IAW mode still exists in the presence of a magnetic field and couples strongly to the ECDI when \( T_e > T_i \). However, they find that the IAW is never unstable, but the Bernstein roots are. However, when the full dispersion relation is solved, the dependence of \( \gamma/\Omega_e \) on \( T_e/T_i \) is even weaker than suggested by Equation 6a. In fact, Forslund et al. [1972] claims that there appears to be no real distinction between the cold and warm plasma solutions since the dependence of \( \gamma/\Omega_e \) on \( T_e/T_i \) is so weak. Even more, when \( T_e \gtrsim T_i \), the largest \( \gamma \)’s occur at the lower harmonics.

Using \( \Omega_e/\omega_{pe} \sim 1/50 \), \( m_i/M_i \sim 1/1836 \), and \( T_i/T_e \sim 1 \), they find that \( \gamma/\Omega_e \) increases rather dramatically for higher harmonics as a function of \( V_{Te}/T_e \).
7.2 Numerical Nonlinear Theory

Consider the case where the electric field parallel to the shock normal is zero, thus the canonical particle momentum in that direction will be a constant for every particle. If we also consider a class of electrons whose undisturbed gyromotion guiding centers result in a velocity, \( \mathbf{v}_n = (\mathbf{x} \cdot \mathbf{V}_d/V_d) \Omega_c. \) Thus the total force along the \( \mathbf{V}_d \) direction (define as \( x \)) is given by:

\[
F_i = -e \left[ -\phi(x) + x\Omega_c B_i/e \right]
\]

which gives an effective combined potential of:

\[
\Phi = -e\phi + m_e/2 \left( \Omega_c x \right)^2
\]

where the second term simply describes the gyromotion, the electrostatic part, \( \phi(x) \), remains tied to the ions and moves with about \( V_d \). The effect could be seen as a superposed ripple on the parabolic potential defined by the second term.

For the situation where \((k \lambda_{De}) < 1, \) and noting that \((k \lambda_{De}) \approx n_i (V_{Te}/V_d), \) if \( V_d \gg V_{Te}, \) few electrons are resonant with the wave. Note that the growth saturates as the perturbed electron velocities reach \( V_d \), which means the electrons break their \textit{frozen-in} trajectories and start to become trapped in the potential wells of the waves. If no magnetic field is present, this occurs at \( e\phi_e \approx m_e V_e^2/2, \) where \( \phi_e \) is the magnitude of oscillations of \( \phi(x). \) However, the addition of a magnetic field reduces the saturation level of \( e\phi \) due to the Lorentz force term \( \mathbf{v} \times \mathbf{B}. \) The equation of motion for an electron in an oscillating electric field is given by:

\[
v_e = -\left( i e E_i/m_e \omega \right) \left[ 1 + \left( \Omega_c \omega \right)^2 \right]^{-1}
\]

where we can replace \( E_i \) with \(-i\phi_e (\omega/V_d) \) for a wave traveling at velocity \( V_d \) with respect to the magnetic field. If we also let \( v_e = -V_d, \) then the trapping saturation estimate goes to:

\[
e\phi_e = m_e V_d^2 \left[ 1 - \left( \Omega_c \omega \right)^2 \right]
\]

Note that in the low density regime one needs \( V_d \gg V_{Te} \) to overcome the Debye-length cutoff and when \( \omega \approx \Omega_c, \) the saturation potential is greatly reduce. When \( \omega \approx \Omega_c, \) the electrons respond by coiling up into ordered spirals in phase space (\textit{i.e.} gyrophase restricted) while the ions suffer considerable heating because of resonant breaking of their \textit{frozen-in} trajectories.

When the dominant modes satisfy \( 1 < (k \lambda_{De}) < 2\pi, \) a transition behavior is observed. There is considerable electron heating in this regime.

