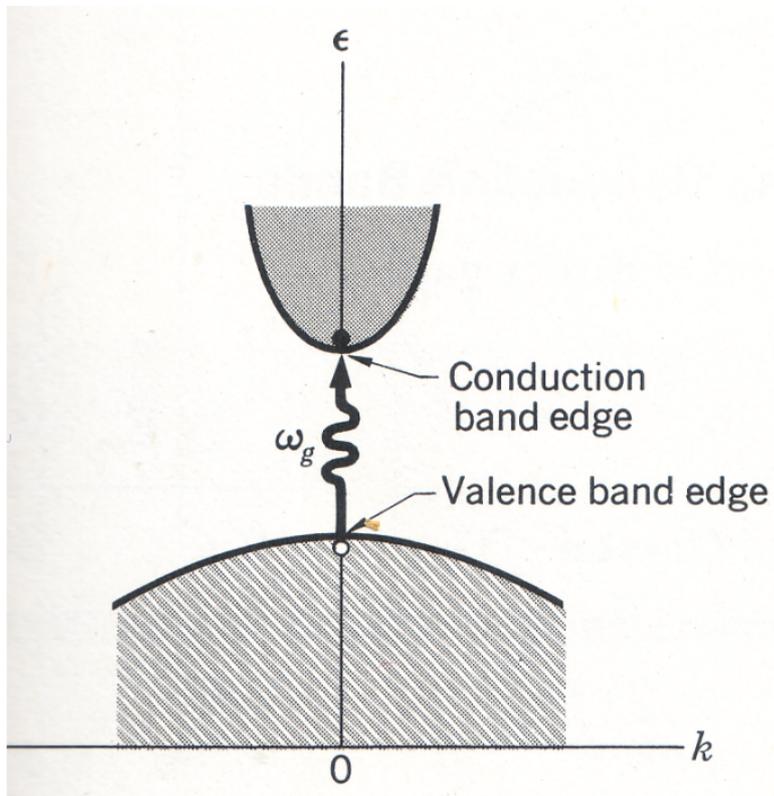


PHYS208  
Elementary Physics of P-N Junctions

Olav Torheim

June 5, 2007



# 1 Intrinsic semiconductors

The lower end of the conduction band is a parabola, just like in the quadratic free electron case ( $E = \frac{\hbar^2 k^2}{2m}$ ).

The density of states for free particles in a box is:

$$g(E) = \frac{V}{2\pi^2} \frac{2m^{3/2}}{\hbar^2} E^{1/2}$$

We assume that the electrons in the conduction band behave like free electrons, and therefore use the the same density of states as for the free electron model. We only correct for the effective mass and that we have to count the energy from the bottom of the conduction band and up:

$$g_c(E) = \frac{V}{2\pi^2} \frac{2m_e^*{}^{3/2}}{\hbar^2} (E - E_c)^{1/2}$$

The number of free electrons per unit volume in the conduction band is then given by the following integral:

$$n_c = \frac{1}{V} \int_{E_c}^{\infty} g_c(E) * f_{FD}(E) dE = 2 * \left( \frac{m_e^* kT}{2\pi\hbar^2} \right)^{3/2} e^{\frac{\mu - E_c}{kT}} = N_c * e^{\frac{\mu - E_c}{kT}}$$

where  $N_c \equiv 2 \left( \frac{m_e^* kT}{2\pi\hbar^2} \right)^{3/2}$ .

We can make the same calculation for the number of holes per unit volume in the valence band if we use the effective mass for holes and integrate from the top of the valence band and down to infinity. Here we also have to take into consideration that  $f_h(E) = 1 - f_e(E)$ .

