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Curie point of ferromagnets

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Abstract. A student experiment is described concerning properties of ferromagnets near the Curie point. Gadolinium provides a good opportunity to determine the magnetic susceptibility above the transition point and to evaluate the critical exponent of the susceptibility. The temperature dependence of the electrical resistance of nickel at enhanced frequencies shows the influence of the magnetic properties of the sample on the skin depth. From the measurements, the phase transition is clearly seen and the temperature dependence of the magnetic permeability below the Curie point is available. Data on the resistance at an enhanced frequency reveal the phase transition in a nickel-based alloy. These items may be added to related student experiments described previously.

Zusammenfassung. Ein Studentenexperiment in Beziehung der Eigenschaften eines Ferromagnetikums nahe des Curie-Punktes ist beschrieben. Gadolinium bietet eine gute Gelegenheit, die magnetische Suszeptibilität über der Übergangstemperatur auszuwerten und den kritischen Exponent der Suszeptibilität berechnen. Die Temperaturabhängigkeit des elektrischen Widerstandes des Nickels in höheren Frequenzen demonstriert den Einfluß der magnetischen Eigenschaften der Probe auf die Skinschicht. Von den Messungen, der Phasenübergang ist klar und die Temperaturabhändigkeit der magnetischen Permeabilität unter der Curie-Temperatur kann berechnet werden. Die Daten des Widerstandes in höheren Frequenzen enthüllen den Phasenübergang in einer Nickellegierung. Vorher beschriebene Studentenexperimente können mit obigen Punkten ergänzt werden.

1. Introduction

At temperatures above the Curie temperature, T_C, a ferromagnet loses its intrinsic magnetization. The transition from the magnetic to the nonmagnetic state is a second-order phase transition. Many properties of a ferromagnet manifest singularities near the transition point, usually with a power-like dependence (e.g. Fisher 1965, 1967, Heller 1967, Kadanoff *et al* 1967). The exponent in such a dependence is referred to as the critical exponent or critical index. For instance, the temperature dependence of the specific heat, C, follows the equation

$$C = A + B|T - T_C|^{-\alpha} \quad (1)$$

where A and B are constants, generally different below and above the Curie point, and α is termed the critical exponent of the specific heat.

Below the transition point, the spontaneous magnetization of a ferromagnet, M, obeys the relation

$$M = A(T_C - T)^{\beta} \quad (2)$$

where β denotes the critical exponent of the spontaneous magnetization.

Close to the Curie point, the magnetic susceptibility, χ, is given by

$$\chi = A|T - T_C|^{-\gamma} \quad (3)$$

where γ is the critical exponent of the susceptibility.

One of the most important objectives of the theory and experiment is to determine the critical exponents. The mean-field theory predicted values of the critical exponents as follows: $\alpha = 0, \beta = 0.5, \gamma = 1$. However, observed dependences appeared to be quite different. The modern theory of critical phenomena can be found elsewhere (Ma 1976, Patashinskii and Pokrovskii 1979, Stanley 1983, Domb 1996). In 1982, Kenneth G Wilson was awarded the Nobel Prize in Physics ‘for his theory for critical phenomena in connection with phase transitions’ (see Wilson 1993). The theory is very complicated but Maris and Kadanoff (1978) have shown how Wilson’s theory could be incorporated into an undergraduate course of statistical physics. At present, the accepted theoretical values are as follows: $\alpha = -0.115 \pm 0.009, \beta = 0.3645 \pm 0.0025, \gamma = 1.386 \pm 0.004$ (Domb 1996). Generally, experimentally determined critical exponents are, to within experimental errors, in agreement with these theoretical values. The theory also predicts some relations between the critical exponents, e.g. a relation fitting the above critical exponents:

$$\alpha + 2\beta + \gamma \geq 2. \quad (4)$$

Both sets of critical exponents given above fulfil this relation even as an equality. The parameters and critical exponents describing the phase transition in a ferromagnet are similar to those for the critical point of the liquid–vapour coexistence curve. The

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magnetization is analogous to the difference between the density of the fluid and the critical density, whereas the magnetic susceptibility is analogous to the isothermal compressibility of the fluid.