When the dominant modes satisfy \( (k \lambda_{De}) \gg 2\pi, \) and if \( V_d < V_{Te} \) and \( \omega_{pe}/\Omega_c \gg 1, \) as in the solar wind, the wave is resonant with the bulk of the electron distribution. To modify the linear growth by a nonlinear distortion of the electron velocity distribution, the electrons must have time execute a trapping oscillation in the potential wells. Unlike the field-free case, the resonant interaction of the electrons with the wave is limited by the smaller of the following two: 1) the time a gyrating electron remains in resonance with a wave \((\sim \Omega_c^{-1}), \) or 2) the time that a well, \( \phi(x), \) remains in existence \((\sim (e\phi_e/m_e) (k\Omega_c V_d)) \). From this, we can estimate the threshold for which electron trapping modifies the linear growth rate as:

\[
e\phi_e \approx m_e V_d^2 \left[ 2\pi \left( \Omega_c/kV_d \right) \right]^{2/3}
\]

Note, however, that the potential estimated by Equation 16 tends to be a very small number. However, if a sufficiently large number of electrons become trapped in the wells, the potentials will enhance an possibly grow to larger than the thermal energy of the electrons. If this happens, the trapped electrons will be carried along by the potentials to be eventually released at a higher energy which increases the energy associated with gyration (since they released into the larger magnetic potential well). In other words, the electrons are first energized along the shock normal and their increased energy perpendicular to the magnetic field, thus they gain energy in the \( x \)-direction too! Also, if \( V_d \ll V_{Te} \) and \( \phi \) is large enough to trap electrons above their thermal energy, then the electrons remain trapped for much longer than a gyroperiod.

Their perturbed charge density is also shifted relative to the potentials.

The ion perturbed charge density, on the other hand, is almost entirely defined by \( \phi(x) \) alone. Thus a phase shift is produced between the perturbed charge density of each species which drives the magnitude of \( \phi \) well beyond the value in
Equation 16. This makes the instability become very efficient at heating electrons. However, the nonlinear instability is no longer resonant with the bulk of the ions but only their high energy tails.

7.3 Discussion
The anomalous resistance is ultimately caused by clumps of trapped electrons being pulled across the magnetic field by the drifting density maxima of the ions. The electrons cause an effective drag on the ion drift causing the ions to lose drift energy and gain thermal energy by breaking. This also causes the electrons to get heated by increasing their velocity along the shock normal until they are pulled out of their potential wells (i.e. perpendicular heating). The requirement for no net current along the shock normal direction causes the convective electric field to adjust itself to give an $\mathbf{E} \times \mathbf{B}$ drift along the shock normal which cancels out the net drift of the electrons in that direction.

8 Lampe et. al., [1971]
Lampe et al. [1971a] examined the nonlinear development of the ECDI. If $V_d > C_s$, then the electron Bernstein modes can couple to the IAWs. They also found that the ECDI saturations after a sufficient amplitude and mode converts to the IAW. For warm plasmas, the IAWs are stabilized by Landau damping (i.e. parallel heating).

9 Matsukiyo and Scholer, [2006]
Matsukiyo and Scholer [2006] investigated microinstabilities at a perpendicular supercritical collisionless shock. They used a 2D PIC simulation with realistic mass ratio ($\sim 1836$), $\beta_{inc} = 0.04$, $\beta_{ref} = 0.01$, $\beta_e = 0.05$, $n_{ref}/n_{inc} = 0.25$, $(\omega_{pe}/\Omega_{ce})^2 = 4$, $\Delta t \sim 0.02 \omega_{pe}^{-1}$, $\Delta x = \Delta y = 0.5 \lambda_{De} \sim 0.04 c/\omega_{pe}$, $U_{inc}/V_A = +2.14$, and $U_{ref}/V_A = -8.57$. There are three instabilities of interest, though they observed 6, ECDI, MTSI-1, MTSI-2.

The simulation geometry is shown as:
The properties of the ECDI observed in this simulation can be seen as:

1. **Free Energy Source:** [reflected ion] - [incident electron] relative drift
2. $k_x \sim n\Omega_c/U_r$, where $U_r$ is the reflected ion speed
3. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 10$, $-2 \gtrsim k_y c/\omega_{pe} \gtrsim 2$
4. 1st two harmonics are seen
5. mostly in $\delta E_x$ and $\delta B_y$, but VERY diffuse in $k_x$-$k_y$ space
6. X-mode polarization $\Rightarrow$ compressional $\delta B$
7. nonlinearly couples to MTSI-1
8. heats electrons strongly perpendicular to magnetic field and slightly heats reflected ions

The properties of the MTSI-1 observed in this simulation can be seen as:

1. **Free Energy Source:** [incident ion] - [locally decelerated electron] relative drift
2. $k$ mostly $\perp B_0$
3. $k_x > 0 \Rightarrow$ anti-$\parallel n$
4. $k_y$ is both positive and negative
5. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 3$, $-0.5 \gtrsim k_y c/\omega_{pe} \gtrsim 0.5$ (seen in $\delta B_z$)
6. nonlinearly couples to ECDI
7. heats incident ions strongly and maintains their density profile
8. drives ES perpendicular whistler waves