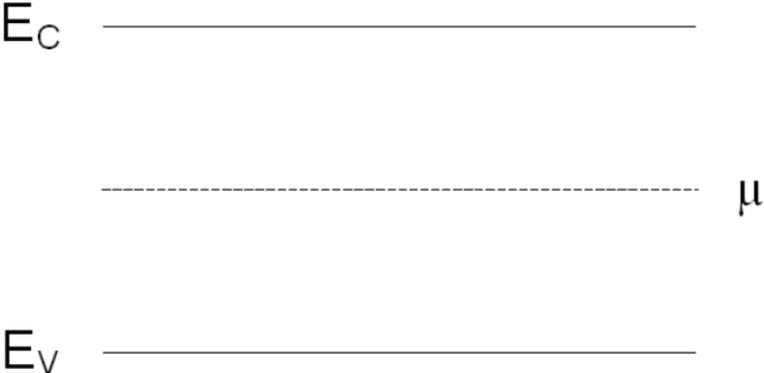
$$n_v = \frac{1}{V} \int_{E_v}^{-\infty} g_v(E) * (1 - f_{FD}(E)) dE = 2 * \left( \frac{m_h^* kT}{2\pi\hbar^2} \right)^{3/2} e^{\frac{E_v - \mu}{kT}} = N_v * e^{\frac{E_v - \mu}{kT}}$$

where  $N_v \equiv 2 \left( \frac{m_h^* kT}{2\pi\hbar^2} \right)^{3/2}$ .

The chemical potential of the intrinsic semiconductor is found by setting  $n_c = n_v$ :

$$\mu = \frac{E_c + E_v}{2} + \frac{3}{4} kT * \ln \left( \frac{m_h^*}{m_e^*} \right)$$

Which shows that the chemical potential must be in the middle of the energy gap:



In the same way, we can find the intrinsic carrier concentration by multiplying  $n_c$  and  $n_v$ :

$$n_i^2 = n_c * n_v = 4 * m_e^* m_h^{*3/4} (kT/2\pi\hbar^2)^3 e^{-E_g/kT}$$

The intrinsic carrier concentration is dependent only on the energy gap and not on the Fermi level (which means that the law of mass action,  $n_c * n_v = n_i^2$ , is valid also for doped semiconductors).

## 2 Doped semiconductors

Impurity atoms are added to provide loosely bound electrons (n-doping with group 5 material) or holes (p-doping with group 3 material) that are thermally excited to the conduction or valence band. The impurity atoms are usually not close enough for the wavefunctions to overlap, therefore the impurity states are localized. Their energy levels are either right below the conduction band or right above the valence band. How close they are can be approximated with a modified hydrogen atom model:

$$E_{ion} = 13.6eV * \frac{m^*}{m} * \left(\frac{\epsilon_0}{\epsilon}\right)^2$$

Here the electron mass  $m$  is replaced by the effective mass  $m^*$  and the dielectricity constant refers to the crystal and not to free space.

Example: Phosphorus n-doping of silicon.  $\epsilon_r = 11.7$  and  $\frac{m^*}{m}$  is 0.19 near the bottom of the conduction band for Si, which gives us  $E_{ion} = 0.019eV$ . The donor impurity therefore requires 0.019eV to become ionized and the position in the band gap of the donor impurity level is just 0.019 eV below the conduction band edge.

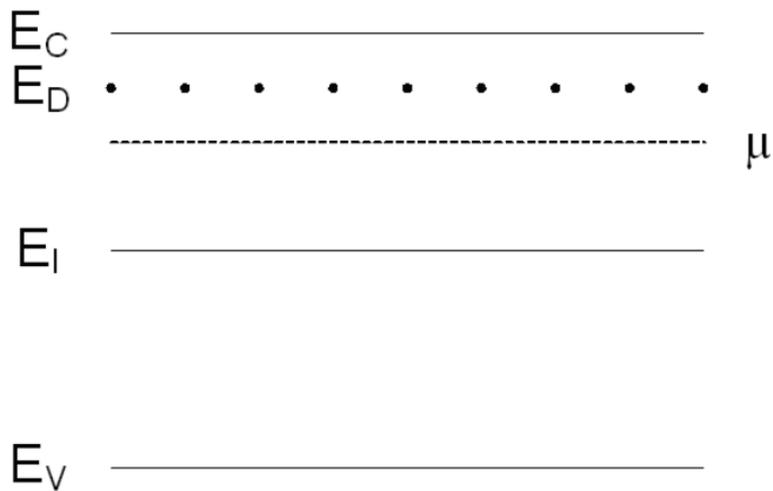
Assuming complete ionization, we set  $n_c = N_D$ . Combining this equation with the already established relation  $n_c = N_c * e^{\frac{\mu - E_c}{kT}}$ , we find the chemical potential for the n-doped semiconductor:

