phys 341 — class 37 #1 } Ch16.pdf
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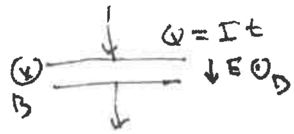
#1) 

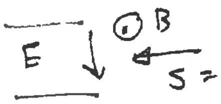
$Q = It$

$E = \frac{Q/A}{\varepsilon_0} = \frac{It}{A\varepsilon_0}$

$J_D = \varepsilon_0 \partial_t E = \frac{I}{A\varepsilon_0}$ ← displacement current density

Ampere: $\oint B \cdot dl = B \cdot 2\pi r = \mu_0 I_{enc} = \mu_0 \frac{I}{A} \pi r^2$

$B = \mu_0 \frac{I}{A\,2} r \Big|_{r=a} = \frac{\mu_0 I a}{2A} = \frac{\mu_0 I}{2\pi a}$ ($A = \pi a^2$)



$S = \frac{1}{\mu_0} E \times B = \frac{It}{A\varepsilon_0} \frac{I}{2\pi a}$

Surface area: circumference of capacitor: $2\pi a \cdot d$

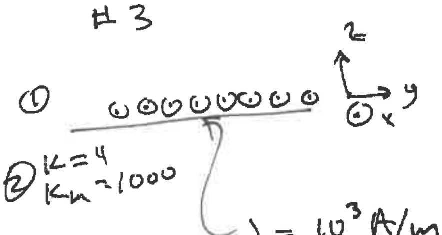
total energy flow in $= \frac{I^2 t}{A\varepsilon_0 2\pi a} \cdot 2\pi a \cdot d = \frac{I^2 t}{A\varepsilon_0} d$

energy stored in capacitor: $\underbrace{\pi a^2 \cdot d}_{\text{volume}} \frac{\varepsilon_0}{2} E^2 = \pi a^2 d \frac{\varepsilon_0}{2}\left(\frac{It}{A\varepsilon_0}\right)^2$

$= d \frac{I^2 t^2}{2A\varepsilon_0}$

time derivative of stored energy $= \frac{d I^2 t}{A\varepsilon_0}$ same

#3



in region ① $\vec{E} = 10^3 \hat{y} + 10^4 \hat{k}$

$\vec{B} = -10^{-4} \hat{y} + 10^{-5} \hat{k}$

conservation of $E_t \Rightarrow$ ② $E_y = 10^3$

$B_n \Rightarrow$ ② $B_z = 10^{-5}$

note: $\varepsilon_0 = 8.85 \times 10^{-12}$

$J = 10^3$ A/m

$\sigma = 8.85 \times 10^{-8} \frac{C}{m^2}$

$\varepsilon_0 {}^{①}E_z - 4\varepsilon_0 {}^{②}E_z = \sigma = 8.85 \times 10^{-8}$

$\Rightarrow {}^{①}E_z - 10^4 = 4\,{}^{②}E_z$

$\frac{1}{4}(10^4 - 10^4) = {}^{②}E_z = 0$

${}^{②}H_y \Delta y - {}^{①}H_y \Delta y = j \Delta y$

$\frac{1}{1000\mu_0}$ ${}^{②}B_y + \frac{1}{\mu_0} 10^{-4} = 10^3$

${}^{②}B_y = 1000\mu_0 \left(10^3 - \frac{1}{\mu_0} 10^{-4}\right) = \mu_0 10^6 - 10^{-4} = 4\pi \cdot 10^{-1} - 10^{-4}$

$= 1.2565$ T

#4) $E = -\left(\frac{pk^2}{4\pi\varepsilon_0}\right)\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}\,\hat{\theta}$

$B = -\left(\frac{p\mu_0\omega k}{4\pi}\right)\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}\,\hat{\phi}$

$\nabla\cdot E = \frac{1}{r\sin\theta}\underbrace{\partial_\theta\left(\sin\theta^2\right)}_{2\sin\theta\cos\theta}\frac{e^{i(kr-\omega t)}}{r} = \frac{2}{r^2}\cos\theta\, e^{i(kr-\omega t)}$

↑ faster fall off

$\nabla\cdot B = \frac{1}{r\sin\theta}\partial_\phi(1)\,\frac{\sin\theta}{r}\, e^{i(kr-\omega t)} = 0$

$\nabla\times E = \hat{\phi}\,\frac{1}{r}\partial_r(rF_\theta) = -\left(\frac{pk^2}{4\pi\varepsilon_0}\right)\frac{\sin\theta}{r}\, ik\, e^{i(kr-\omega t)}\,\hat{\phi}$

$-\partial_t B = -\left(\frac{p\mu_0\omega k}{4\pi}\right)\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}\,\hat{\phi}\,(i\omega)$

Compare coefs: $+\left(\frac{\cancel{p}k^2}{\cancel{4\pi}\varepsilon_0}\right)\cancel{i}\cancel{k} \stackrel{?}{=} +\left(\frac{\cancel{p}\mu_0\omega\cancel{k}}{\cancel{4\pi}}\right)\cancel{i}\omega$

