21-49. The resultant electric field is the vector sum of the fields of the individual charges. The magnitude of the electric field due to a single charge is: $E = kq/d^2$. In this problem q_1 is located at $(.15, 0)$, q_2 is at $(-.15, 0)$, and $q_1 = q_2 = 6$ nC.

(a) The origin is symmetrically located. Since it is the same distance to q_1 as to q_2, E_1 and E_2 have the same magnitude, but $\vec{\mathbf{E}}_1$ points to the left and $\vec{\mathbf{E}}_2$ points to the right. The net electric field (the vector sum of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$) is zero.

(b) The point (.3,0) and the charges q_1 and q_2, all lie on the x-axis. The distance to q_1 is $d_1 = .15$ m; the distance to q_2 is $d_2 = .45$ m. $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ both point in the same direction (to the right), so vector addition is just normal addition and the net electric field is:

$$\vec{\mathbf{E}}_{\text{net}} = (E_1{+}E_2)\,\hat{\mathbf{i}} = \left(\frac{kq_1}{d_1^2} + \frac{kq_2}{d_2^2}\right)\hat{\mathbf{i}} = 8.99{\times}10^9{\cdot}6{\times}10^{-9}\left(\frac{1}{.15^2} + \frac{1}{.45^2}\right)\hat{\mathbf{i}} = 2260\,\hat{\mathbf{i}}\text{ N/C}$$

(c) The point $(.15, -.40)$ lies a distance $d_1 = .4$ m directly below q_1 and a distance $d_2 = \sqrt{.3^2 + .4^2} = .5$ m from q_2. We can find the magnitude of the corresponding electric fields from Coulomb's Law:

$$\begin{aligned} E_1 &= \frac{kq_1}{d_1^2} = \frac{8.99 \times 10^9 \cdot 6 \times 10^{-9}}{.4^2} = 337 \text{ N/C} \\ E_2 &= \frac{kq_2}{d_2^2} = \frac{8.99 \times 10^9 \cdot 6 \times 10^{-9}}{.5^2} = 216 \text{ N/C} \end{aligned}$$

Clearly $\vec{\mathbf{E}}_1 = (0, -337)$ N/C. The direction for $\vec{\mathbf{E}}_2$ is the same as the vector pointing outward from q_2 towards the point of interest $(.15, -.4)$, which is itself the hypotenuse of a 3-4-5 right triangle. Notice immediately that we expect the x component of $\vec{\mathbf{E}}_2$ to be positive and the y component to be negative. The standard angle is negative: $-\arctan(.4/.3) = -53.1°$. Using a vector-capable calculator we immediately have:

$$\vec{\mathbf{E}}_2 = (130, -173)\text{ N/C}$$

Thus:

$$\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = (130, -510)\text{ N/C}$$

or: $526\text{ N/C} \angle -75.8°$

(d) The point $(0, .2)$ is again symmetrically located with respect to q_1 and q_2. The distance, d, to either q_1 or q_2 is: $\sqrt{.15^2 + .2^2} = .25$ m (this is a non-obvious 3-4-5 triangle). The electric field of q_1 points up and to the left; the electric field of q_2 points symmetrically up and to the right... the x components of these vectors are equal but opposite, so the net electric field is just in the y direction. The electric field magnitudes E_1 and E_2 are equal: $kq/d^2 = 863$ N/C. The net electric field is just $2E_1 \sin\theta\,\hat{\mathbf{j}}$, where $\theta = \arctan(.2/.15) = 53.1°$. Using this value of θ or the equivalent parts of the 3-4-5 triangle, we find $\sin\theta = .2/.25 = .8$, with result:

$$\vec{\mathbf{E}}_{\text{net}} = (0, 1380)\text{ N/C}$$

21-51. The resultant electric field is the vector sum of the fields of the individual charges. The magnitude of the electric field due to a single charge is: $E = kq/d^2$. In this problem q_1 "$(+q)$" is located at $(.15, 0)$, q_2 "$(-q)$" is at $(-.15, 0)$, and $q_1 = -q_2 = 6$ nC.

