Electret

Unlike a dielectric (where the polarization is proportional to $\mathbf{E}$) an *electret* can be created with a specified unchanging polarization $\mathbf{P}(\mathbf{r})$. For example, in the past we've considered a sphere with uniform polarization. Now consider an infinite cylinder (radius R) electret where (in cylindrical coordinates) the polarization points radially outward with magnitude proportional to $\mathbf{r}$: $\mathbf{P} = \alpha\mathbf{r}$, where α is constant.

A. What are the units of α?

B. Find ρ_b, the bound volume charge density, and σ_b, the bound charge on the surface of the cylinder. (The results should be constants.) Show that the total bound charge is zero (as it must be). Show that your formulas for ρ_b & σ_b produce the appropriate units.

C. Using Gauss' Law for E (which cares about all—including bound—charge) find the electric field both for $r < R$ and $r > R$. Record the direction of $\mathbf{E}$. Note that electrically the electret seems to be in a high energy state as the molecular dipoles are not aligned with $\mathbf{E}$.

D. Using the definition: $\mathbf{D} \equiv \epsilon_0\mathbf{E} + \mathbf{P}$, find $\mathbf{D}$ both for $r < R$ and $r > R$. (You should find that $\mathbf{D} = 0$.) This should come as no surprise as coaxial Gaussian cylinders will enclose zero free charge. (There is no free charge in this problem.) Comment: Recall that in the uniformly polarized sphere, D was not zero any where and, in that case also, there was no free charge. (I.e., an absence of free charge does not guarantee that $D = 0$, but that happens in this problem.)

E. If you calculate the energy density $\frac{1}{2}\mathbf{E} \cdot \mathbf{D}$ you will of course get zero, but I claim that if you have electric fields you must have energy. The proof of our energy density formula starts in section 6-2. Report how the assumptions of this problem violate the hypotheses of that 'proof'. Report exactly which sentences in the textbook fail to match the conditions of this problem.