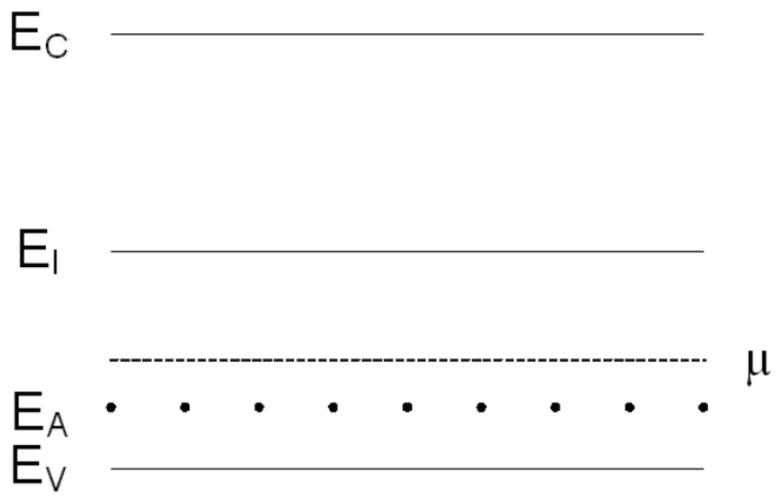
$$\mu = E_c - kT * \ln\left(\frac{N_c}{N_D}\right) \quad (1)$$

It is seen that the chemical potential is moved closer to the conduction band as the doping concentration increases.

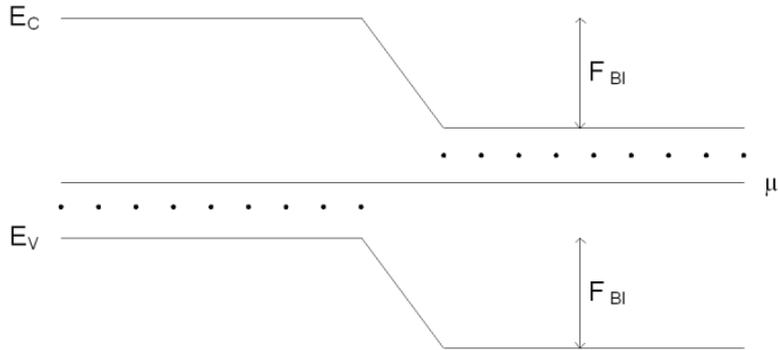
The same argument can be applied to the p-doped case to show that

$$\mu = E_v + kT * \ln\left(\frac{N_v}{N_A}\right) \quad (2)$$





When p-doped and n-doped materials are joined, there is an imbalance between the n-type and p-type carrier concentrations on each side because of the different Fermi levels in the two kinds of materials. N-type and p-type majority carriers will therefore diffuse into each others regions and recombine until a potential is built up that opposes the diffusion currents. The potential is called the built-in-potential,  $\phi_{BI}$ , and it raises the band edges on the p-side so the Fermi levels on each side are aligned:



After equilibrium is established, there is balance between the diffusion current caused by the carrier inhomogeneity (given by Ficks law) and the drift current caused by the electric field (given by simple kinetic free electron theory):

$$n(x)\mu E(x) = -D \frac{dn}{dx}$$

which leads to

$$E_x(x) = -\frac{D}{\mu} * \frac{1}{n} * \frac{dn}{dx}$$

The built-in potential can now be calculated:

$$\phi_{BI} = \phi(\infty) - \phi(-\infty) = - \int_{-\infty}^{\infty} E_x dx = \frac{D}{\mu} \int_{-\infty}^{\infty} \frac{1}{n(x)} * \frac{dn}{dx} dx = \frac{D}{\mu} \ln \frac{n(\infty)}{n(-\infty)}$$

Now we have the condition for the energy difference, but  $\frac{D}{\mu}$ ,  $n(\infty)$  and  $n(-\infty)$  are still unknown. It is therefore time to introduce Einstein-Nernst.

