```
$Assumptions=$Assumptions && {a>0}
f[x_]=1/Cosh[a x]
ke=Integrate[f'[x]^2,{x,-Infinity,Infinity}]
        2 a
Out[3]= ---
         3

pe=Integrate[x^2 f[x]^2,{x,-Infinity,Infinity}]
          2
        Pi
Out[4]= ----
           3
        6 a

n=Integrate[f[x]^2,{x,-Infinity,Infinity}]
        2
Out[5]= -
        a

f[x_]=Exp[-a (x-2)^2]+Exp[-a (x+2)^2]

ke=Integrate[f'[x]^2,{x,-Infinity,Infinity}]
                             8 a
        Sqrt[a] (1 - 16 a + E   ) Sqrt[2 Pi]
Out[7]= ------------------------------------
                         8 a
                        E

pe=-2(1+Exp[-a 4^2])
direct=0, exchange=-Exp[-a 4^2], self=-1
n=Integrate[f[x]^2,{x,-Infinity,Infinity}]
              8 a
        (1 + E   ) Sqrt[2 Pi]
Out[9]= ---------------------
                     8 a
            Sqrt[a] E

(ke/2+pe)/n
Plot[%,{a,.1,2}]
FindMinimum[%%,{a,.5}]
Out[12]= {-0.340238, {a -> 0.490313}}

f[x_]=Exp[-a (x-2)^2]-Exp[-a (x+2)^2]
ke=Integrate[f'[x]^2,{x,-Infinity,Infinity}]
                               8 a
         Sqrt[a] (-1 + 16 a + E   ) Sqrt[2 Pi]
Out[15]= -------------------------------------
                          8 a
                         E
pe=-2(1-Exp[-a 4^2])
direct=0, exchange=+Exp[-a 4^2], self=-1
n=Integrate[f[x]^2,{x,-Infinity,Infinity}]
                8 a
         (-1 + E   ) Sqrt[2 Pi]
Out[17]= ----------------------
                       8 a
              Sqrt[a] E

(ke/2+pe)/n
Plot[%,{a,.1,2}]
FindMinimum[%%,{a,.75}]
Out[20]= {-0.306583, {a -> 0.766212}}

exact: -0.480369 & -0.517251
```