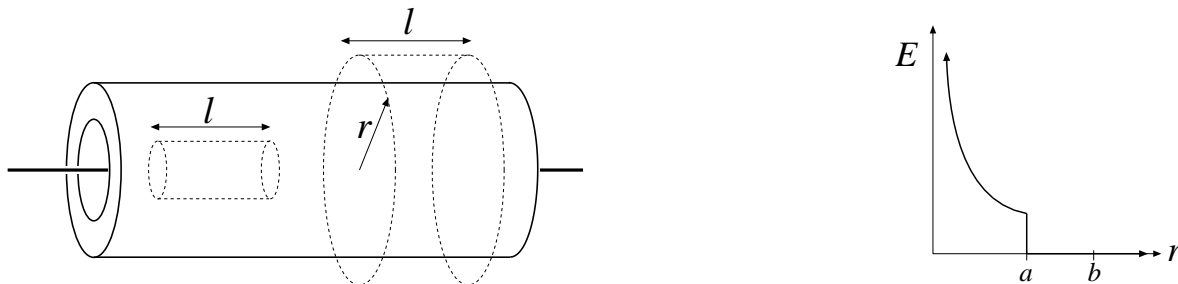


22-39. Apply Gauss's law using coaxial, cylindrical Gaussian surfaces of length ℓ and radius r . By symmetry the electric field must be radially outward and a function of just r . As a result the end caps of the Gaussian surface have zero flux ($\vec{\mathbf{E}} \perp \hat{\mathbf{n}}$) and the flux through the curved surface is easy to evaluate: $EA = E2\pi r\ell$. SO:

$$\Phi_E = E2\pi r\ell = Q_{\text{encl}}/\epsilon_0 \implies E = \frac{Q_{\text{encl}}}{\ell 2\pi\epsilon_0 r}$$

The enclosed charge can be found from the net charge per length. Do note that the electric field is zero inside any conductor. Below we show two such Gaussian surfaces: the one on the left has $r < a$, the one on the right has $r > b$.



- (a) i. For $r < a$ the enclosed charge is just that of the axial line-of-charge: $Q_{\text{encl}} = \alpha\ell$, with result:

$$E = \frac{\alpha}{2\pi\epsilon_0 r}$$

Note that the surrounding charged conductor has no net effect on the electric field inside of it!

- ii. $a < r < b$ we are inside a conductor so $E = 0$.
 iii. For $b < r$ the net enclosed charge is zero because the charge on the conductor is equal but opposite to the charge on the central line-of-charge. We conclude: $E = 0$.
 (b) We can calculate the charge on the conductor's surfaces two different ways: via Eq. 22-10: $\sigma = \epsilon_0 E_{\perp}$ or by using the above calculated Q_{encl} and subtracting the charge on the central line-of-charge.

- i. For a Gaussian surface just inside the conductor (i.e., $r = a^+$), $E = 0$, so the enclosed charge is zero. The charge per length on the inner surface of the conductor must exactly balance the charge per length due to the central line-of-charge. Hence the charge per length on the inner surface must be $-\alpha$. Alternatively, the electric field just adjacent to the inner surface (i.e., at $r = a^-$) is:

$$\vec{\mathbf{E}} = \frac{\alpha}{2\pi\epsilon_0 a} \hat{\mathbf{r}}$$

Since $\vec{\mathbf{E}}$ is in the opposite direction as $\hat{\mathbf{n}}$ for the surface, we find:

$$\sigma = -\frac{\alpha}{2\pi a}$$

In a length ℓ of this surface, there will be a total charge of $\sigma A = \sigma 2\pi a\ell = -\alpha\ell$, so we see that the charge per length must be $-\alpha$.

- ii. The total charge per length on the conductor was to be $-\alpha$, and we have exhausted that on the inner surface so there must be zero surface charge on the outer surface of the conductor. That agrees with the fact that $E = 0$ for $r = b^+$.

