Select the letter of the single best answer. Each answer is worth 1 point.

Physical Constants:
proton charge $= e = 1.602 \times 10^{-19}$ C
proton mass $= m_p = 1.673 \times 10^{-27}$ kg
electron mass $= m_e = 9.109 \times 10^{-31}$ kg
permittivity $= \epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$
Coulomb constant $= k = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$

1. How many of the below numbers display exactly 3 significant digits?

   - 0.03
   - 0.030
   - 0.0300
   - 0.72
   - 0.601×10^{24}
   - 1230
   - 0.007
   - 0.720
   - 70.0
   - 3.912×10^{14}

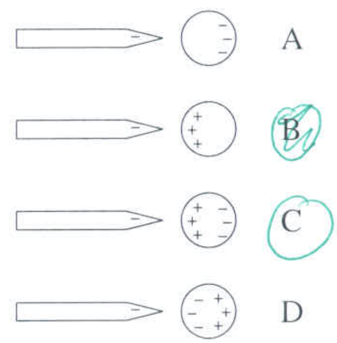
   A. five
   B. six
   C. seven
   D. none of the above

2. Consider the below list of physical quantities with associated units. How many of these statements correctly reports the units of the named physical quantity?

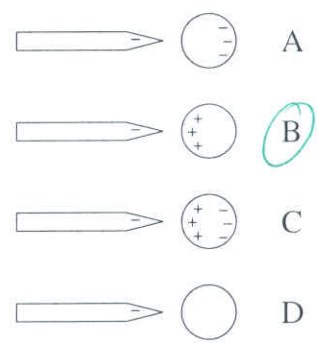
   - electric potential (V): J/C
   - electric field (E): N/C
   - electric field (E): V/m
   - surface charge density (σ): C/m^2
   - surface charge density (σ): C/m^3

   A. one
   B. two
   C. three
   D. four

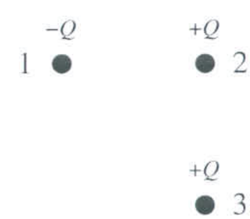
3. A negatively charged rod approaches a previously neutral conducting sphere. Which picture best displays the arrangement of the surface charges on the sphere?



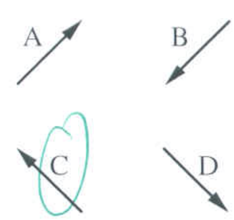
4. The sphere described the previous problem is now grounded. Which picture now best displays the arrangement of the surface charges on the sphere?



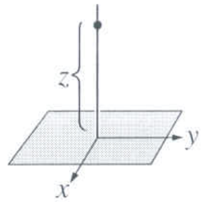
5. Three charges all carry the same magnitude of charge Q, but with different signs as shown below.



Which of the arrows is in the direction of the net force on charge 2?

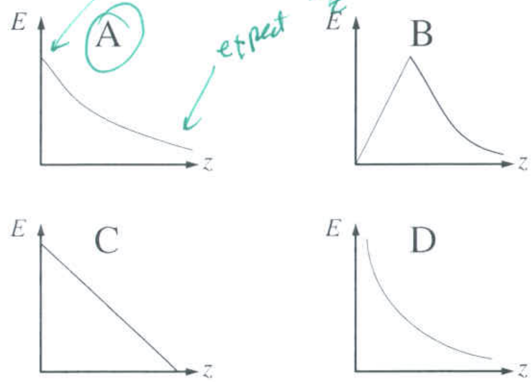


6. A card-of-charge sits in the xy plane with center at the origin. The card-of-charge has uniform surface charge density (σ) and is a square with side s. We seek the magnitude of the resulting electric field (E) a distance z directly above the center of the card-of-charge. Which of the below graphs of E vs. z properly displays this relationship.

