

21-70. The plates produce a uniform electric field in the space between them. This field exerts torque on a dipole and gives it potential energy. The electric field between the plates is given by $E = \sigma/\epsilon_0 = 125 \times 10^{-6}/8.8542 \times 10^{-12} = 1.41 \times 10^7$ N/C. The dipole moment is $p = ed = 1.6022 \times 10^{-19} \cdot 220 \times 10^{-9} = 3.52 \times 10^{-26}$ C · m. The potential energy of the dipole due to the field is $U = -\vec{\mathbf{E}} \cdot \vec{\mathbf{p}} = -E p \cos \phi$, and the torque the field exerts on it is $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$, so $|\vec{\tau}| = \tau = E p \sin \phi$. Clearly U is maximized by $\phi = 180^\circ$ ($\vec{\mathbf{E}}$ and $\vec{\mathbf{p}}$ in opposite directions) and τ is maximized by $\phi = 90^\circ$ ($\vec{\mathbf{E}}$ and $\vec{\mathbf{p}}$ perpendicular). In either case that maximum is $E p = 4.98 \times 10^{-19}$ with unit J commonly used for energy and N · m for torque. The net force on a dipole in a uniform electric field is zero.

21-95. Recall the results of old exam #14: a distance d from (and along the extended line) of a line charge (linear charge density: λ , length: a), the electric field has magnitude:

$$E = k\lambda \left(\frac{1}{d} - \frac{1}{a+d} \right)$$

Recall the results of problem 21-90: a distance x perpendicularly from the end of such a line charge the electric field is given by:

$$\vec{\mathbf{E}} = \left(\frac{k\lambda a}{x(x^2 + a^2)^{1/2}}, -k\lambda \left[\frac{1}{x} - \frac{1}{(x^2 + a^2)^{1/2}} \right] \right)$$

The above result was derived assuming the line-of-charge was on the y -axis. If we rotate the situation so the line-of-charge lies on the x -axis and seek the electric field at a point on the y -axis, we find:

$$\vec{\mathbf{E}} = \left(-k\lambda \left[\frac{1}{y} - \frac{1}{(y^2 + a^2)^{1/2}} \right], \frac{k\lambda a}{y(y^2 + a^2)^{1/2}} \right)$$

- (a) If we had a negatively charged line-of-charge on the x -axis between $(0, -a)$ and $(0, 0)$, we will have an attractive electric field symmetrically related to the field described above. If we have both line-of-charges present, the y components for the electric fields will cancel, whereas the x components will add, with result:

$$\vec{\mathbf{E}} = \left(-2k\lambda \left[\frac{1}{y} - \frac{1}{(y^2 + a^2)^{1/2}} \right], 0 \right)$$

Following our work on 21-90, see that for $y \gg a$, this becomes:

$$\vec{\mathbf{E}} = \left(-\frac{2kQ}{ay} \left[\frac{1}{2}(a/y)^2 \right], 0 \right) = \left(-\frac{kQa}{y^3}, 0 \right)$$

Since the force is given by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ it is clear that the force is in the $-\hat{\mathbf{i}}$ direction with magnitude qE .

- (b) At the location $(x, 0)$ we are a distance $d_+ = x - a$ from the end of the positive line-of-charge, and a distance $d_- = x$ from the end of the negative line-of-charge. The positive line-of-charge produces an electric field in the $+\hat{\mathbf{i}}$ direction; the negative line-of-charge produces a (smaller) electric field in the $-\hat{\mathbf{i}}$ direction. Adding these

two fields we find that the net electric field has magnitude:

$$\begin{aligned}
 E &= k\lambda \left(\frac{1}{x-a} - \frac{1}{x} \right) - k\lambda \left(\frac{1}{x} - \frac{1}{x+a} \right) \\
 &= k\lambda \left\{ \frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a} \right\} \\
 &= k\lambda \left\{ \frac{2x}{(x-a)(x+a)} - \frac{2}{x} \right\} \\
 &= \frac{2k\lambda}{x} \left\{ \frac{1}{1-(a/x)^2} - 1 \right\} = \frac{2k\lambda}{x} \left\{ (1-(a/x)^2)^{-1} - 1 \right\} \\
 &\rightarrow \frac{2k\lambda}{x} \left\{ (a/x)^2 \right\} = \frac{2kQa}{x^3} \text{ for: } x \gg a
 \end{aligned}$$

Since the force is given by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ it is clear that the force is in the $+\hat{\mathbf{i}}$ direction with magnitude qE .

21-104. There are a couple of easy approaches to this problem. Going back to the original derivation of the disk's electric field (p. 732) all you need to do is switch the range of integration from \int_0^R to $\int_{R_1}^{R_2}$. However the approach I'll take here is to view the annulus as the superposition of a uniformly charged disk with surface charge density $+\sigma$ and radius R_2 with a uniformly charged disk with surface charge density $-\sigma$ and radius R_1 . Combining the resulting electric fields gives the net electric field:

$$\begin{aligned}
 \vec{\mathbf{E}} &= \hat{\mathbf{i}} \frac{\sigma x}{2\epsilon_0} \left\{ \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R_2^2}} \right] - \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R_1^2}} \right] \right\} \\
 &= \hat{\mathbf{i}} \frac{\sigma x}{2\epsilon_0} \left\{ \frac{1}{\sqrt{x^2 + R_1^2}} - \frac{1}{\sqrt{x^2 + R_2^2}} \right\} \\
 &\rightarrow \hat{\mathbf{i}} \frac{\sigma x}{2\epsilon_0} \left\{ \frac{1}{R_1} - \frac{1}{R_2} \right\} \text{ for: } x \ll R_1
 \end{aligned}$$

Note that by this result if $x > 0$ $\vec{\mathbf{E}}$ is in the $+\hat{\mathbf{i}}$ direction, whereas $x < 0$ produces $\vec{\mathbf{E}}$ in the $-\hat{\mathbf{i}}$ direction.

The total charge on the annulus is: $\sigma(\pi R_2^2 - \pi R_1^2)$. FYI: part (d) of this problem aims to remind you that if Hook's law ($F = -kx$) applies, the resulting motion is harmonic with frequency:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$