**Physical Constants:**

Earth's free-fall acceleration $= g = 9.80$ m/s$^2$

Circle the letter of the single best answer. Each question is worth 1 point

1. A quartz fiber resists being twisted. The restoring torque ($\tau$) is in proportion to the angle of twist $\theta$ (measured in radians):

$$\tau = -\kappa\theta$$

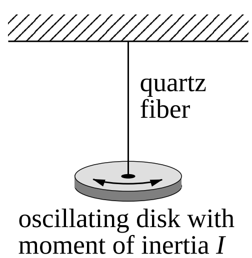
where the proportionality constant is $\kappa$. In a torsional oscillator a disk of moment of inertia $I$ oscillates first one way (twisting the fiber) and then the other (untwisting the fiber) taking a time $T$ to complete a cycle. Which of the below is a possible formula for $T$ (i.e., has units of time)?

A. $T = \sqrt{\frac{\kappa}{I}}$

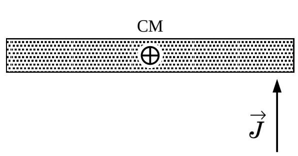
B. $T = \kappa \cdot I$

C. $T = \kappa^{\frac{3}{2}} \cdot I^{\frac{1}{2}}$

D. $T = \sqrt{\frac{I}{\kappa}}$




2. A uniform bar resting on frictionless ice is kicked near its end providing a horizontal impulse $\vec{J}$ as shown below. The center of mass (labeled below as CM) will then:



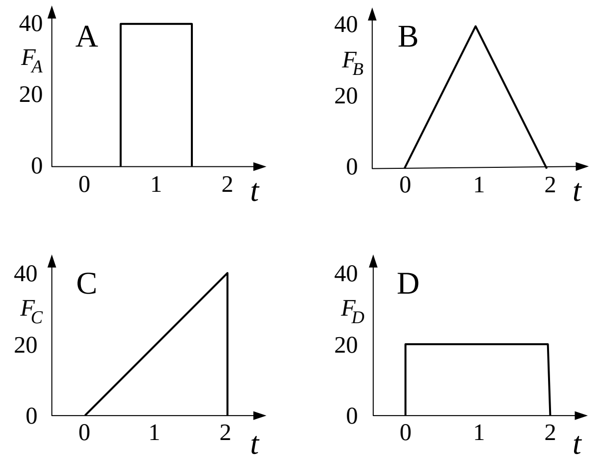

A. remain at rest with the bar rotating around it.

B. move in a straight line at constant velocity.

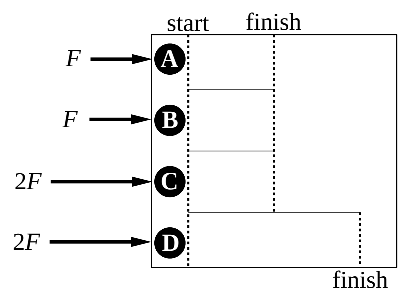
C. wobble due to the unbalanced impulse.

D. none of the above

3. Four different objects with masses: $m_A = 1$ kg, $m_B = 2$ kg, $m_C = 3$ kg, $m_D = 4$ kg, were at rest until forces acted on them. The graphs of force (in kN) versus time (in $\mu$s) for each object are displayed below. ($F_A$ is the force acting on object $A$, etc.) Which object ends up with the most <u>kinetic energy</u>?




4. Four pucks race on a frictionless surface. The pucks have masses: $M_A = 1$ kg, $M_B = 2$ kg, $M_C = 3$ kg, and $M_D = 4$ kg. As shown below, they are subjected to different forces and race lengths. Circle the puck with the largest <u>momentum</u> when it crosses its finish line.