We start by writing the electric field as the gradient of the potential:

$$E_x(x) = -\frac{d\phi}{dx}$$

Therefore

$$-\mu * n * e * \frac{d\phi}{dx} = eD \frac{dn}{dx}$$

So

$$\frac{dn}{d\phi} = -\frac{\mu * n}{D}$$

The differential equation has the solution

$$n = n_0 e^{-\frac{\mu}{D}\phi}$$

But because this non-uniform electron distribution has to be maintained against the potential  $\phi$  using thermal energy, we must also have

$$n = n_0 e^{-\frac{e\phi}{kT}}$$

Thus we finally arrive at the Einstein-Nernst relation:

$$\frac{\mu}{D} = \frac{e}{kT}$$

Now we continue to evaluate  $\ln \frac{n(\infty)}{n(-\infty)}$ . In the n-region the density of free electrons is approximately  $n(\infty) = N_D$ . In the p-region, the negative carrier concentration is taken from the law of mass action,  $n(-\infty) = \frac{n_i^2}{N_A}$ .

From this we obtain the built-in potential:

$$\phi_{BI} = \frac{D}{\mu} \ln \frac{n(\infty)}{n(-\infty)} = \frac{kT}{e} \ln \frac{N_A N_D}{n_i^2}$$

After establishing equilibrium between drift and diffusion currents, we are left with a depletion region in the middle of the junction where the holes and

electrons have recombined and we have approximately no free carriers. Starting out with the Poisson equation in one dimension, we can make a model of this depletion region characterized by the parameters  $\frac{kT}{e}$ ,  $N_A$  and  $N_D$ :

$$-\frac{d^2V}{dx^2} = C\rho(x)$$

( $C$  depends on which system of units. In SI units  $C = \frac{1}{\epsilon}$  where  $\epsilon = K_S\epsilon_0$ .)

The net charge density,  $\rho$ , is approximated as zero outside the depletion region. Inside the depletion region, we have approximately  $\rho = -qN_A$  on the p side ( $-d_p \leq x \leq 0$ ) and  $\rho = qN_D$  on the n-side ( $0 \leq x \leq d_n$ ).

Starting on the p side of the depletion region, integration of the Poisson equation yields

$$\epsilon = -\frac{dV}{dx} = -eN_A C(d_p + x)$$

Another integration yields

$$V = eN_A C\left(\frac{x^2}{2} + d_p x + \frac{d_p^2}{2}\right) = \frac{1}{2}eN_A C(d_p + x)^2$$

for  $-d_p \leq x \leq 0$ , so that  $V(x = 0) = V_1 = eN_A C \frac{d_p^2}{2}$ .

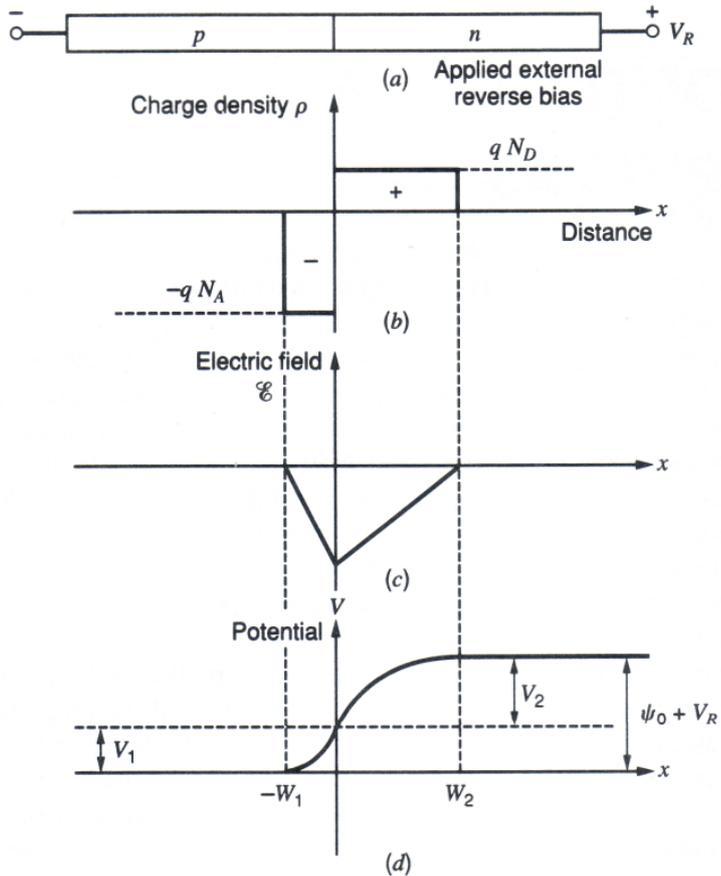
In the same way we have for the n side that  $V(x = d_p) - V(x = 0) = V_2 = eN_D C \frac{d_n^2}{2}$ .

