341 – class 24

The entire *xy* plane consists of a uniform current moving in the +*y* direction. Through any small segment Δx on the *x* axis, a current $\Delta I = j\,\Delta x$ is flowing in the +*y* direction. This surface current density produces magnetic fields just as in Eq. 8-27 or 8-29, except, instead of $I d\mathbf{l}$ or $\mathbf{J}\, dV$, you use $\mathbf{j}\, da$. In this problem, the vector surface current is: $\mathbf{j}=\langle 0,K,0\rangle$, where K is some constant. You seek the magnetic field at a point a distance d above the sheet due to all of this current flowing in the *y* direction. Proceed as usual: $\mathbf{r}'=(x',y',0)$, $\mathbf{r}=(0, 0, d)$, in rectangular coordinates symmetry should show all but one component of $\mathbf{B}$ is zero. The easiest way to calculate that one non-zero integral is in polar coordinates.

SO...integrate to find the magnetic field above this current sheet.

$$\vec{j} \times (\vec{r}-\vec{r}') = \begin{vmatrix} i & j & k \\ 0 & K & 0 \\ -x' & -y' & d \end{vmatrix} = (Kd,\ 0,\ Kx')$$

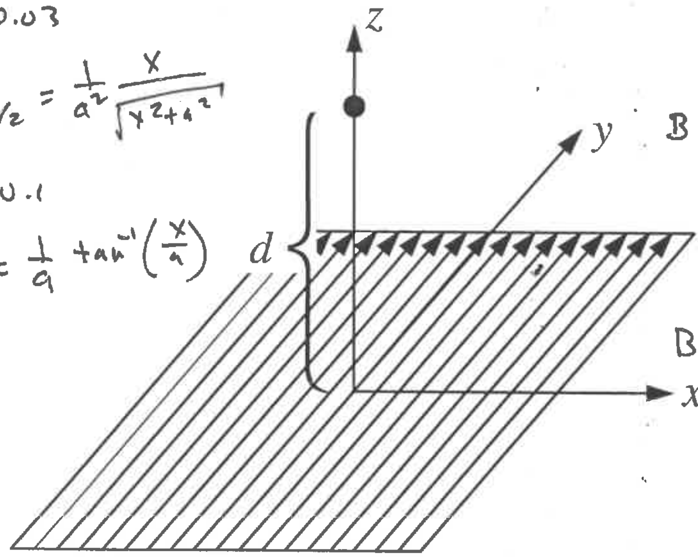
Integrals used:

Dwight 200.03

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

Dwight 120.1

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$



$$B = \frac{\mu_0}{4\pi}\iint_{-\infty}^{\infty} \frac{(Kd, 0, Kx')\, dx'\, dy'}{(x'^2+y'^2+d^2)^{3/2}}$$

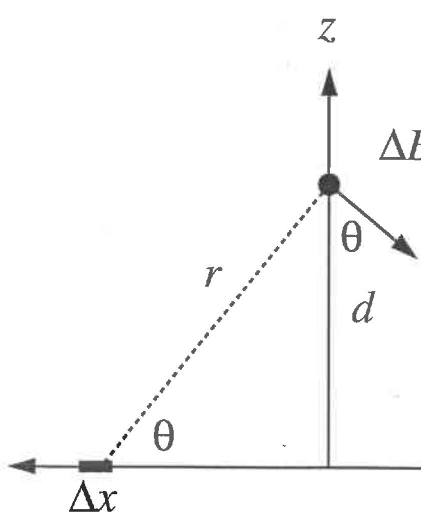
the $B_z = 0$ as $\displaystyle\int_{-\infty}^{\infty} \frac{x'\, dx'}{(x'^2+y'^2+d^2)^{3/2}} = 0$

$$B_x = \frac{\mu_0 K d}{4\pi}\iint \frac{dx'\, dy'}{(x'^2+y'^2+d^2)^{3/2}}$$

$$= \frac{\mu_0 K d}{4\pi}\int \frac{2}{(x'^2+d^2)}\, dx'$$

$$= \frac{\mu_0 K}{2}$$

Additionally calculate $\mathbf{B}$ by dividing up the sheet into an infinite number of 'wires' parallel to the *y* axis. The current of such a wire would be: $\Delta x K$, and each such wire makes a circular magnetic field $\Delta B = \mu_0 \Delta I / 2\pi r$. These magnetic fields point in different directions: for example, for a wire at the far left ($x=-\infty$), $\mathbf{B}$ points basically down whereas for a wire at the far right ($x=+\infty$), $\mathbf{B}$ points basically up. We need to add up (integrate) the magnetic fields from all the 'wires' to find the net magnetic field at the requested point. Do notice that the positive component B_z produced by a wire at $x>0$ will be exactly cancelled by a negative value for B_z from the diametrically opposite wire at $x<0$. Because of this symmetry, the sheet's net magnetic field will point in the *x* direction, and so we need only add the *x* component of $\mathbf{B}$ (which requires using $\sin\theta$).



$$\Delta B_x = \frac{\mu_0 \Delta I}{2\pi r}\sin\theta = \frac{\mu_0 K \Delta x}{2\pi r}\frac{d}{r}$$

$$B_x = \frac{\mu_0 K d}{2\pi}\int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)} = \frac{\mu_0 K}{2}$$