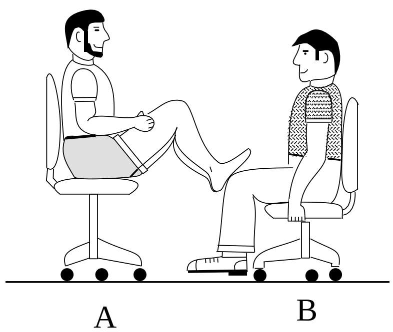
5. In the below figure, student $A$ has a mass of 100 kg and student $B$ has a mass of 75 kg. They sit in identical rolling office chairs facing each other. Student $A$ places his bare feet on the knees of student $B$, as shown. Student $A$ then suddenly pushes outward with his feet; as a result, both chairs to roll frictionlessly away from each other. *During* the push (while the students are still in contact):

   A. student $A$ exerts a force on student $B$, but $B$ is not exerting any force on $A$.

   B. the students exert forces on each other, with the larger force on $B$.

   C. the students exert forces on each other, with the larger force on $A$.

   D. none of the above




6. Let $\vec{p}_A$ and $\vec{v}_A$ denote respectively the momentum and velocity of student $A$ after the push; let $\vec{p}_B$ and $\vec{v}_B$ similarly denote the momentum and velocity of student $B$. Which of the below statements is correct?

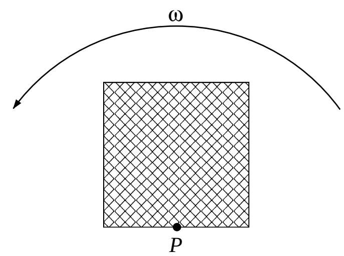
   A. $|\vec{p}_A| > |\vec{p}_B|$ and $|\vec{v}_A| > |\vec{v}_B|$

   B. $|\vec{p}_A| = |\vec{p}_B|$ and $|\vec{v}_A| > |\vec{v}_B|$

   C. $|\vec{p}_A| = |\vec{p}_B|$ and $|\vec{v}_A| < |\vec{v}_B|$

   D. $|\vec{p}_A| < |\vec{p}_B|$ and $|\vec{v}_A| < |\vec{v}_B|$

7. A uniform square is spinning at a constant rate about the pivot point $P$ on a frictionless surface. No forces are acting on square, except perhaps at the pivot point. The force at the pivot point:



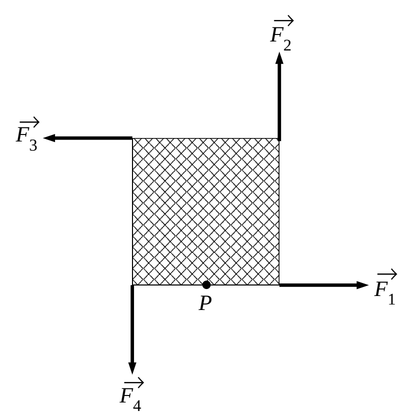

   A. provides a torque to maintain the spin.

   B. points down the page at the instant shown.

   C. points to the left at the instant shown.

   D. is in fact zero.

8. Four forces of equal magnitude but with directions as shown below act at the corners of a square. The square has a fixed pivot point $P$ and is similar to the square in the previous problem, Rank (from least to greatest) the torque produced by these forces about the pivot point. We define a positive torque as one in the counter-clockwise direction. Note: negative numbers are smaller than any positive number. (The torque produced by $\vec{F}_1$ is denoted $\tau_1$, etc.)



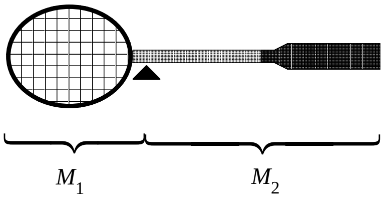

   A. $\tau_1 = \tau_2 = \tau_3 = \tau_4$

   B. $\tau_1 < \tau_2 = \tau_4 < \tau_3$

   C. $\tau_1 < \tau_2 = \tau_3 = \tau_4$

   D. $\tau_4 < \tau_1 < \tau_2 < \tau_3$

9. The below diagram displays a racket balanced on a point. How does the mass on one side of the fulcrum compare to the mass on the other?

