

28-64. The right and left sides of the rectangle experience exactly the same magnetic fields, but with reversed current; the forces on these two sides are equal but opposite. The magnetic field due to the current in the AB wire is into-the-page where the rectangular loop is located. For the top of the loop $d\vec{\ell} \times \vec{B}$ is up-page (\hat{j}) whereas for the bottom of the loop $d\vec{\ell} \times \vec{B}$ is down-page ($-\hat{j}$) and weaker due to the reduced B . The net magnetic force is then:

$$F = \frac{\mu_0 I_{AB}}{2\pi} \ell I_{\text{loop}} \left(\frac{1}{r_{\text{top}}} - \frac{1}{r_{\text{bottom}}} \right) = \frac{4\pi \times 10^{-7} \cdot 14}{2\pi} \cdot 2.5 \left(\frac{1}{.026} - \frac{1}{.1} \right) = 7.97 \times 10^{-5} \text{ N}$$

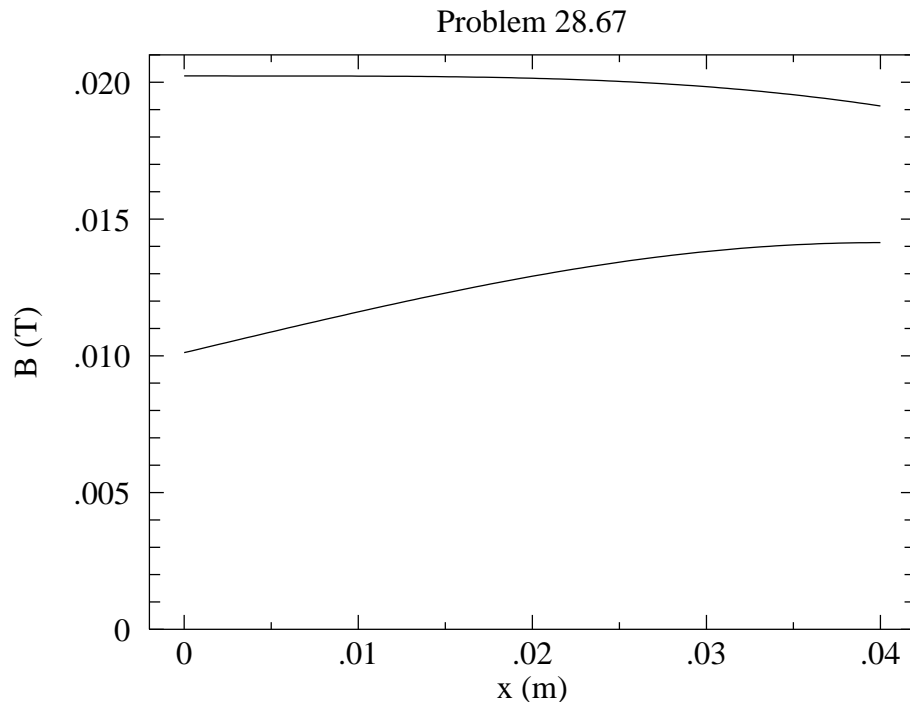
28-67. (a) According to Eq. 28-16 the magnetic field due to a single coil-set is:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$

For the left hand coil: $x \rightarrow a/2 + x$ for the right hand coil: $x \rightarrow a/2 - x$. Combining these magnetic fields yields:

$$B_x = \frac{\mu_0 N I a^2}{2} \left[\left((a/2 + x)^2 + a^2 \right)^{-3/2} + \left((a/2 - x)^2 + a^2 \right)^{-3/2} \right]$$

(b) Plotted for the data of part (d).



(c) At $x = 0$ we have:

$$B_x = \mu_0 N I a^2 \left[\left(\frac{5a^2}{4} \right)^{-3/2} \right] = \frac{\mu_0 N I}{a} \left(\frac{4}{5} \right)^{3/2}$$

(d)

$$B_x = \frac{\mu_0 NI}{a} \left(\frac{4}{5}\right)^{3/2} = \frac{4\pi \times 10^{-7} \cdot 300 \cdot 6}{.08} \left(\frac{4}{5}\right)^{3/2} = 0.0202 \text{ T}$$

(e)

$$\frac{dB}{dx} = -3 \frac{\mu_0 NI a^2}{2} \left[(a/2 + x) \left((a/2 + x)^2 + a^2 \right)^{-5/2} - (a/2 - x) \left((a/2 - x)^2 + a^2 \right)^{-5/2} \right]$$

At $x = 0$ this is zero.

$$\begin{aligned} \frac{d^2B}{dx^2} = & -3 \frac{\mu_0 NI a^2}{2} \left\{ \left[\left((a/2 + x)^2 + a^2 \right)^{-5/2} + \left((a/2 - x)^2 + a^2 \right)^{-5/2} \right] \right. \\ & \left. - 5 \left[(a/2 + x)^2 \left((a/2 + x)^2 + a^2 \right)^{-7/2} + (a/2 - x)^2 \left((a/2 - x)^2 + a^2 \right)^{-7/2} \right] \right\} \end{aligned}$$

Plugging in $x = 0$ yields:

$$\frac{d^2B}{dx^2} = -3 \frac{\mu_0 NI a^2}{2} \left[2 \left(5a^2/4 \right)^{-5/2} \right] \left\{ 1 - 5 \frac{(a/2)^2}{(5a^2/4)} \right\} = 0$$

The results $dB/dx = 0$ and $d^2B/dx^2 = 0$ say the slope is zero and the change in slope is zero: clearly this means flat.

28-79. The suggested integral is essentially $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$, as the 'loop-return' integral from $x = +\infty$ to $x = -\infty$ at $r = \infty$ is zero as $B \sim 1/r^3$ through path length πr yields an integral $\sim 1/r^2 \rightarrow 0$ if $r \rightarrow \infty$.

$$\int_{-\infty}^{+\infty} B_x dx = \frac{\mu_0 I a^2}{2} \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I}{2} \left[\frac{x}{\sqrt{x^2 + a^2}} \right]_{-\infty}^{+\infty} = \frac{\mu_0 I}{2} [1 - -1] = \mu_0 I$$

28-82. In the context of the current sheet problem (assigned 8-3), $K = nI$, so the magnetic field due to one alone sheet is $B = \mu_0 nI/2$ directed either to the right or left. At P and S the magnetic fields due to the bisheet cancel: $B = 0$. At R the fields add with result: $B = \mu_0 nI$ with direction to the right.