And we have an equation for the total potential across the junction:

$$V_C = \phi_{BI} + V_R = e \frac{C}{2} (N_A d_p^2 + N_D d_n^2).$$

We now define  $d = d_p + d_n$ . And since  $N_D d_n = N_A d_p$  (charge balance), we finally arrive at an equation that combines the potential across the junction with the geometry and the doping profile of the depletion region:

$$V_C = \frac{1}{2} C e d^2 \frac{N_D N_A}{N_D + N_A}.$$



We say that it is impossible to stop the drift current, since there always will be minority carriers on each side of the junction that are thermally excited and swept across the electric field (which is on the order of 10.000 V/cm). When it comes to the diffusing majority carriers, there will be a Boltzmann distribution on how many carriers that have energy enough to break through the potential barrier. This recombination current is therefore proportional to the Boltzmann factor  $e^{-qV_C/kT}$ .

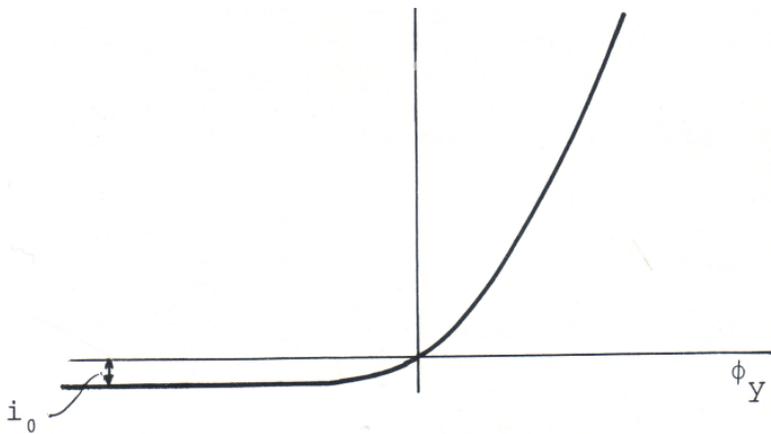
When no external potential is applied, the generation and recombination currents are in equilibrium:

$$i_{tot} = i_{rec} - i_{gen} = i_{gen} - i_{gen} = 0 \quad (3)$$

When an external potential,  $\phi_{ext}$ , is applied, the recombination current is increased with the Boltzmann factor  $e^{q\phi_{ext}/kT}$ , while the generation current remains the same:

$$i_{tot} = i_{gen} * e^{q\phi_{ext}/kT} - i_{gen} = i_{gen}(e^{q\phi_{ext}/kT} - 1) \quad (4)$$

which shows us that the p-n junction acts as a rectifier.



# References

- [1] P.C. Hemmer, Faste stoffers fysikk, Tapir Forlag, 1987.
- [2] C. Kittel, Introduction to Solid State Physics, Wiley, 5th Edition, 1976.
- [3] P. R. Gray, R. G. Meyer, P. J. Hurst, S. H. Lewis, Analysis and Design of Analog Integrated Circuits, 4th Edition, 2001.
- [4] G. Burns, Solid State Physics, Academic Pr, 1985.
- [5] L. G. Johansen, Radiation hard silicon microstrip detectors for use in ATLAS at CERN, 2005.