

- 23-10. The work done by the electric field on the alpha particle is minus the change in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides; the change in potential energy can be related to the change in electric potential (a.k.a.: “voltage”):

$$W_{\text{center} \rightarrow \text{side}} = -\Delta U = U_{\text{initial}} - U_{\text{final}} = U_{\text{center}} - U_{\text{side}} = q(V_{\text{center}} - V_{\text{side}})$$

where the charge on the alpha particle is $q = 2e$. The center of the square is a distance $s/\sqrt{2}$ from the electrons. The corresponding voltage is:

$$V_{\text{center}} = \frac{-4 \times ke}{(s/\sqrt{2})} = \frac{-4ke}{s} \sqrt{2} = \frac{-4 \cdot 8.99 \times 10^9 \times 1.6 \times 10^{-19} \cdot \sqrt{2}}{10 \times 10^{-9}} = -.81457 \text{ V}$$

The side midpoint is a distance $s/2$ from two electrons and $\sqrt{s^2 + (s/2)^2} = \sqrt{5}s/2$ from the other two electrons. The corresponding voltage is:

$$V_{\text{side}} = \frac{-2ke}{(s/2)} + \frac{-2ke}{(\sqrt{5}s/2)} = \frac{-4ke}{s} \left(1 + \frac{1}{\sqrt{5}}\right) = -.83357 \text{ V}$$

$V_{\text{center}} - V_{\text{side}} = +0.0190 \text{ V}$. So the resulting work is:

$$W_{\text{center} \rightarrow \text{side}} = 2 \cdot 1.6 \times 10^{-19} \cdot 0.019 = 6.09 \times 10^{-21} \text{ J}$$

Since the work is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side; the particle would gain kinetic energy during this motion. Note that the problem actually asks for the work “needed to move” the alpha particle. Those words mean the work an external force would need to do; that external force would need to cancel the electric force in order to move the particle freely, so the answer to this question is actually: $+\Delta U = -6.09 \times 10^{-21} \text{ J}$; that is the required external force is actually a brake! I’m reasonably sure the authors intended you to calculate the work done by the electric field (as that is what the text mostly discusses). I’ll accept either answer as correct now, but do note the precise meanings of these phrases!

- 23-21. The electric potential (a.k.a.: “voltage”) of a point charge is given by: $V = kq/r$ where we have arranged the potential at infinity to be zero. With two point charges, we need to add up the potential of each:

(a)

$$V_A = V_1 + V_2 = \frac{kq_1}{.05 \text{ m}} + \frac{kq_2}{.05 \text{ m}} = -737.0 \text{ V}$$

(b)

$$V_B = V_1 + V_2 = \frac{kq_1}{.08 \text{ m}} + \frac{kq_2}{.06 \text{ m}} = -704.0 \text{ V}$$

(c)

$$W_{B \rightarrow A} = q(V_B - V_A) = 2.5 \times 10^{-9}(-704 + 737) = 8.24 \times 10^{-8} \text{ J}$$

23-63. The motion of this electron is exactly analogous to a projectile on Earth.

- (a) The force on the electron is $\vec{F} = q\vec{E}$ where $q = -e$ and \vec{E} is upward. As a result the force is downward with magnitude:

$$F = |qE| = 1.6 \times 10^{-19} \cdot 1100 = 1.76 \times 10^{-16} \text{ N}$$

- (b) The (downward) acceleration (analogous to g) is:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{1.6 \times 10^{-19} \cdot 1100}{9.11 \times 10^{-31}} = 1.93 \times 10^{14} \text{ m/s}^2$$

- (c) As usual for projectile motion, the x and y motions can be considered separately. As there is no acceleration in the x direction, v_x remains constant. The electron will take a time:

$$t = \frac{d}{v_x} = \frac{.06}{6.5 \times 10^6} = 9.23 \times 10^{-9} \text{ s}$$

to traverse the plates. During this period, the y motion is accelerated. The initial y velocity is zero; we take the initial y position to be zero, so we have:

$$\begin{aligned} y &= -\frac{1}{2} at^2 \\ v_y &= -at \end{aligned}$$

We can then calculate the y position at the time the electron is exiting the plates:

$$\begin{aligned} y &= -\frac{1}{2} at^2 = -\frac{1}{2} 1.93 \times 10^{14} \cdot (9.23 \times 10^{-9})^2 = 8.22 \times 10^{-3} \text{ m} \\ v_y &= -at = -1.93 \times 10^{14} \cdot 9.23 \times 10^{-9} = -1.79 \times 10^6 \text{ m/s} \end{aligned}$$

- (d) The angle of its motion at the time the electron is exiting the plates can be calculated from its velocity vector \vec{v} :

$$\theta = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{-at}{v_x}\right) = \arctan\left(\frac{-1.93 \times 10^{14} \cdot 9.23 \times 10^{-9}}{6.5 \times 10^6}\right) = -15.3^\circ$$

- (e) On exiting the plates, the electron will move with a constant \vec{v} . It will take an additional time:

$$t = \frac{d}{v_x} = \frac{.12}{6.5 \times 10^6} = 18.5 \times 10^{-9} \text{ s}$$

to reach the screen. During this time it will have an additional y displacement of:

$$\Delta y = v_y t = 0.0330 \text{ m}$$

The total y displacement is then: $0.0330 + 0.0082 = 0.0412 \text{ m}$.