22-40. Following the formulation of the previous problem, we have:

$$E = \frac{Q_{\text{encl}}}{\ell 2\pi\epsilon_0 r}$$

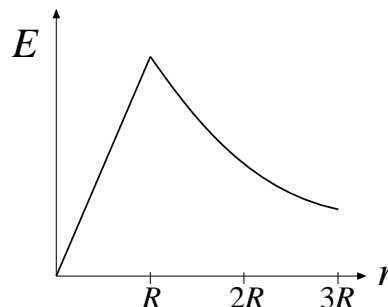
- (a) If we consider a cylindrical Gaussian surface that fully lies within the charge: $Q_{\text{encl}} = \rho V = \rho\pi r^2\ell$ which results in:

$$E = \frac{\rho r}{2\epsilon_0}$$

- (b) If we consider a cylindrical Gaussian surface that fully lies outside the charge: $Q_{\text{encl}} = \rho V = \rho\pi R^2\ell$ which results in:

$$E = \frac{\rho R^2}{2r\epsilon_0}$$

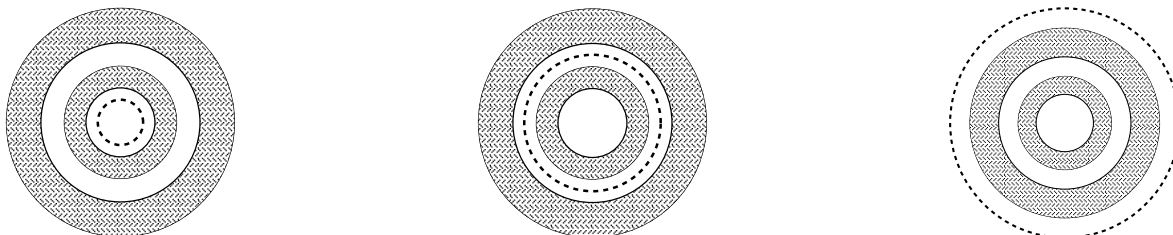
- (c) The answers (a) and (b) are consistent if $r = R$, namely $E = \rho R/2\epsilon_0$



22-45. Apply Gauss's law using spherical Gaussian surfaces of radius r . By symmetry the electric field must be radially outward and a function of just r . The flux through the Gaussian surface is easy to evaluate: $EA = E4\pi r^2$. SO:

$$\Phi_E = E4\pi r^2 = Q_{\text{encl}}/\epsilon_0 \implies E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

Do note that the electric field is zero inside any conductor. Below we show three such Gaussian surfaces: $r < a$, $b < r < c$. and $d < r$ (respectively).



- (a)
- i. For $r < a$ the enclosed charge is zero so $E = 0$.
 - ii. $a < r < b$ we are inside a conductor so $E = 0$.
 - iii. For $b < r < c$ the net enclosed charge is $2q$. so:

$$E = \frac{2q}{4\pi\epsilon_0 r^2}$$

- iv. $c < r < d$ we are inside a conductor so $E = 0$.

- v. For $d < r$ the net enclosed charge is $6q$. so:

$$E = \frac{6q}{4\pi\epsilon_0 r^2}$$



(b) We can calculate the charge on the conductor's surfaces two different ways: via Eq. 22-10: $\sigma = \epsilon_0 E_{\perp}$ or by using the Q_{encl} and subtracting any known enclosed charge.

i. For a Gaussian surface just inside the conductor (i.e., $r = a^+$), $E = 0$, so the enclosed charge is zero. Alternatively, the electric field just adjacent to the inner surface (i.e., at $r = a^-$) is zero so σ is zero.

ii. Since there is no charge on the inner surface, there must be $2q$ on the outer surface. Alternatively, the electric field just adjacent to the outer surface (i.e., at $r = b^+$) is:

$$E = \frac{2q}{4\pi\epsilon_0 b^2} \implies \sigma = \frac{2q}{4\pi b^2}$$

so the total charge on this surface is $\sigma A = \sigma 4\pi b^2 = 2q$.

iii. For a Gaussian surface just inside the conductor (i.e., $r = c^+$), $E = 0$, so the enclosed charge is zero, but $2q$ of charge is known on the inner conductor, so the total charge on the $r = c$ surface must be $-2q$. Alternatively, the electric field just adjacent to the inner surface

$$E = \frac{2q}{4\pi\epsilon_0 c^2} \implies \sigma = -\frac{2q}{4\pi c^2}$$

(the minus sign results from the fact that $\vec{\mathbf{E}}$ is in the opposite direction as $\hat{\mathbf{n}}$ for the surface). The total charge on this surface is $\sigma A = \sigma 4\pi c^2 = -2q$.

iv. Since the total charge on this conductor must be $4q$, there must be $6q$ on the $r = d$ surface as there is $-2q$ on the inner surface.