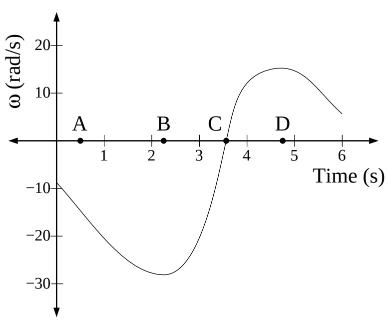



A. $M_1 < M_2$

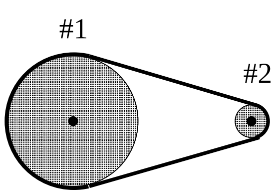
B. $M_1 = M_2$

C. $M_1 > M_2$

10. (Mark two letters!) The below graph displays the angular velocity, $\omega$, of a rotating object as a function of time. Consider a point on the rim of the rotating object. Circle the letter where the magnitude of the point's centripetal acceleration is the largest. X the letter where the magnitude of the point's tangential acceleration is the largest.




11. A belt drives (without slipping) a large radius pulley (#1) from a small radius pulley (#2) as shown below. Please compare the angular velocity of each pulley ($\omega_1$, $\omega_2$) and the speed at the edge of each pulley ($v_1$, $v_2$). Which combination of statements is correct?




A. $\omega_1 < \omega_2$, $v_1 = v_2$

B. $\omega_1 = \omega_2$, $v_1 = v_2$

C. $\omega_1 < \omega_2$, $v_1 > v_2$

D. $\omega_1 = \omega_2$, $v_1 > v_2$

12. Consider the moment of inertia of a bicycle wheel rotated about different rotation axes:

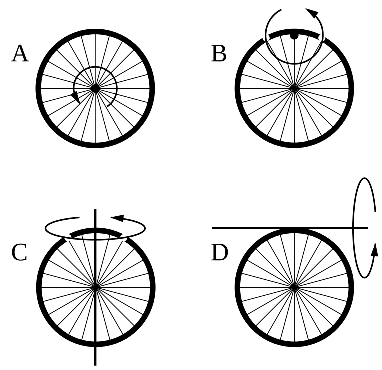
A. its usual axle

B. about a parallel axis on the edge

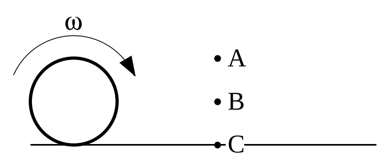
C. a diameter

D. an axis parallel to a diameter, but on an edge.

Which rotation axis has the smallest moment of inertia?




13. A hoop ($I_{\text{hoop}} = MR^2$) is rolling without slipping on a horizontal surface. Which choice of fixed origin results in a total angular momentum of zero?

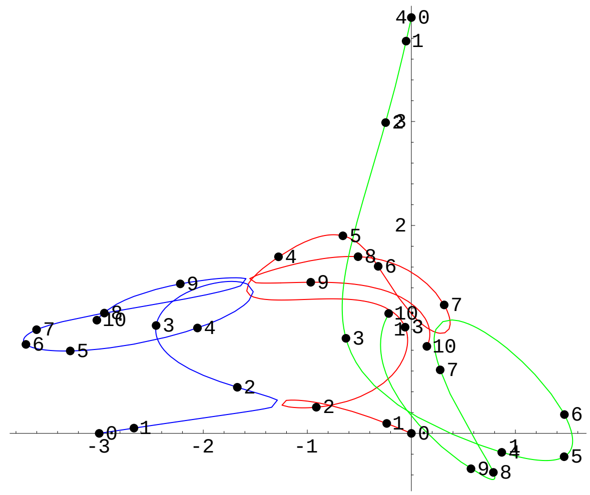



D: no origin produces zero angular momentum

**The following questions are worth 5 pts each**

14. Ernst Meissel (1826–1895) invented a standard problem in celestial mechanics. He considered three stars of mass $m_1 = 3\ M_\odot$, $m_2 = 4\ M_\odot$ and $m_3 = 5\ M_\odot$ ($M_\odot$ is the mass of the Sun) released from rest at the initial locations described below. (Distances are measured in AU, which is the average distance between the Earth and the Sun.) The stars are assumed to be in deep space: the only forces are those due to the gravitational attraction between these three stars. The below picture shows how the stars move during the first 10 years.

| mass ($M_\odot$) | position (AU) |
|---|---|
| $m_1 = 3$ | $\vec{r}_1 = 4\,\hat{\jmath}$ |
| $m_2 = 4$ | $\vec{r}_2 = -3\,\hat{\imath}$ |
| $m_3 = 5$ | $\vec{r}_3 = \vec{0}$ |




A. Find the location of the center of mass at the time of release: $\vec{R}_{\text{cm}}$ (i.e., both $x$ and $y$ components).