The properties of the MTSI-2 observed in this simulation can be seen as:

1. **Free Energy Source:** [reflected ion] - [incident electron] relative drift
2. $k$ oblique-$B_0$
3. $k_x < 0 \Rightarrow \parallel n$
4. $k_y$ is both positive and negative
5. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 2$, $-1 \gtrsim k_y c/\omega_{pe} \gtrsim 1$ (seen in $\delta B_z$)
6. little to no heating of reflected ions
7. drives oblique EM whistler waves, electron holes
8. through a two-step heating process the effects of the MTSI-2 cause tremendous electron heating

The two-step heating process occurs in the following manner:

1. ECDI drives a perpendicular anisotropy in the electrons which is unstable to whistlers (seen in $\delta B_z$ and $\delta B_x$)
2. the MTSI-2 drives oblique EM whistler waves and electron holes
3. the electron holes produce double-peaked electron velocity distribution functions which are unstable to EAWs (seen in $\delta E_y$) which strongly heat electrons $\parallel B_0$
4. the electron temperature is observed to increase by a factor of $\sim 5$ while the ions only increase by $\sim 5/4$ for reflected and $\sim 2$ for incident
where \( k = k_{\omega} \hat{x} + k_{\perp} \hat{z} \), \( k_{\omega}, k_{\perp} \gg k_{\omega} \), and an electron velocity distribution, \( f_e = f_{\omega} + \delta f_e \), where \( f_{\omega} \) is a Maxwellian and \( \delta f_e \) is governed by the linearized Vlasov equation:

\[
\frac{\partial \delta f_e}{\partial t} + v \cdot \nabla (\delta f_e) = -\frac{e}{m_e} \nabla \phi \cdot \frac{\partial f_{\omega}}{\partial v}
\]

which results in a solution for \( \delta f_e \) going as:

\[
\delta f_e = \frac{2ie}{m_e V_{ce}^2} (f_{\omega} \phi_e) I
\]

where:

\[
I = \sum_{p=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \frac{e J_l(\alpha)}{(\omega_o - p\Omega_{ce} - k_{\perp} v_{\perp})} \left[ \frac{ik_{\omega} v_{\perp}}{2} (J_{p+1}(\alpha) + J_{p-1}(\alpha)) + ik_{\perp} v_{\perp} J_p(\alpha) \right]
\]

where \( \alpha = k_{\omega} v_{\perp}/\Omega_{ce}, \theta = \tan^{-1}(v_{\perp}/v_o) \) is the gyrophase angle, \( v_{\perp} = (v_o^2 + v_{\perp}^2)^{1/2} \), and they used the identity:

\[
e^{i\alpha \sin \theta} = \sum_{l=-\infty}^{\infty} J_l(\alpha) e^{il\theta}
\]

From this, we can find the electron drift velocity due to the Bernstein mode as:

\[
v_o = \int_0^\infty \int_{-\infty}^{\infty} dv_{\perp} dv_o d\theta \, v_{\perp} \, \delta f_e = u_o \phi_e
\]

where \( u_o \) can be represented as:

\[
u_{ox} = \left( \frac{2\omega_o^2 \Omega_{ce}^2}{e^2 V_{ce}^2 k_{\perp} k_{\omega}} \right) \left[ (l+1)I_l(b)e^{-b} + I_l + \frac{V_{ce} k_{\perp}}{4\Omega_{ce}} \left( 1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) l I_l(b)e^{-b} \right]
\]

\[
u_{oy} = \left( \frac{\omega_o^2}{2\pi e k_{\perp} V_{ce}} \right) \left[ k_{\perp} (I_l - I_{l-1}) + (k_{\perp} - I_k) \left( 1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) \right]
\]

\[
u_{oz} = \left( \frac{\omega_o^2 \Omega_{ce}}{2\pi e k_{\perp} V_{ce}^2} \right) \left( 1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) l I_l(b)e^{-b}
\]
where the terms $I'$, $I_h$, $I_d$, and $I_e$ are given as:

$$I' = -\frac{2k_{\perp} \Omega_{\perp} V_{Te}}{\Omega_{e} V_{Te}} I_h - \left( \frac{3k_{\perp}^2}{4 \Omega_{e}^2 V_{Te}} \right) (I_h + I_e)$$  \hspace{1cm} (24a)$$

