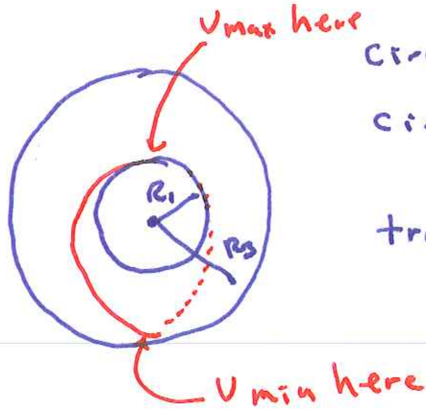
Hohmann transfer orbits (circular case)



Circular orbit speed ① $= \sqrt{\dfrac{\alpha}{\mu R_1}}$

circular orbit speed ③ $= \sqrt{\dfrac{\alpha}{\mu R_3}}$

transfer orbit semi major axis $= a = \dfrac{R_1+R_3}{2}$

eccentricity: $ae = a - R_1$

$$e = \frac{a-R_1}{a} = \frac{R_3-R_1}{R_3+R_1}$$

$$V_{max} = (1+e)\,V_0$$

$$= \sqrt{\frac{(1+e)}{(1-e)}\,\frac{\alpha}{\mu a}}$$

$$\frac{(1+e)}{(1-e)} = \frac{R_3}{R_1}$$

$$1+e = 1 + \frac{R_3-R_1}{R_3+R_1} = \frac{2R_3}{R_3+R_1}$$

$$1-e = 1 - \frac{R_3-R_1}{R_3+R_1} = \frac{2R_1}{R_3+R_1}$$

$$\lambda = \frac{V_{max}}{\text{orbit speed 1}} = \sqrt{\frac{R_3}{R_1}\,\frac{R_1}{(R_1+R_3)/2}} = \sqrt{\frac{2R_3}{R_1+R_3}}$$

$$V_{min} = \sqrt{\frac{(1-e)}{(1+e)}\,\frac{\alpha}{\mu a}} = \sqrt{\frac{R_1}{R_3}\,\frac{\alpha}{\mu(R_1+R_3)/2}}$$

$$\lambda' = \frac{\text{orbit speed 3}}{V_{min}} = \frac{\sqrt{\dfrac{\alpha}{\mu R_3}}}{\sqrt{\dfrac{R_1}{R_3}\,\dfrac{2\alpha}{\mu(R_1+R_3)}}} = \sqrt{\frac{(R_1+R_3)}{2R_1}}$$

Note: in practice more interested in Δv than $\dfrac{V_{max}}{v_1}$

: HW deals with Mars — not a circular orbit
things get complex so I suggest computer
calculation not calculator — example hohmann.cal
is online.

Oberth Effect - Power = F·v so rocket engines have more power if moving fast -

numbers for trip to Mercury @ perihelion

burn to transfer: 9.4 km/s   burn to join Mercury: 7.5 km/s

low earth orbit velocity = 7.7 $\frac{km}{s}$ escape velocity = 11.2 km/s

the implication seems to be that to reach transfer orbit we need $(11.2 - 7.7) + 9.4 = 12.9 \frac{km}{s}$

But combine escape + transfer: velocity needed at LEO:

$$\frac{1}{2} m V_{need}^2 \underbrace{- 2 \frac{m}{2} V_{orbit}^2}_{\text{this is } -\frac{\alpha}{r} \text{ expressed in terms of } V_{orbit}} = \frac{1}{2} m V_{trans}^2$$

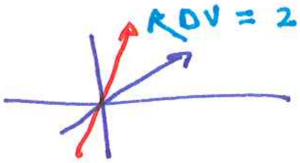
$$V_{need} = \sqrt{V_{trans}^2 + 2 V_{orbit}^2}$$

$$\Delta V = V_{need} - V_{orbit} = \sqrt{V_{trans}^2 + 2 V_{orbit}^2} - V_{orbit}$$

$$= 6.7 \frac{km}{s}$$

← much less than straightforward transfer calculation which did not include escape!

Orbital plane changes - at equator



$\Delta V = 2 V_{orbit} \sin(\theta/2)$

to change from FL to Baikonur ~ 2.4 $\frac{km}{s}$

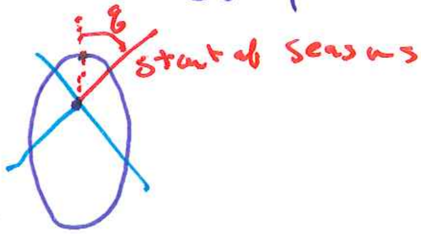
↑ a good fraction of what it takes to go to Moon.

Seasons on Earth (determined by Earth's tilt not orbit) are not of equal length because the Earth orbit is not exactly ~~c~~ circular

2013	Dec 21	
2014	March 20	88.99
	June 21	92.75
	Sept 23	93.65
	Dec 21	89.86

short so sun near perihelin at these times.

The effect depends on eccentricity & start point of seasons compared to perihelim.



The times of these orbital events can be found from Keplers Eq: $u - e\sin u = \omega t$

Mathematica code displays predictions (given e & θ) and actual — adjust until match.

Thus did pre BC greek astronomers know e of Earth even though they thought Sun orbited Earth ("geocentric")