Several student experiments related to the Curie point have already been described. Two of these employ the mutual inductance technique for determining the Curie point of ferromagnets (Edgar and Quilty 1993, Fisher and Franz 1995). Other experiments include measurements of the electrical resistivity of nickel (Kamal *et al* 1983, Fox *et al* 1986, Sullivan *et al* 1987) and observations of changes in the resonant frequency of an LC circuit when the sample placed inside the inductor undergoes the transition (Fox *et al* 1986).

Two items described below may be added to the student experiments reported earlier, namely: (i) the magnetic susceptibility of a ferromagnet above the Curie point; (ii) the influence of the magnetic properties of a ferromagnet on the skin depth when an AC current passes through the sample.

In the first part, the magnetic susceptibility of gadolinium is measured above the Curie point. When approaching the Curie point, the magnetic susceptibility depends on the quality of the sample and on the approach to the transition point. In magnetic measurements, this approach was limited by the value of $|T - T_C|/T_C$ in the range 10^{-4}–10^{-3}. With such a proximity, the magnetic susceptibility above the Curie point may be much smaller than that in the ferromagnetic phase (e.g. Heller 1967, Herzum *et al* 1974). Gadolinium is rather an exception that possesses a high magnetic susceptibility above the Curie point. It allows one to perform the measurements over a wide temperature interval in the nonmagnetic phase and to evaluate the critical exponent of the magnetic susceptibility.

In the second part, the temperature dependence of the electrical resistance of nickel is measured using a DC current and an AC current of enhanced frequency. The transition to the nonmagnetic state becomes evident owing to a change in the skin depth. Moreover, the temperature dependence of the magnetic permeability of the sample can be evaluated from the ratio of the electrical impedance of the sample to its DC resistance. Similar measurements reveal the phase transition in a nickel-based alloy.

2. Magnetic susceptibility of gadolinium

The aim of this experiment is to determine the magnetic susceptibility of gadolinium above the Curie point. Gadolinium provides a very convenient temperature of the phase transition and a relatively large interval where the magnetic susceptibility can be measured.

For the measurements, an E-shaped transformer is employed. The core and the coils have been purchased from PASCO, catalogue numbers SF-8615, SF-8610 and SF-8611. The primary winding ($n_1 = 800$) is connected to a low-frequency oscillator (figure 1). The

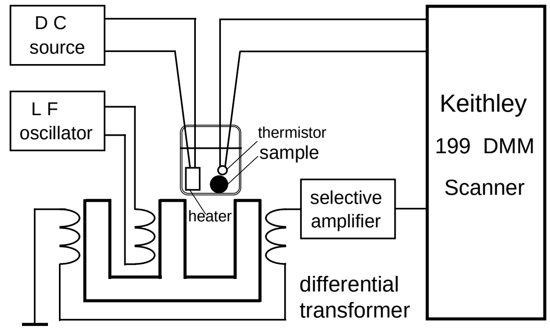

Figure 1. Set-up to measure the magnetic susceptibility of gadolinium.

AC current creates a magnetic flux in the magnetic core. Two secondary windings ($n_2 = 400$) are connected in opposition; thus without a sample no voltage appears at the transformer's output. A small additional coil connected in series with the secondary windings is used to finely balance the transformer. This coil is placed near the core to obtain the necessary compensation voltage. With a magnetic sample in the gap of the core, the magnetic flux through this part of the core increases. This causes an increase in the voltage induced in the corresponding secondary winding. To avoid the influence of eddy currents in the sample, the operating frequency is reduced to 30 Hz.

A spherical gadolinium sample (99.9%), 5 mm in diameter, is placed into a small glass container filled with oil to reduce temperature gradients and to prevent oxidation of the sample. A small thermistor (Fenwal Electronics, model UUA 35J3, 5000 Ω at 25 °C) is attached to the sample. Its resistance, R, relates to the absolute temperature as $T = 3895/\ln(R/0.0106)$. A bifilar electrical heater is also inserted in the oil. The heater and the thermistor contain no magnetic parts.

The internal magnetic field in the sample, H_i, should be calculated taking into account the demagnetizing factor, α. Thus, the magnetization is $M = \chi H_i = \chi(H_e - \alpha M)$, where H_e is the external field. Hence, $M = H_e/(\alpha + 1/\chi)$. The output voltage of the differential transformer is proportional to the magnetization of the sample: $V = KM = KH_e/(\alpha + 1/\chi)$. Below the Curie point, $1/\chi \ll \alpha$. This assumption allows one to determine the external magnetic field: $H_e = \alpha V_0/K$, where V_0 is the output voltage below T_C. For a sphere, $\alpha = \frac{1}{3}$ (SI units). Hence, $\chi = 3V/(V_0 - V)$. The absolute value of the magnetic susceptibility is thus available (Heller 1967).

