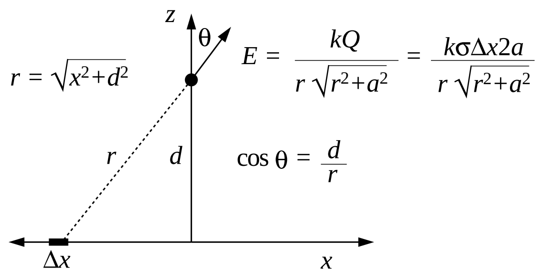
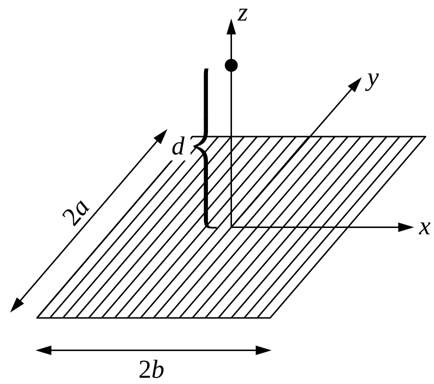
Challenge Problem

not to be graded!





A card-of-charge sits in the xy plane with center at the origin. The card-of-charge has uniform surface charge density (σ) and has dimensions $2a \times 2b$. We seek the magnitude of the resulting electric field (E) a distance d directly above the center of the card-of-charge. From symmetry is it clear that $\vec{\mathbf{E}}$ points directly out from the card: along the z axis. We can think of the card-of-charge as being divided into a series of strips, each of thickness Δx and length $2a$. In Example 21.11 we found that the electric field a distance r from such a 'wire' points directly away from the wire and has magnitude:

$$E = \frac{kQ}{r\sqrt{r^2 + a^2}}$$

We need to add up the z component of $\vec{\mathbf{E}}$ from every 'wire' that makes up the sheet. You are unlikely to recognize the resulting integral, but looking in a table of integrals I found the following entry which may be of use:

$$\int \frac{dx}{(x^2 + a^2)\sqrt{x^2 + b^2}} = \frac{1}{a\sqrt{b^2 - a^2}} \arctan\left(\frac{\sqrt{b^2 - a^2}x}{a\sqrt{b^2 + x^2}}\right)$$

The Web `http://www.oberlin.edu/physics/dstyer/P111/Problems.pdf` claims the answer is:

$$E(z) = \frac{1}{4\pi\epsilon_0} \frac{q}{ab} \arctan\left[\frac{ab}{z\sqrt{z^2 + a^2 + b^2}}\right]$$

where q is the total charge on the card and z is the distance from the card... but then you know how reliable the web can be.