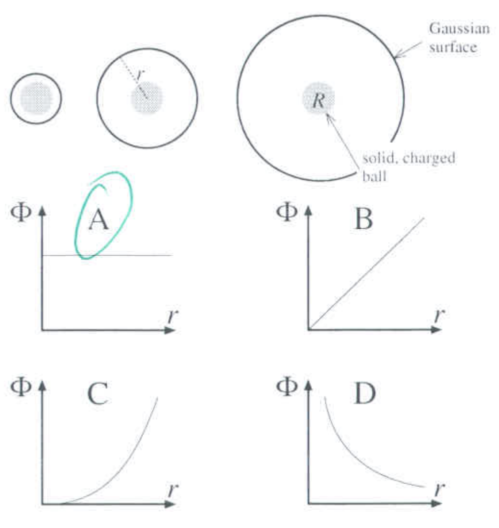


expect $\frac{\sigma}{2\varepsilon_0}$

expect $\frac{kQ}{z^2}$



7. The below figure shows three identical, uniformly-charged solid spheres (radius R, volume charge density ρ). Consider the electric flux Φ through the surface of Gaussian spheres of radius r. (Note that in all cases: $r > R$, i.e., the Gaussian sphere lies fully outside the charged ball.) Which of the the below graphs best displays how Φ depends on the radius r of the Gaussian sphere?

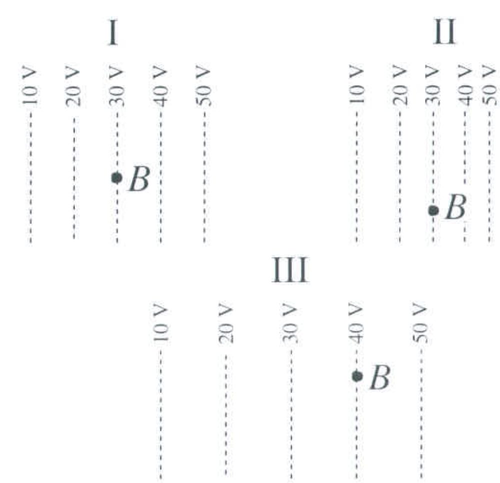


8. The electric field $\vec{\mathbf{E}}$ just outside of a positively charged conductor may be best characterized by which combination of the following conditions:

   I. tangent to the surface X normal!
   II. zero magnitude X not outside
   III. largest on the parts of the conductor at the highest voltage X ↳ equipotential
   IV. largest on the parts of the conductor with the smallest radius of curvature ✓
   V. largest on the parts of the conductor with the largest surface charge density ✓

   A. I, III, IV
   B. IV, V
   C. II
   D. II, IV, V

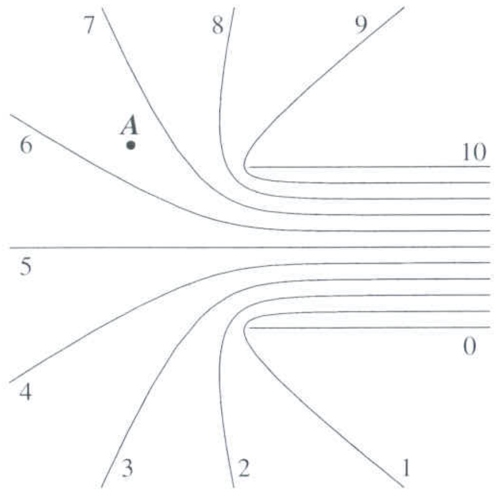
9. The following figures faithfully show (using the same scale) the location of equipotential lines (displayed as the dotted lines with corresponding voltages). How does the magnitude of the electric field at B compare in the three cases? (E_I denotes the magnitude of electric field at B in figure I, etc.)



   A. $E_{III} > E_{II} > E_I$
   B. $E_I > E_{III} > E_{II}$
   C. $E_I = E_{II} > E_{III}$
   D. $E_{II} > E_I > E_{III}$

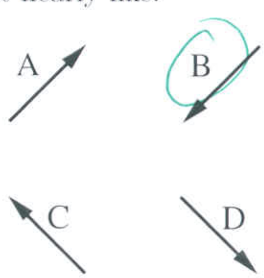
small $\Delta x \Rightarrow$ big E

10. The below diagram shows the equipotentials that result from parallel line conductors. (This is very similar to the equipotentials you studied in the last lab, although here we have a different configuration of conductors.) The bottom conductor is at 0 V; the top is at 10 V. The voltage on each equipotential is labeled.