The output voltage of the differential transformer is fed to an amplifier, PAR model 124A. The amplifier operates in the selective mode and is tuned to the signal frequency. The amplified voltage is monitored by an oscilloscope. For initial balancing of the transformer, it is more convenient to observe the Lissajous pattern on the oscilloscope's screen. For this purpose, the voltage drop across a resistor connected in series with

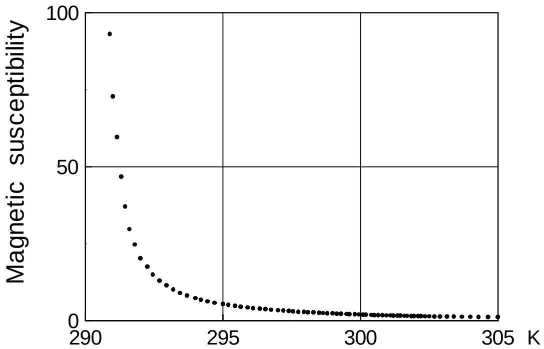

Figure 2. Temperature dependence of the magnetic susceptibility of gadolinium.

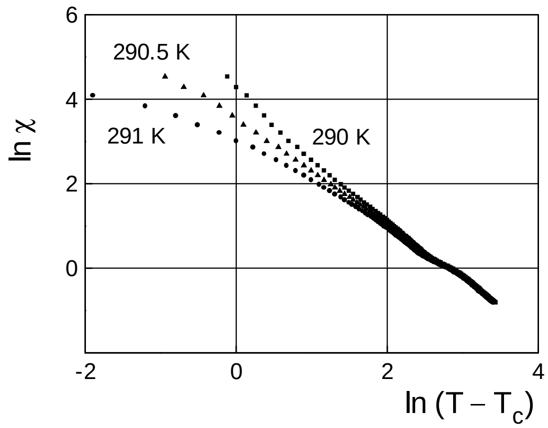

Figure 3. Plot of $\ln \chi$ versus $\ln(T - T_C)$. Determination of γ includes the choice of the transition temperature. The best value of T_C is that which provides a linear dependence over the broadest temperature range.

the primary winding of the transformer is fed to the X-input of the oscilloscope.

The temperature range of the measurements is 285–325 K. Cold water is used to cool the glass with the sample before the measurements. After cooling, the temperature of the sample starts to increase due to heat exchange with the environment. Close to the Curie point, the heating rate should be about 2–3 K min^{-1}. The low heating rate is necessary to reduce temperature gradients in the sample and obtain a sharp transition. In the vicinity of the transition, the data are taken every 0.2–0.3 K. The electrical heater inside the glass heats the sample above room temperature.

The resistance of the thermistor and the output voltage of the amplifier are measured and stored by the data-acquisition system VIEWDAC with the Keithley 199 DMM/Scanner. The program ORIGIN serves to process the data. The magnetic susceptibility is presented as a function of temperature (figure 2). The critical exponent is available from the plot of $\ln \chi$ versus $\ln(T - T_C)$. This plot and the value of γ depend on T_C. In our case, the plot is linear over the broadest temperature range when T_C is taken as 290.5 K, while the values 290 and 291 K lead to significant curvature of the plot (figure 3). The critical exponent γ equals 1.25. A more rigorous treatment should include determinations of both T_C and γ by the least-squares method.

The magnetic susceptibility of gadolinium above the Curie point has been reported in many papers (e.g. Graham 1965, Wantenaar *et al* 1984, Hargraves *et al* 1988). For gadolinium, experimentally obtained γ values appeared to be somewhat smaller than those for other ferromagnets.

3. Skin effect in nickel

Near the Curie point, the electrical resistivity of a ferromagnetic metal manifests a specific behaviour that becomes clearer when the temperature derivative of the resistivity is measured directly (Kraftmakher 1967). This derivative behaves like the specific heat (e.g. Kawatra and Budnick 1972). The Curie point can thus be identified as a point where the slope of the temperature dependence of the electrical resistivity has a maximum. However, now we employ quite another approach.