(a) The origin is symmetrically located. Since it is the same distance to q_1 as to q_2, E_1 and E_2 have the same magnitude, and both $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ point to the left. The net electric field (the vector sum of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$) is just $-2E_1\,\hat{\mathbf{i}}$. The result is:

$$\vec{\mathbf{E}}_{\text{net}} = -2E_1\,\hat{\mathbf{i}} = -\frac{2 \cdot 8.99 \times 10^9 \cdot 6 \times 10^{-9}}{.15^2}\,\hat{\mathbf{i}} = -4800\,\hat{\mathbf{i}}\text{ N/C}$$

(b) The point (.3,0) and the charges q_1 and q_2, all lie on the x-axis. The distance to q_1 is $d_1 = .15$ m; the distance to q_2 is $d_2 = .45$ m. $\vec{\mathbf{E}}_1$ points right and $\vec{\mathbf{E}}_2$ points left, so vector addition is just normal subtraction and the net electric field is:

$$\vec{\mathbf{E}}_{\text{net}} = (E_1 - E_2)\,\hat{\mathbf{i}} = \left(\frac{kq_1}{d_1^2} - \frac{k|q_2|}{d_2^2}\right)\hat{\mathbf{i}} = 8.99 \times 10^9 \cdot 6 \times 10^{-9}\left(\frac{1}{.15^2} - \frac{1}{.45^2}\right)\hat{\mathbf{i}} = 2130\,\hat{\mathbf{i}}\text{ N/C}$$

(c) The point $(.15, -.40)$ lies a distance $d_1 = .4$ m directly below q_1 and a distance $d_2 = \sqrt{.3^2 + .4^2} = .5$ m from q_2. We can find the magnitude of the corresponding electric fields from Coulomb's Law:

$$\begin{aligned} E_1 &= \frac{kq_1}{d_1^2} = \frac{8.99 \times 10^9 \cdot 6 \times 10^{-9}}{.4^2} = 337\text{ N/C} \\ E_2 &= \frac{k|q_2|}{d_2^2} = \frac{8.99 \times 10^9 \cdot 6 \times 10^{-9}}{.5^2} = 216\text{ N/C} \end{aligned}$$

Clearly $\vec{\mathbf{E}}_1 = (0, -337)$ N/C. The direction for $\vec{\mathbf{E}}_2$ is the same as the vector pointing inward towards q_2 from the point of interest $(.15, -.4)$, which is itself the hypotenuse of a 3-4-5 right triangle. Notice immediately that we expect the x component of $\vec{\mathbf{E}}_2$ to be negative and the y component to be positive. The standard angle is: $180° - \arctan(.4/.3) = 127°$. Using a vector-capable calculator we immediately have:

$$\vec{\mathbf{E}}_2 = (-130, +173)\text{ N/C}$$

Thus:

$$\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = (-130, -165)\text{ N/C}$$

or: $209\text{ N/C} \angle -128°$

(d) The point $(0, .2)$ is again symmetrically located with respect to q_1 and q_2. The distance, d, to either q_1 or q_2 is: $\sqrt{.15^2 + .2^2} = .25$ m (this is a non-obvious 3-4-5 triangle). The electric field of q_1 points up and to the left; the electric field of q_2 points symmetrically down and to the left…the y components of these vectors are equal but opposite, so the net electric field is just in the $-x$ direction. The electric field magnitudes E_1 and E_2 are equal: $kq/d^2 = 863$ N/C. The net electric field is just $-2E_1 \cos\theta\,\hat{\mathbf{i}}$, where $\theta = \arctan(.2/.15) = 53.1°$. Using this value of θ or the equivalent parts of the 3-4-5 triangle, we find $\cos\theta = .15/.25 = .6$, with result:

$$\vec{\mathbf{E}}_{\text{net}} = (-1040, 0)\text{ N/C}$$

old exam: 11A Note that the distance to q_1 or q_2 is $d = .3$ m. The (fully vertical) distance to q_3 is $d_3 = .6 \sin 60° = .520$ m. The electric field due to q_1 is in the positive x direction; the electric field due to q_2 is in the negative x direction; the electric field due to q_3 is in the negative y direction. If E_1 is the magnitude of the electric field due to q_1 (and respectively), then the net electric field is:

$$\vec{\mathbf{E}}_{\text{net}} = (E_1 - E_2, -E_3)$$

(This result is not exactly obvious: make sure you understand how adding electric field vectors produced this simple result.) One can quickly calculate:

$$\begin{aligned} E_1 &= 300 \text{ N/C} \\ E_2 &= 500 \text{ N/C} \\ E_3 &= 233 \text{ N/C} \end{aligned}$$

So the result is:

$$\vec{\mathbf{E}}_{\text{net}} = (-200, -233) \text{ N/C}$$

which can also be written: $307 \text{ N/C} \angle -131°$