1. By direct integration of current sources, we have shown (Biot-Savart Law):
   (a) For an infinitely long cylindrical shell (radius R) carrying a total current I parallel to the cylinder's axis: $\mathbf{B} = 0$ inside and $\mathbf{B} = \mu_0 I \hat{\boldsymbol{\phi}}/2\pi r$ outside.
   (b) For an infinitely long solenoid (radius R) carrying surface current $\mathbf{j} = NI\hat{\boldsymbol{\phi}}$ around the circumference: $\mathbf{B} = 0$ outside and $\mathbf{B} = \mu_0 NI\hat{\boldsymbol{z}}$ inside.
   (c) For the infinite $z = 0$ plane with a uniform surface current $\mathbf{j} = K\hat{\boldsymbol{y}}$, $B_x = \mu_o K/2$ for $z > 0$ and $B_x = -\mu_0 K/2$ for $z < 0$

   For each of the above provide an amperian loop argument that reproduces these correct results.

2. For a circular current loop (radius R) sitting in the xy plane with center at the origin, we have found $\mathbf{B}$ on the z-axis to be:

$$B_z = \frac{\mu_0 I R^2}{2\sqrt{z^2 + R^2}^3}$$

   Consider the 'loop' that travels from $z = -\infty$ to $z = +\infty$ along the z-axis and then loops around at $r = \infty$ where $B \propto r^{-3}$ so even over a half-circumference of πr, the return loop integral is 'vanishingly small'. This loop encloses the current loop, so Ampere promises:

$$\int_{-\infty}^{+\infty} B_z dz = \mu_0 I$$

   Show this.

3. For each of the following vector potentials $\mathbf{A}$, calculate the corresponding $\mathbf{B}$ and additionally confirm $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$.
   (a) $\mathbf{A} = Bx\hat{\mathbf{j}}$
   (b) $\mathbf{A} = \frac{1}{2}\ \hat{\mathbf{k}} \times \mathbf{r}$ (in cylindrical coordinates this is $A_\phi = r/2$ (explain why!) or use rectangular)
   (c) $\mathbf{A} = -\frac{\mu_0 I \ln(r)}{2\pi}\ \hat{\mathbf{k}}$ (cylindrical coordinates)
   (d) Note that the below $\mathbf{A}$ (in cylindrical coordinates) is continuous

$$\mathbf{A} = \begin{cases} \dfrac{\mu_0 N I r}{2}\ \hat{\boldsymbol{\phi}} & \text{for: } r < R \\[2ex] \dfrac{\mu_0 N I R^2}{2r}\ \hat{\boldsymbol{\phi}} & \text{for: } r > R \end{cases}$$

   (e)
$$\mathbf{A} = \frac{\mu_0}{4\pi}\ \frac{m\mathbf{k} \times \mathbf{r}}{r^3}$$
   In spherical coordinates this is $A_\phi = \mu_0 m \sin\theta/(4\pi r^2)$.

4. Find the sold angle subtended by the Moon as seen from Earth. Prove/find the general formula for the the solid angle subtended by a sphere as a function of the distance from the center of that sphere. (Think about Neil Armstrong traveling to and landing on the Moon.)