When an AC current passes through a sample, the skin depth depends on the magnetic permeability of the sample, μ, its electrical resistivity, ρ, and on the frequency of the current. The skin depth is given by $\delta = (2\rho/\omega\mu_0\mu)^{1/2}$ (SI units), where $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} is the permeability of free space, and ω is the angular frequency of the current. The theory of the skin effect can be found elsewhere (e.g. Stratton 1941, Scott 1966, Landau and Lifshitz 1984). In the ferromagnetic phase, the skin depth decreases strongly because of the high magnetic permeability of the sample. Measurements of the resistance with an AC current of a proper frequency provide an additional opportunity to observe the transition to the nonmagnetic state (e.g. Kraftmakher and Pinegina 1974). Moreover, the magnetic permeability of the sample can be evaluated as shown below.

The ratio of the impedance of a round wire, $\boldsymbol{Z}$, to its DC resistance, R, is expressed through Bessel functions as follows (e.g. Irving and Mullineux 1959, Relton 1965):

$$\boldsymbol{Z}/R = kaI_0(ka)/2I_0'(ka) \qquad (5)$$

where $k^2 = \mathrm{i}\omega\mu_0\mu/\rho$.

The ratio of Z, the modulus of the impedance, to the DC resistance R is thus a function of the quantity α/δ (figure 4). Numerical data for this plot were taken from Abramowitz and Stegun (1965). The frequency of 10 kHz was chosen to neglect the skin effect in the nonmagnetic state and obtain a significant difference between Z and R in the magnetic phase.

Direct measurements of the absolute values of Z and R are accompanied by unavoidable errors, so that

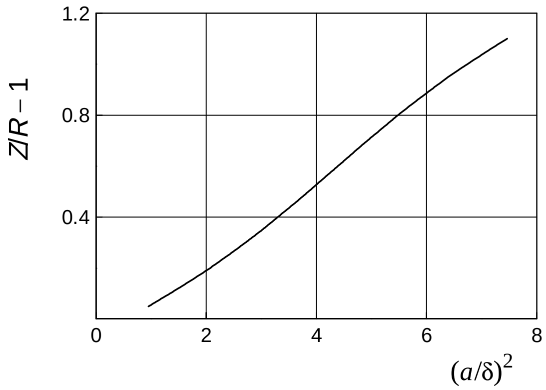

Figure 4. Theoretical dependence of the quantity $Z/R - 1$ versus $(a/\delta)^2$.

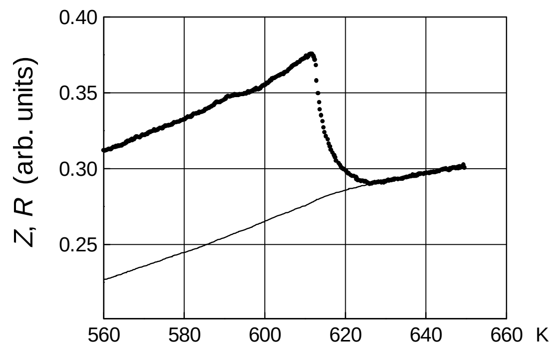

Figure 5. DC resistance (——) and impedance of the nickel sample at 10 kHz (•) fitted to the resistivity at 400 °C.

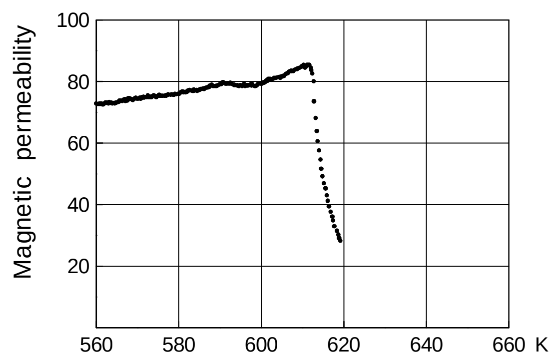

Figure 6. Magnetic permeability of nickel evaluated from the measured Z/R ratio.

