22-7. The easy way to do this problem is to immediately use Gauss' law since $\Phi_E = Q_{\rm encl}/\epsilon_0 = \lambda\ell/\epsilon_0$, however I'm sure the intent is to use the definition of flux: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ where:

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{\mathbf{r}}$$

Note that for the disks that make up the end caps of the cylinder, $\vec{\mathbf{E}} \perp d\vec{\mathbf{A}}$ (i.e., $\vec{\mathbf{E}}$ points parallel to the surface of the disk, whereas $d\vec{\mathbf{A}}$ points along the axis of the cylinder), so $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$. For the curved part of the cylinder, both $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ point in the same direction (radially outward) so $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA$, where the area of the curved part of the cylinder $A = 2\pi r\ell$. Thus:

$$\Phi_E = \frac{\lambda}{2\pi\epsilon_0 r}\,2\pi r\ell = \frac{\lambda\ell}{\epsilon_0}$$

Note particularly that the final answer does not depend on r and echos Gauss' law.

(a)

$$\Phi_E = \lambda\ell/\epsilon_0 = \frac{6\times 10^{-6}\cdot .4}{8.8542\times 10^{-12}} = 2.71\times 10^5\ \mathrm{N\cdot m^2/C}$$

(b) As noted above Φ_E does not depend on r: so the answer is same as (a).

(c)

$$\Phi_E = \lambda\ell/\epsilon_0 = \frac{6\times 10^{-6}\cdot .8}{8.8542\times 10^{-12}} = 5.42\times 10^5\ \mathrm{N\cdot m^2/C}$$

22-33. The problem poses an electric field of the form: $\vec{\mathbf{E}} = f(x)\hat{\mathbf{i}}$, where $f(0) = 125$ N/C, and a 1 m × 2 m × 3 m box with with faces aligned with the xyz axes and one corner at the origin. Under these circumstances the flux through every face is zero, except face I and the face opposite to I. The area of these non-zero faces is $A = 6\ \mathrm{m}^2$, so the flux through I is $Af(0) = 6\cdot 125 = 750\ \mathrm{N\cdot m^2/C}$ and the flux through it's opposite is $-Af(-1)$ (since $\hat{\mathbf{n}}$ is in the $-\hat{\mathbf{i}}$ direction). The total electric flux is:

$$\Phi_E = A(f(0) - f(-1)) = Q_{\rm encl}/\epsilon_0$$

Rearranging this a bit we have:

$$f(0) - Q_{\rm encl}/A\epsilon_0 = f(-1)$$

or

$$f(-1) = 125 - \left(-24\times 10^{-9}/(6\cdot 8.8542\times 10^{-12})\right) = 577\ \mathrm{N/C}$$

The electric field lines leaving the object (for example, those for $x > 0$) must end on charge, which clearly must lie outside of the box since $\vec{\mathbf{E}} = f(x)\hat{\mathbf{i}}$ removes the possibility of the lines circling around and connecting up with charges inside the box. (The assumption that every electric field line starts and ends on charge—rather than just extending to infinity—is equivalent to the assumption that the universe has zero net charge. Under that assumption, of course if there is charge inside the box there must be charge outside the box.) Better answer: If $\vec{\mathbf{E}} = f(x)\hat{\mathbf{i}}$, Gauss's Law says the charge distribution must also just be a function of x, so charge at say $x = -.5$ inside the box, must also be echoed outside the box at that same x value.

old exam #12:

A. Under the cited conditions the cube in (i) lies totally within the sphere whereas the cube in (ii) totally surrounds the sphere. In either case we can determine the total electric flux Φ_E by determining the total enclosed charge and using Gauss' law. Note that a direct calculation of flux, while possible, is much more difficult because the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ is not constant and $|\vec{\mathbf{E}}|$ is not constant.

i.

$$\Phi_E = Q_{\text{encl}}/\epsilon_0 = \rho V/\epsilon_0 = 1 \times 10^{-6} \cdot (.01)^3/8.8542 \times 10^{-12} = 0.113 \text{ N} \cdot \text{m}^2/\text{C}$$

ii.

$$\Phi_E = Q_{\text{encl}}/\epsilon_0 = \rho V/\epsilon_0 = 1 \times 10^{-6} \cdot \frac{4}{3}\pi .04^3/8.8542 \times 10^{-12} = 30.3 \text{ N} \cdot \text{m}^2/\text{C}$$

B. Under the cited conditions the cube in (i) encloses a square (side 1 cm) of charge whereas the cube in (ii) totally surrounds the disk. In either case we can determine the total electric flux Φ_E by determining the total enclosed charge and using Gauss' law. In this case we do not have a formula for $\vec{\mathbf{E}}$ off axis, so a direct calculation of flux would be overwhelmingly complex.

i.

$$\Phi_E = Q_{\text{encl}}/\epsilon_0 = \sigma A/\epsilon_0 = 1 \times 10^{-6} \cdot (.01)^2/8.8542 \times 10^{-12} = 11.3 \text{ N} \cdot \text{m}^2/\text{C}$$

ii.

$$\Phi_E = Q_{\text{encl}}/\epsilon_0 = \sigma A/\epsilon_0 = 1 \times 10^{-6} \cdot \pi .04^2/8.8542 \times 10^{-12} = 568 \text{ N} \cdot \text{m}^2/\text{C}$$