$\vec{p} \parallel \vec{E}$

A water molecule (which has an electric dipole moment $\vec{\mathbf{p}}$) is placed at location A. Because of the electric force, the molecule orientates itself in a situation of minimum potential energy and zero torque. The electric dipole vector ends up oriented most nearly like:



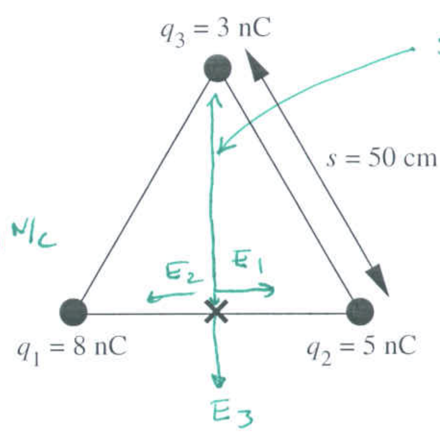
The following problems are worth 10 points each

11. As shown below three charges are arranged in an equilateral triangle with side 50 cm.

   A. Find the electric field vector at the spot marked X (i.e., the midpoint of the horizontal segment).

   B. Find the voltage at the spot marked X. (Assume as usual: $V(\infty) = 0$.)

$r_3 = 50\cos 30^\circ =$



$$E_1 = \frac{kq_1}{(.25)^2} = 1150 \text{ N/C}$$

$$E_2 = \frac{kq_2}{(.25)^2} = 719 \text{ N/C}$$

$$E_x = E_1 - E_2 = 431 \text{ N/C}$$

$$E_3 = kq_3/r_3^2 = 144 \text{ N/C}$$

$$\vec{E} = (431, -144)$$

$$V_1 = \frac{kq_1}{(.25)} = 287.6 \text{ V}$$

$$V_2 = \frac{kq_2}{(.25)} = 179.8$$

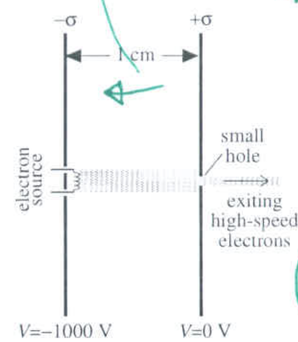
$$V_3 = \frac{kq_3}{r_3} = 623$$

530 V

$E = \frac{\Delta V}{\Delta x} = \frac{1000\,V}{cm}$

$= 10^5\,V/m$

12. An electron 'gun' consists of two oppositely charged parallel plates separated by 1 cm. An electron source (a hot wire) is at the voltage of the negatively charged plate (i.e., $V = -1000$ V). The ejected electrons initially have zero velocity; they are accelerated as they approach the positively charged plate. The positively charge plate (which is grounded, i.e., $V = 0$ V) has a small hole through which some of the fast moving electrons can exit.



A. How fast are the electrons moving when they exit the hole?

B. What is the direction and magnitude of the electric field between the plates. (Seeking a number not a formula like σ/ϵ_0.)

$\frac{eE}{m}$

E is constant inside plates ⇒ a constant so can use $v^2 - v_0^2 = 2a(x-x$

instead I used conservation of energy $-\Delta U = \Delta K = K_f - K_i = K_f$

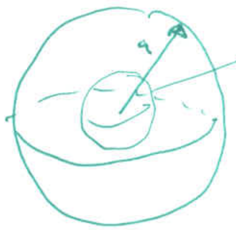
$U = qV$ $\qquad K_i = 0$

so $K_f = -(-e)\left[0 - -1000\right] = 1000 \cdot e = \frac{1}{2}mv^2$

$e = 1.6\times10^{-19}\,C$ $\qquad q$

$v = \sqrt{\frac{2 \cdot 1000 \cdot e}{m_e}} = \sqrt{\frac{2 \cdot 1000 \cdot 1.6\times10^{-19}}{9.11\times10^{-31}}} = 1.88\times10^7\ m/s$

13. Consider a solid ball (radius a) of constant charge density $\rho > 0$ material. Using Gauss' Law derive the formula for electric field inside the ball (i.e., for $r < a$). Don't forget to report the $\underbrace{\text{direction of } \vec{\mathbf{E}}}_{\text{outward}}$!



