

27-35.  $\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}}$ ;  $F = IlB \sin \phi$ . For all segments here  $\phi = 90^\circ$ . From the right hand rule for cross products, the horizontal segments produce a force in the  $-\hat{\mathbf{j}}$  direction whereas the vertical section produces a force in the  $+\hat{\mathbf{i}}$  direction. Putting together those pieces, we have:

$$\vec{\mathbf{F}} = (Il_y B, -Il_x B) = IB(\ell_y, \ell_x) = 4.5 \cdot 0.24(.3, -.6) = (0.324, -0.648) \text{ N}$$

A vector capable calculator finds  $\vec{\mathbf{F}} = .725 \text{ N } \angle -63.4^\circ$

27-72. (a)  $\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}}$  reports that the force is outward with magnitude  $F = ILB$  (since  $\sin \phi = 1$ ).

(b)  $a_x = F/m$ ,  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ ,  $v_{0x} = 0$ ,  $d = x - x_0$ :

$$\begin{aligned} v_x^2 &= 2a_x d \\ \frac{v_x^2 m}{2ILB} &= d \end{aligned}$$

(c)

$$d = \frac{v_x^2 m}{2ILB} = \frac{(11.2 \times 10^3)^2 \cdot 25}{2 \cdot 2000 \cdot 0.5 \cdot 0.5} = 3.14 \times 10^6 \text{ m}$$

27-78. Begin by noting that since  $\vec{\ell}$  and  $\vec{\mathbf{B}}$  lie in the plane of the page,  $\vec{\ell} \times \vec{\mathbf{B}}$  always points into or out of the page. I define forces directed out-of-the-page to be positive; into-the-page to be negative. Also note that the cited problem (27.77) suggests finding torques by assuming magnetic forces on straight segments act as if the force were concentrated on the center of the segment. Finally this triangle is a 3-4-5 right triangle with hypotenuse  $\sqrt{.6^2 + .8^2} = 1 \text{ m}$ .

- (a) i. The force is zero on segment  $PQ$  as  $\vec{\ell} \parallel \vec{\mathbf{B}}$ .  
 ii. The angle,  $\theta$ , between segment  $QR$  and  $\vec{\mathbf{B}}$  is just the supplement of  $\angle PQR$  (i.e.,  $\theta = 180^\circ - \angle PQR$ );  $\angle PQR$  can be found from the triangle relationships, e.g.,  $\tan^{-1}(.8/.6) = 53.13^\circ$ . (Note: the sin of supplementary angles are equal  $\sin \theta = \sin(\angle PQR)$ .)  $\vec{\ell} \times \vec{\mathbf{B}}$  is out-of-page, so we have the force on segment  $QR$ :

$$F = IlB \sin \theta = 5 \cdot 1 \cdot 3 \cdot \sin(\theta) = 15 \cdot .8 = 12 \text{ N}$$

- iii. The force on segment  $RP$  is into-the-page (negative) and simply calculated since  $\vec{\ell} \perp \vec{\mathbf{B}}$ :

$$F = -IlB = -5 \cdot 0.8 \cdot 3 = -12 \text{ N}$$

(b) zero ( $12 - 12 = 0$ )

- (c) i. Since the force is everywhere zero on segment  $PQ$ , so is the torque.  
 ii. With a moment arm (a.k.a. lever arm) of .3 m, the torque for segment  $QR$  is

$$\tau = .3 \cdot 12 = 3.6 \text{ N} \cdot \text{m}$$

This torque is trying to rotate  $Q$  out of the page.

- iii. Since segment  $RP$  is on the axis, the moment arm is zero, so the torque is also zero.

(d) Since the area vector is perpendicular to  $\vec{\mathbf{B}}$ ,  $\phi = 90^\circ$  and the torque given by Eq. 27-28 is:

$$\tau = IAB = 5 \left( \frac{1}{2} \cdot 0.6 \cdot 0.8 \right) 3 = 3.6 \text{ N} \cdot \text{m}$$

This is the same as the sum of the (mostly zero) torques given above.

(e)  $Q$  is being rotated out of the page.

27-79. Application of the right hand rule on any small arc ( $d\vec{\ell}$ ) of the coil finds that the force  $\vec{F}$  on that arc is down and outward. Since  $\vec{F} \perp \vec{B}$  and  $\vec{B}$  is directed  $60^\circ$  outwards (from the positive  $y$  axis),  $\vec{F}$  is directed  $30^\circ$  outwards from the negative  $y$  axis. The outward components of  $\vec{F}$  must cancel around the circle so the net force will be in the  $-\hat{j}$  direction. Since  $d\vec{\ell} \perp \vec{B}$ ,  $F = Id\ell B$  and  $F_y = -Id\ell B \cos 30^\circ$ . It is easy to add these forces on small arcs to get the net force:

$$F_y = -IlB \cos 30^\circ = -0.95 \cdot 50 \pi \cdot 0.0156 \cdot 0.22 \cdot \cos 30^\circ = -.444 \text{ N}$$