the values above the Curie point may appear to be somewhat different. A good alternative approach is to fit both quantities, Z and R, to the resistivity of nickel at a selected temperature above the Curie point (e.g. 0.31 $\mu\Omega$ m at 400 °C). In this case, one immediately obtains data on the DC resistivity over the whole temperature range that will be necessary for calculations of the magnetic permeability of the sample. Since a, ω and ρ are now known, the magnetic permeability is available from the determined Z/R ratio. In the range $1 < a/\delta < 3$, the quantity $Y = (a/\delta)^2 = a^2\omega\mu_0\ \mu/2\rho$ can be taken as a function of $X = Z/R - 1$ as follows:

$$Y = 0.53 + 8.7X - 5.7X^2 + 3.2X^3. \qquad (6)$$

The magnetic permeability of the sample is available from the relation

$$\mu = 2Y\rho/a^2\omega\mu_0. \qquad (7)$$

A nickel wire (99.99%), 1 mm in diameter, is placed in an oven and connected to the outputs of a DC source and an oscillator. A DC current and an AC current pass through the sample simultaneously. The four-probe technique is used to measure the DC and AC voltage drops across the central portion of the sample, of about 1 cm long. Nickel wires welded to the sample serve as the potential probes. The DC voltage is fed to the data-acquisition system. The AC component is fed, through a transformer, to an amplifier, PAR model 124A. It operates as a selective amplifier tuned to the frequency of the current. Since the AC voltage to be measured is small, of the order of 1 mV, it is very important to reduce the magnetic coupling between the current leads and the potential leads. The areas of both loops should be kept to a minimum. The amplified AC voltage is monitored with an oscilloscope.

To increase the thermal inertia and avoid temperature gradients, the sample is placed in an aluminium block inside the oven. The measurements are carried out during cooling of the oven. After heating up to 400 °C, the heating current is changed to obtain a desired cooling rate near the Curie point, of about 3–5 K min^{-1}. The temperature of the sample is measured with a thermocouple. For the iron versus (Cu + 43% Ni) thermocouple used, the temperature relates to the thermal EMF U, expressed in mV, as $T = 6.3 + 17.89U + 0.006U^2$. This equation is valid in the range 300–400 °C. The data-acquisition system stores the EMF of the thermocouple, the DC voltage drop across the central portion of the sample and the amplified AC voltage for processing the data by the program ORIGIN.

From the change of the impedance, the phase transition is quite evident (figure 5). The evaluated magnetic permeability of the sample appeared to be in reasonable agreement with the available data (e.g. Herzum *et al* 1974). The increase in the permeability close to the Curie point is clearly seen (figure 6).

4. Phase transition in a nickel-based alloy

A nickel-based alloy, monel 400 (Ni65/Cu33/Fe2), is used in the measurements. The sample, 1 mm in diameter, is placed in a metallic can provided by an electrical heater and a small platinum thermometer (100 Ω at 0 °C). In the range 250–320 K, the absolute

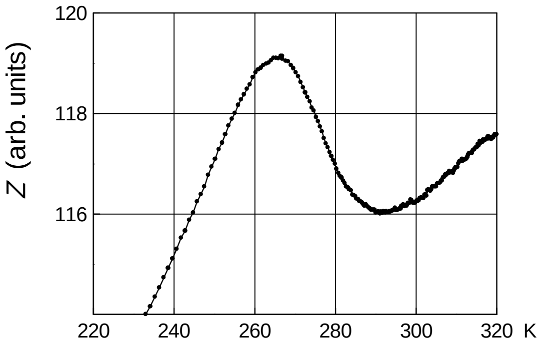

Figure 7. Impedance of the monel sample at 30 kHz.

temperature relates to the resistance of the thermometer, R, as $T = 31.3 + 2.32R + 9.85 \times 10^{-4}R^2$.

The measurements at 30 kHz are carried out using the four-probe technique. Thin copper wires soldered to the sample serve as the potential probes. First, the metallic can containing the sample and the thermometer is cooled by placing it into a Dewar vessel with liquid nitrogen. Then the can is raised above the liquid and the temperature of the sample starts to increase. The electrical heater heats the sample to above room temperature. Close to the transition, the heating rate should be about 3–5 K min^{-1}. The data obtained are stored by the data-acquisition system. The change in the resistance measured at the high frequency shows the phase transition (figure 7). The magnetic permeability of the alloy is smaller than that of pure nickel. The frequency necessary to observe the skin effect in the alloy is therefore higher than that in pure nickel.

All the materials for the measurements (gadolinium, nickel and monel) were purchased from the Goodfellow Company.

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