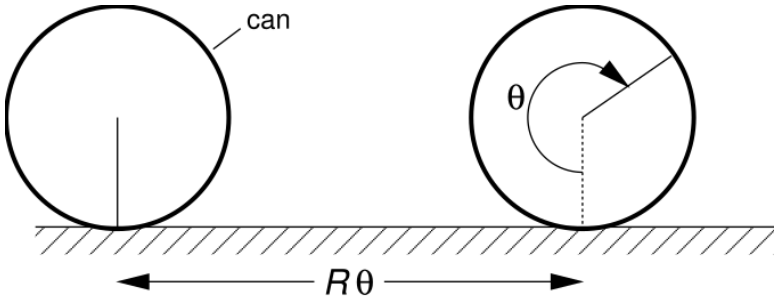
Rolling Can

A large empty coffee can has been turned into a cylinder (radius R) by removing its top and bottom. Consider a spot on the cylindrical surface of the can as it rolls along at constant speed on a flat table. The location of the spot (when attached to the cylinder) is described by the angle θ shown below. So when $\theta = 0$ the spot was in contact with the table. Technically speaking the path followed by the spot is part of a cycloid. Report the reasoning and display the algebra to show the following



$$\begin{array}{lll} x: & x = R\theta - R\sin\theta & \dot{x} = R(1-\cos\theta)\dot{\theta} \\ y: & y = -R\cos\theta & \dot{y} = R\sin\theta\,\dot{\theta} \end{array}$$

$$v^2: \qquad v^2 = \dot{x}^2 + \dot{y}^2 = 2R^2\left(1-\cos\theta\right)\dot{\theta}^2$$

The origin's (fixed) location is where the can's center was when $\theta = 0$; x is horizontal, y is vertical.

1. If the speed of the can's center is V, report $\dot{\theta}$ in terms of V.
2. What is the spot's velocity when in contact with the table?
3. What is the spot's velocity when it reaches maximum height?
4. What is the spot's velocity when $\theta = \pi/2$?
5. If the spot disconnects from the rolling can when $\theta = \pi/2$, how far does it travel horizontally before it hits the table? Is the released spot in danger of hitting the rolling can? Explain!