B. Find the velocity of the center of mass 20 years after the release: $\vec{V}_{\text{cm}}$ (i.e., both $x$ and $y$ components).

15. Particles 1 and 2 collide in space where no external forces are present. Particle 1, with mass $m_1 = 2$ kg, moves parallel to the $x$ axis and collides with particle 2 (which has mass $m_2 = 3$ kg). The below lists a pre-collision (unprimed) and a post-collision (primed) position (in m) and velocity (in m/s).

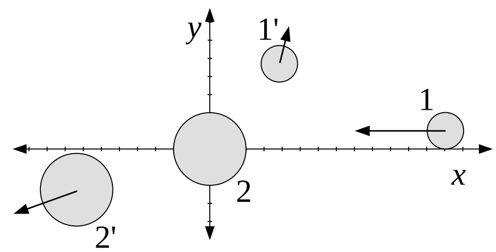
| particle mass | pre-collision | | post-collision | |
|---|---|---|---|---|
| $m_1 = 2$ | $\vec{r}_1 = 13\,\hat{\imath} + \hat{\jmath}$ | $\vec{v}_1 = -5\,\hat{\imath}$ | $\vec{r}\,'_1 = \frac{19}{5}\,\hat{\imath} + \frac{23}{5}\,\hat{\jmath}$ | $\vec{v}\,'_1 = \frac{2}{5}\,\hat{\imath} + \frac{9}{5}\,\hat{\jmath}$ |
| $m_2 = 3$ | $\vec{r}_2 = \vec{0}$ | $\vec{v}_2 = \vec{0}$ | $\vec{r}\,'_2 = -\frac{36}{5}\,\hat{\imath} - \frac{12}{5}\,\hat{\jmath}$ | $\vec{v}\,'_2 = -\frac{18}{5}\,\hat{\imath} - \frac{6}{5}\,\hat{\jmath}$ |

A. Show that the initial momentum in the $x$ direction equals the final momentum in the $x$ direction.

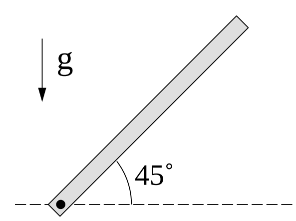
B. Show that the initial momentum in the $y$ direction equals the final momentum in the $y$ direction.

C. Calculate the total kinetic energy in the pre-collision state and in the post-collision state. Is this an elastic collision?

D. Calculate the relative velocity vector: $\vec{v} = \vec{v}_1 - \vec{v}_2$ in the pre-collision and post-collision states. Should the relative *speeds* be equal?




16. The thin uniform rod in the below figure has a mass of 2 kg and a length of 1 m and can pivot about a horizontal, frictionless pin through one end. It is released from an angle of 45° above the horizontal. Use the principle of conservation of energy to determine the angular speed of the rod as it passes through the horizontal position. Note: $I = \frac{1}{3}ML^2$ for a rod rotated about one end.



17. The platter of a turntable is a uniform disk with a mass of 3 kg and a diameter of 30 cm. The platter had an angular speed of 45 rpm the instant it was turned off. A constant frictional torque results in the platter coming to rest 5 seconds later.

   A. What is the magnitude of the frictional torque? (Note: $I_{\text{disk}} = \frac{1}{2}MR^2$.)

   B. How many radians did the platter turn during the first second of the deacceleration?

18. An old stone well consists of a bucket (mass $M$) and a reel with crank (moment of inertia $I$ and radius $R$ ) to pull the bucket up. The bucket is released from the top of the well; as it falls it pulls the rope down and, as a result, the crank assembly runs backward. Find the acceleration of the bucket. Of course, your answer will include two free body diagrams: one of the bucket and one of the reel.