$\frac{k^2}{\varepsilon_0} = \mu_0\omega^2$

$\nabla\times B = \underbrace{\hat{r}\,\frac{1}{r\sin\theta}\partial_\theta(F_\phi\sin\theta)}_{} - \hat{\theta}\,\frac{1}{r}\partial_r(rF_\phi)$

there is no such term in $\vec{E}$ but is $\frac{1}{r^2}$ vs $\frac{1}{r}$
just focus on other term

$= \left(\frac{p\mu_0\omega k}{4\pi}\right) ik\,\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}\,\hat{\theta}$

$\frac{1}{c^2}\partial_t E = \left(\frac{pk^2}{4\pi\varepsilon_0}\right)\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}\,\hat{\theta}\,(+i\omega)$

Compare coefs: $\left(\frac{\cancel{p}\mu_0\cancel{\omega}k}{\cancel{4\pi}}\right)\cancel{i}\cancel{k} \stackrel{?}{=} \frac{\cancel{p}k^{\cancel{2}}}{\cancel{4\pi}\varepsilon_0}\cancel{i}\cancel{\omega}\,\frac{1}{c^2}$

$\mu_0 = \frac{1}{\varepsilon_0}\frac{1}{c^2}$

Conclude:

$\left.\begin{array}{l}\nabla\times E = -\partial_t B\\ \nabla\cdot B = 0\end{array}\right\}$ satisfied

$\left.\begin{array}{l}\nabla\cdot E = 0\\ \nabla\times B = \frac{1}{c^2}\partial_t E\end{array}\right\}$ not satisfied cuz of extra $\frac{1}{r^2}$ term
argue for large r this is small.

c) $\vec{S} = \frac{1}{\mu_0}\overrightarrow{E \times B}$; $\hat{\theta} \times \hat{\phi} = \hat{r}$ so its outward flow

$$= \underbrace{\frac{1}{\mu_0}\left(\frac{pk^2}{4\pi\varepsilon_0}\right)\left(\frac{p\mu_0\omega k}{4\pi}\right)} \frac{\sin^2\theta}{r^2} \cos^2(kr-\omega t)$$

$$\frac{p^2 c^2 k^3 \omega \mu_0}{(4\pi)^2} \underset{k=\frac{\omega}{c}}{\uparrow}{=} \frac{p^2 \omega^4 \mu_0}{(4\pi)^2 c}$$

area thru which goes $= r^2 d\Omega$ & $\langle \cos^2 \rangle = \frac{1}{2}$

$$\frac{dP}{d\Omega} = \frac{1}{2}\frac{p^2\omega^4\mu_0}{(4\pi)^2 c}\sin^2\theta$$ ← so nothing into poles mostly into equator $\Rightarrow \theta = 90°$

to get total power: integrate $\int d\Omega = \int_0^{\pi}\int_0^{2\pi} \sin\theta \, d\theta \, d\phi$

$$\int \sin^2\theta \sin\theta \, d\theta \, d\phi = 2\pi \int_{-1}^{1} (1-c^2)\, dc = 2\pi\left(2 - \frac{2}{3}\right) = \frac{8\pi}{3}$$

$$P = \frac{p^2 \omega^4 \mu_0}{4\pi c \, 3}$$

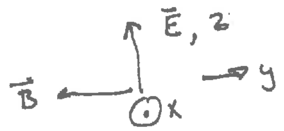
remark: if dipole is moving charge: $p = qz$ $p\omega^2 = qa$

$$P = \frac{q^2 a^2 \mu_0}{4\pi c\, 3} = \frac{q^2 a^2}{4\pi\varepsilon_0 c^3 3}$$

$\mu_0 = \frac{1}{\varepsilon_0 c^2}$

if we "undo" the time average → $P = \underbrace{\frac{q^2}{4\pi\varepsilon_0 c^3}\frac{2}{3}a^2}_{\text{Larmor Eq}}$

d) on x axis: $kr = kx$ − $\hat{\theta}$ direction $= \hat{z}$
− $\hat{\phi}$ direction $= -\hat{y}$



$\overrightarrow{E \times B}$ is going down x axis as expected

#5: clearly $\nabla \cdot E_{new} = 0$ & $\nabla \cdot B_{new} = 0$

$$\nabla \times B_{new} \overset{?}{=} \frac{1}{c^2} \partial_t E_{new}$$

$$\nabla \times \frac{E}{c} \overset{?}{=} \frac{1}{c^2} \partial_t (-cB)$$

$$\nabla \times E = -\partial_t B \checkmark$$

$$\nabla \times E_{new} \overset{?}{=} -\partial_t B_{new}$$

$$\nabla \times (-cB) = -\partial_t E/c$$

$$\nabla \times B = \frac{1}{c^2} \partial_t E \checkmark$$