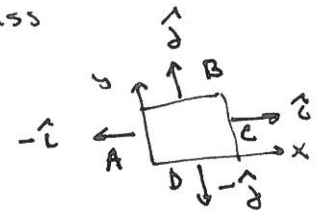
class 2

Gauss

1) 

top normal: $\hat{k}$ E

bottom normal $-\hat{k}$ F

Six faces : A – F

A: $x = 0$ D: $y = 0$

B: $y = 1$ E: $z = 1$

C: $x = 1$ F: $z = 0$

$\vec{E}$ has no $\hat{z}$ component so $\vec{E}\cdot\hat{n} = 0$ on E & F

A: $x=0 \Rightarrow \vec{E} = (0, y, 0)$ $\hat{n} = -\hat{i}$ $\vec{E}\cdot\hat{n} = 0$

B: $y=1 \Rightarrow \vec{E} = (x, 1, 0)$ $\hat{n} = \hat{j}$ $\vec{E}\cdot\hat{n} = 1$

C: $x=1 \Rightarrow \vec{E} = (1, y, 0)$ $\hat{n} = \hat{i}$ $E\cdot\hat{n} = 1$

D: $y=0 \Rightarrow \vec{E} = (x, 0, 0)$ $\hat{n} = -\hat{j}$ $\vec{E}\cdot\hat{n} = 0$

only non-zero $\vec{E}\cdot\hat{n}$ are B & C, for both integrand = 1

$\int \vec{E}\cdot\hat{n}\, dA = E\cdot n\, A = 1 \Rightarrow B + C = 2$ same

$\nabla\cdot E = 2$ $\int \nabla\cdot E\, dV = \nabla\cdot E \int dV = \nabla\cdot E \cdot V = 2$

2) $\vec{E}$ has no $\hat{y}$ or $\hat{z}$ component $\Rightarrow$ only A & C possible non zero

on A $x = 0 \Rightarrow \vec{E} = \vec{0} \Rightarrow \int \vec{E}\cdot\hat{n}\, dA = 0$

on C $x = 1 \Rightarrow \vec{E} = (1, 0, 0) \Rightarrow \int E\cdot n\, dA = 1$ same

$\nabla\cdot E = 2x$ $\int_0^1\int_0^1\int_0^1 2x\, dx\,dy\,dz = \int_0^1 2x\, dx = x^2\Big|_0^1 = 1$

1-20 $\vec{\nabla}(\vec{A}\cdot\vec{r}) = \vec{\nabla}(A_x x + A_y y + A_z z)$

$= \left(\frac{\partial}{\partial x}(\ ), \frac{\partial}{\partial y}(\ ), \frac{\partial}{\partial z}(\ )\right)$

$= (A_x, A_y, A_z) = \vec{A}$

Longer way: 1-1-6: $\vec{\nabla}(\vec{F}\cdot\vec{G}) = (F\cdot\nabla)\vec{G} + (G\cdot\nabla)\vec{F} + \vec{F}\times\overrightarrow{\nabla\times G} + \vec{G}\times\overrightarrow{\nabla\times F}$

derivatives $\vec{A} = 0$ as constant

$\overrightarrow{\nabla\times r} = 0$

$\Rightarrow \vec{\nabla}(\vec{A}\cdot\vec{r}) = (A\cdot\nabla)\vec{r} = \vec{A}$

in class proof