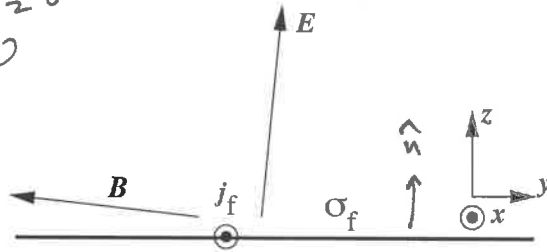


The reason for these sketches is so you have an idea what to expect

$$E = \frac{\sigma}{2\epsilon_0} = 5000 \text{ V/m}$$

The  $z = 0$  plane is the boundary between two materials: the region of space with  $z > 0$  is vacuum, the region with  $z < 0$  has  $\epsilon = 4\epsilon_0$  and  $\mu = 1000\mu_0$ . The boundary carries a surface charge density of  $\sigma_f = 8.85 \times 10^{-8} \text{ C/m}^2$  and a surface current (flowing in the  $x$  direction) of  $j_f = 10^3 \text{ A/m}$ . On the vacuum side of the boundary  $\vec{E} = 10^3\hat{j} + 10^4\hat{k} \text{ V/m}$ , and  $\vec{B} = -10^{-4}\hat{j} + 10^{-5}\hat{k} \text{ T}$ . Start by making a sketch showing the directions of the  $\vec{E}$  (on both sides) that would result from  $\sigma_f$  in a vacuum. Show on your sketch the direction you are taking for the boundary's normal. Make a sketch showing the direction of  $\vec{B}$  (on both sides) that would result from  $\vec{j}_f$  in a vacuum. Show on your sketch the direction you are taking for the boundary's tangent. Now find  $\vec{E}$  and  $\vec{B}$  inside the material. ( $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$ ,  $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ )

$$B = \frac{\mu_0}{2} j = 2\pi \cdot 10^{-7} \cdot 10^3 = 2\pi \cdot 10^{-4} = 6.28 \times 10^{-4} \text{ T}$$



$E_x$  unchanged so  $E_y = 10^3 \text{ V/m}$

$$D_z^+ - D_z^- = \sigma_f \Rightarrow D_z^- = D_z^+ - \sigma_f = \epsilon_0 \cdot 10^4 - 8.85 \times 10^{-8}$$

$$= 0 = \epsilon E_z^-$$

$$\vec{E}^- = 10^3 \hat{j} + 0$$

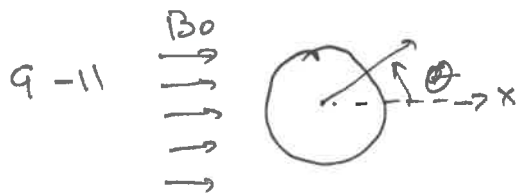
$B_x^+ : H_x^+$  zero - so then  $B_x^- : H_x^-$  are zero.

$$H_y^- - H_y^+ = j_f ; H_y^- = H_y^+ + j_f = \frac{-10^{-4}}{\mu_0} + 10^3 = \left(1 - \frac{1}{4\pi}\right) 10^3$$

$$B_y^- = 1000 \mu_0 H_y^- = 10^6 (4\pi - 1) \cdot 10^{-7} = (4\pi - 1) 10^{-1} \text{ T} = 1.16 \text{ T}$$

$$B_z^- = B_z^+ = 10^{-5} \text{ T}$$

p341 - class 30 - 9-11 & BC.pdf



problem is even about  $x$  axis  $\theta$

inside:  $\mu_0 \frac{I}{r^n}$

outside:  $-\mu_0 \frac{I}{r^n}$  - otherwise  $\mu_0 r^n$  case

$$\phi_{in} = \sum A_n r^n \cos n\theta$$

$$\phi_{out} = -H_0 r \cos\theta + \sum \frac{C_n}{r^n} \cos(n\theta)$$

problem & experience suggests only need  $n=1$

$$\phi_{in} = A_1 r \cos\theta$$

$$\phi_{out} = -H_0 r \cos\theta + \frac{C_1}{r} \cos\theta$$

$$H_t \text{ continuous} \Rightarrow \phi_{in} = \phi_{out} \Big|_{r=R} \Rightarrow A_1 R = -H_0 R + \frac{C_1}{R}$$

$$\text{or } A_1 = -H_0 + \frac{C_1}{R^2}$$

$$B_r \text{ continuous} \Rightarrow k \partial_r \phi_{in} = \partial_r \phi_{out} \Big|_{r=R} \Rightarrow k A_1 = -H_0 - \frac{C_1}{R^2}$$

$$A_1 = -H_0 + (-H_0 - k A_1)$$

$$(k+1) A_1 = -2 H_0 \Rightarrow A_1 = \frac{-2}{k+1} H_0$$

$$A_1 = \frac{-2}{k+1} H_0 = -H_0 + \frac{C_1}{R^2} \Rightarrow \frac{k-1}{k+1} H_0 = \frac{C_1}{R^2}$$

check: if  $k=1$   $C_1=0$  &  $A_1 = -H_0$  i.e. no effect

$$H_{inside} = -\nabla \phi_{inside} = \frac{2}{k+1} H_0 \Rightarrow B_{inside} = \frac{2k}{k+1} \underbrace{\mu_0 H_0}_{B_{outside}}$$

remark: if  $k \rightarrow \infty$ ; cylinder has "sucked in" enough flux to double the  $B$  inside cf. outside.