



1. As described in problem 7-23, the above circuit (called a Wheatstone bridge), can be used to determine an unknown resistance (say, R_5) in terms of known resistances (R_1, R_2, R_4). In brief one of the three known resistors is a variable (adjustable) resistor whose value is adjusted until $I_3 = 0$. If $I_3 = 0$ then $V_A = V_B$ and hence $R_4/R_2 = R_5/R_1$ allowing R_5 to be calculated in terms of the knowns. ‘Null’ methods like this can be extremely sensitive which is explored in problems 7-23 & 7-24 (but neither of those are assigned). Your assignment is just to write down the equations that would be needed to solve this circuit.

(a) Using the usual form of Kirchhoff’s Rules, write down three linear equations that are required to solve for the three currents I_1, I_2, I_3 . Express the results as a matrix equation:

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$$

I won’t again write out such details; instead I’ll write the above as $\overset{\leftrightarrow}{B} \cdot \vec{I} = \vec{A}$ where \leftrightarrow denotes a matrix and an arrow denotes a vector or even more simply: $B \cdot I = A$ where it’s up to you to figure out the quantities involved. (Clearly these vectors are unrelated to xyz and ‘tensors’ rather vectors in the sense of a vector space.) Symbolically the solution to such a set of linear equations is: $I = B^{-1} \cdot A$.

(b) Using the nodal form of Kirchhoff’s Rules, write down two linear equations that are required to solve for the two voltages V_A, V_B . Express the results as a matrix equation.

2. On the class web site the image `xkcd_circuit_full.png` shows a circuit with many in-jokes by Randall Munroe of the geeky web site `xkcd.com`. (You can find a subset of his comics that I’ve enjoyed at `www.physics.csbsju.edu/xkcd`.) A detail in this circuit: `xkcd_circuit.png` shows a challenge circuit for students of Electrical Engineering 201. Solve this challenge circuit using the nodal method and the node labeling on `xkcd_circuitB.png`; assume a voltage V on the top and ground (0 V) at the bottom. Use *Mathematica* to solve the resulting linear equations. A single number summary of the result is the equivalent resistance (V/I) of the circuit; report it! Remark: careful inspection will show node 9 has 8 connections.