27-43. (a)

$$T = \frac{2\pi r}{v} = \frac{2\pi \cdot 5.3 \times 10^{-11}}{2.2 \times 10^{6}} = 1.51 \times 10^{-16} \text{ s}$$

(b)

$$I = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{1.514 \times 10^{-16}} = 1.06 \times 10^{-3} \text{ A}$$

(c)

$$\mu = I\pi r^2 = 9.34 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

27-44. We have shown that the force on a current loop in a uniform magnetic field is zero and the torque is $\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$.

(a) $\vec{\mathbf{F}} = 0$. The area vector $\vec{\mathbf{A}}$ points out-of-page and hence is anti-parallel to $\vec{\mathbf{B}}$, as a result the torque is zero.

(b) $\vec{\mathbf{F}} = 0$. The area vector $\vec{\mathbf{A}}$ now is at an angle of $180° - 30° = 150°$ to $\vec{\mathbf{B}}$. The right hand rule gives the direction of the torque as directly up-the-page. The magnitude of the torque is $\tau = IAB\sin(150°)$, where the area of the square loop is $A = 0.35 \cdot 0.22 = 0.077 \text{ m}^2$. The result is:

$$\tau = 1.4 \cdot 0.077 \cdot 1.5 \cdot \sin(150°) = 0.0809 \text{ N} \cdot \text{m}$$

27-46. $\vec{\boldsymbol{\tau}} = NI\vec{\mathbf{A}} \times \vec{\mathbf{B}}$; $U = -NI\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

(a) The direction of $\vec{\mathbf{B}}$ is up-page ($\hat{\mathbf{j}}$); the direction of $\vec{\mathbf{A}}$ is out-of-page ($\hat{\mathbf{k}}$), The angle between these vectors is 90° so $U = 0$ and τ is at its maximum value: $NIAB$. The direction of $\vec{\boldsymbol{\tau}}$ is left ($-\hat{\mathbf{i}}$).

(b) The direction of $\vec{\mathbf{B}}$ is into-page ($\hat{\mathbf{j}}$); the direction of $\vec{\mathbf{A}}$ is into-page ($\hat{\mathbf{j}}$), The angle between these vectors is 0° so U is at its minimum: $-NIAB$ and $\tau = 0$.

(c) The direction of $\vec{\mathbf{B}}$ is up-page ($\hat{\mathbf{j}}$); the direction of $\vec{\mathbf{A}}$ is into-page ($-\hat{\mathbf{k}}$), The angle between these vectors is 90° so $U = 0$ and τ is at its maximum value: $NIAB$. The direction of $\vec{\boldsymbol{\tau}}$ is right ($\hat{\mathbf{i}}$).

(d) The direction of $\vec{\mathbf{B}}$ is into-page ($\hat{\mathbf{j}}$); the direction of $\vec{\mathbf{A}}$ is out-of-page ($-\hat{\mathbf{j}}$), The angle between these vectors is 180° so U is at its maximum: $+NIAB$ and $\tau = 0$.

27-62. Our basic force result: $\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$ directly applies to the top and bottom straight sections. Example 27.8 (also done in class) shows that the arc produces the same net force as a diameter. (Integration or application of our theorem on the net force on current loops in uniform magnetic fields was required to prove this result.) Putting together these three segments we conclude that the net force is the same as on a single, straight 3 m segment. For such a segment the right hand rule shows $\vec{\mathbf{F}}$ points right with magnitude:

$$F = I\ell B = 3.4 \cdot 3 \cdot 2.2 = 22.4 \text{ N}$$

If we reproduce work we did in class, we have:

$$\begin{aligned}
\vec{\boldsymbol{\ell}} &= R(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}) \\
d\vec{\boldsymbol{\ell}} &= Rd\theta(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}) \\
\vec{\mathbf{B}} &= -B_0\,\hat{\mathbf{k}} \\
d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} &= Rd\theta \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -B_0 \end{vmatrix} = Rd\theta(-\cos\theta B_0\,\hat{\mathbf{i}} - \sin\theta B_0\,\hat{\mathbf{j}})
\end{aligned}$$

Note that here θ runs backwards: from $+\pi/2$ to $-\pi/2$.

$$\begin{aligned}
\vec{\mathbf{F}} &= I\int_{+\pi/2}^{-\pi/2} d\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = IRB_0 \int_{+\pi/2}^{-\pi/2} \Big[-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}}\Big] d\theta = IRB_0\Big[-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}\Big]_{+\pi/2}^{-\pi/2} \\
&= I2RB_0\,\hat{\mathbf{i}}
\end{aligned}$$

so the result just depends on the diameter $2R$ of the circle.