$$I_h = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) J_{i-1}(\alpha) v_{\perp}^2 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24b)$$

$$I_b = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) v_{\perp}^3 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24c)$$

$$I_c = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) J_{i+2}(\alpha) v_{\perp}^3 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24d)$$

$$I_d = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) J_{i-2}(\alpha) v_{\perp}^3 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24e)$$

$$I_e = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) J_{i+1}(\alpha) v_{\perp}^2 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24f)$$

$$I_e = \frac{1}{V_{Te}^2} \int_0^\infty dv_{\perp} J_i(\alpha) J_{i-1}(\alpha) v_{\perp}^2 e^{-(v_{\perp}/V_{Te})^2}$$  \hspace{1cm} (24g)$$

where they assumed that $\omega_o \approx 1 \Omega_e$ and they retained only one term in Equation 20.

In addition to the Bernstein mode, there also exists a low frequency IAW or lower hybrid mode with potential, $\phi$, of the same form as Equation 17 and an electron density perturbation of the form:

$$\delta n_e = \frac{k^2}{4\pi e^2} \chi \phi$$  \hspace{1cm} (25)$$

where $\chi$ is the electron susceptibility and can be of the form:

$$\chi_{IAW} = 2 \left( \frac{\omega_m}{kV_{Te}} \right)^2$$  \hspace{1cm} (26a)$$

$$\chi_{LHW} = \frac{2}{\Omega_{e} \omega_m} \left( \frac{\omega_m k_i}{\Omega_{e} \omega} \right)^2 - \frac{1}{\omega_m} \frac{1}{k}$$  \hspace{1cm} (26b)$$

The density perturbation couples with the $v_o$ to produce a current, $J^{NL}_i = -\frac{1}{2} n_e e v_o$, with frequency, $\omega_i = \omega - \omega_o$, and wave vector, $k_i = k + k_o$. The frequency of the IAW needed for resonant coupling can be shown as:

$$\omega_{IAW} = k C_i = C_i \sqrt{(k_i^2 + k_o^2 - 2k_i k_o \sin \psi)}$$  \hspace{1cm} (27)$$

where $C_i$ is the ion-acoustic speed ($\approx \sqrt{k_i T_i/M_i}$) and $\psi = \cos^{-1}(k_i \cdot B/k_i B_i)$. The electric field of the EM wave produced by the coupling can be written as (after considerable approximations) as:

$$E_1 = \frac{2\pi l_e}{\omega_i^2 - (k_i e)^2} \left( v_o - \frac{c^2 k_i (v_o \cdot v_i)}{\omega_i^2} \right)$$  \hspace{1cm} (28)$$

### 11.1 Parametric Conversion of EM Wave to Bernstein Wave

Consider a circularly polarized EM wave near a cyclotron harmonic in a plasma with an electric field given by:

$$E_i = A_i e^{i(\omega_i t - k_i z)}$$  \hspace{1cm} (29)$$

where $A_i = A_{i,x} \hat{x} + A_{i,y} \hat{y}$, $A_{i,x} = -i A_{i,y}$, $k_i = \omega_i / c [1 - \omega_m^2/\omega_i \{ \omega_i + \Omega_{e} \}]$. This wave oscillates the electrons at a velocity given by:

$$v_i = \frac{e A_{i,x} (\hat{x} - i \hat{y})}{m_e (\omega_i + \Omega_{e})}$$  \hspace{1cm} (30)$$

The pump wave decays into an IAW and Bernstein wave or LHW and Bernstein wave, each with potentials of the form seen in Equation 17 and $\omega_o = \omega - \omega_i$, $k_o = k - k_i$. The Bernstein wave produces an oscillatory electron velocity, $v_o = u_o \phi_o$, which was derived earlier. The density perturbation, $\delta n_e$, due to the IAW couples to the oscillatory velocity $v_i$ produced by the pump EM wave. This coupling between the density perturbation and oscillatory velocity produce a nonlinear current density which drives the Bernstein wave at $(\omega_o, k_o)$. 
11.2 Discussion

Electron Bernstein waves, in the presence of low frequency high(low) wave number(length) IAWs, will efficiently mode convert into EM radiation at the cyclotron harmonics. This radiation can far exceed background if $v_e \sim C_s$. The reverse process, parametric excitation of electron Bernstein waves, is also efficient for high temperature plasmas with optimal growth for $k_o r_e \sim 2$ and $\gamma/\Omega_{ci} \sim 1$.

References