Gaussian sphere radius r inside ball

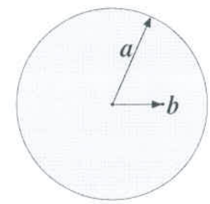
$$\Phi_E = E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0}$$

14. Consider the problem of finding the electric potential inside a sphere (radius: a) with uniform charge density ρ. The plan is to break the sphere up into concentric shells (radius r, thickness dr) and integrate to find the potential (at a point a distance b from the sphere's center) due to these shells. Note that if a shell has a radius less then b (i.e., b is outside the shell) the potential of the shell at b would be: $k\ dQ/b$, whereas inside such a shell the potential is the constant $k\ dQ/r$.

A. Explain why the following integrals give the electric potential at b:

$$V(b) = \int_0^b \frac{k\rho\, 4\pi r^2\, dr}{b} + \int_b^a \frac{k\rho\, 4\pi r^2\, dr}{r}$$



Your explanation of this result should include answers to the following questions:

i. What does $4\pi r^2$ represent? → surface area of spherical shell

ii. What is dQ? $= \rho\, 4\pi r^2\, dr = \rho\, dV$ where dV = volume of shell

iii. Why does the first integral have a b in the denominator whereas the second integral has an r in the denominator?

iv. Why does the first integral have a range of integration $r \in (0, b)$ and the second have a range of integration $r \in (b, a)$?

B. Evaluate the integrals to find a formula for $V(b)$.

C. Find the electric field by taking the derivative of V:

$$E = -\frac{dV}{db}$$

in the first integral we are considering shells with $r < b$
if b is outside of shell $dV = \frac{k\,dQ}{b}$
in the second integral we are considering shells with $r > b$
if b is inside shell $dV = \frac{k\,dQ}{r}$

$$V(b) = \frac{k\rho 4\pi}{b}\int_0^b r^2 dr + k\rho 4\pi \int_b^a r\, dr = k\rho 4\pi \left[\frac{b^2}{3} + \frac{1}{2}(a^2 - b^2)\right]$$

$$= k\rho 4\pi \left[\frac{1}{2}a^2 - \frac{1}{6}b^2\right]$$

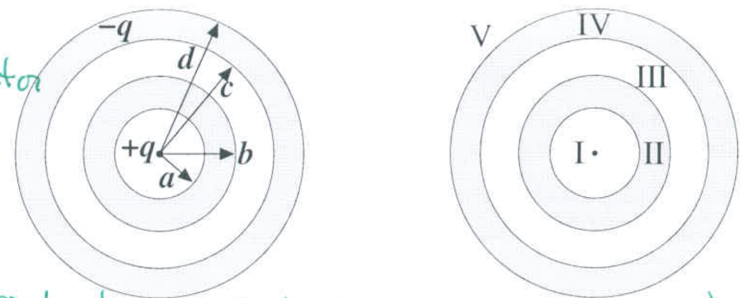
$$E = -\frac{dV}{db} = \underbrace{k}_{\frac{1}{4\pi\varepsilon_0}}\rho 4\pi\, \frac{1}{3}\, b = \frac{\rho b}{3\varepsilon_0}$$ which agrees with previous problem!

15. Two concentric conducting spherical shells surround a point charge $+q$ at the shells' center creating five regions: I: $r < a$, II: $a < r < b$, …, V: $d < r$ (see diagram). The inner shell (region II) has an inner radius of a and an outer radius of b. The outer shell (region IV) has an inner radius of c and an outer radius of d. The inner shell has no net charge; the outer shell has a net charge of $-q$.

A. In which regions will the electric field be zero? For each case, explain why.

B. For each of the four surfaces (i.e., at $r = a, b, c, d$) report the net charge on that surface and your reasoning for each result.

C. Report the formula for the electric field in region I. Explain your reasoning.



A: II & IV since inside conductor

V since net enclosed charge is zero: $+q-q$

B: $r=a \rightarrow -q$ (zero enclosed charge but know $+q$ is inside)

$r=b \rightarrow +q$ (as II has no net charge)

$r=c \rightarrow -q$ (as net enclosed charge must be zero & $+q$ is at origin & II has zero net charge)

$r=d \rightarrow 0$ ($E=0$ in region V)

C: Gauss: $4\pi r^2 E = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$

When finished: insert your formula sheet inside this booklet, make sure your name is on the front cover, and place the resulting packet in the pile at the front of the classroom. You are free to leave when you have turned in your exam.