

**Answer 10 of the following 14 questions**

1. According to Zombeck's *Handbook of Space Astronomy & Astrophysics* the altitude of a star can be determined from:

$$\sin a = \sin \delta \sin \phi + \cos \delta \cos h \cos \phi$$

where:  $a$  is altitude,  $\delta$  is declination,  $\phi$  is latitude, and  $h$  is hour angle. Prove this formula. Begin by considering the celestial sphere oriented so that the north celestial pole (NCP) (and the  $z$  axis) is at zenith and the  $x$  axis (from which  $h$  is measured) points due south. Given  $h$  and  $\delta$ , find the three components of the unit vector that points at the star. Now, in fact, the NCP is not at our zenith, rather it's tipped towards north by an angle related to our latitude. What is the (simple!) formula for this tipping angle as determined from  $\phi$ ? Evidently we must rotate the previously determined unit vector about the  $y$  axis by the amount determined above. Write down the rotation matrix and calculate the components of the rotated unit vector. By dotting this vector with the zenith vector (0,0,1) you should be able to find the altitude.

2. At the SJU observatory (latitude=  $45^\circ 34' 58''$ , longitude= $94^\circ 23' 25''$ ) you observe the star Sirius when it is on the meridian, and then again 1 hour, 2 hours and 3 hours later producing photon counts of: 8273, 8102, 7727, and 7132. How many photons would you estimate you'd get without the Earth's atmosphere in the way. What is your estimated error in that count?
3. Find the clock time when the star Regulus is on the meridian at the SJU observatory on April 1. What altitude/azimuth will it have when it's on the meridian.
4. You have probably forgotten the formula for the radius of curvature you learned in calculus (I did!). But if you apply it to an ellipse you'll find that the ends of the major axis have radius of curvature  $r_C = b^2/a$ . That means that, *when the planet is at the ends, i.e., at aphelion or perihelion*, the planet feels like it's going in a circle with radius  $r_C$  and so should undergo the normal centripetal acceleration. Thus  $F = ma$  reads:

$$\frac{GMm}{r^2} = m \frac{v^2}{r_C}$$

where  $r$  is the actual distance the planet is from the Sun. Pick aphelion or perihelion, calculate both sides of the above equation, and show the above equality. Hint: use conservation of energy to find  $v^2$ .

5. The orbit of Halley's comet has semi-major axis,  $a = 17.94$  AU and eccentricity  $e = .9673$ . Halley's comet was last near the Sun in 1986.112. When will it again be near the Sun? Using the material in <http://www.physics.csbsju.edu/orbit/>, have *Mathematica* plot the orbit, with dots showing where it is at intervals of one year.

6. There is an old joke which implies that a semi-truck full of pigeons weighs less if the pigeons are flying than if they are roosting. Clearly if that were the case (and it's not) an empty and full gas cylinder would weigh the same (since gas molecules are much better flyers than pigeons). Thus the gas must be providing a net force on the container. What is that force? Show that the net force of the gas on the container is equal to the weight of the gas.
7. Enoch is a planet just like Venus—except (1) it's cool enough to have an ocean on its surface and (2) Enoch has a moon whose orbit is similar—but not identical—to the Moon's orbit around the Earth. The little green enoids of Enoch cannot see the stars because of the continuous cloud cover, however, the enoids have noticed that periodically the beaches are covered with water. They call these times nobeach times, and there are several per day. The below chart shows the nobeach times for more than a year. Note that enoids divide a day into 24 crepos much as we divide a day into 24 hours, but they count in base 16 so they'd say there are 18 crepos in a day.

Date	Times of nobeach
1 Kopow	1:00, 3:2C, 6:1D, 9:0D, B:39, E:2A, 11:1A, 14:0A, 16:37
2 Kopow	1:28, 4:17, 7:08, 9:34, C:24, F:25, 12:05, 14:31, 17:21

What is the sidereal period of Enoch's moon? How distant is Enoch's moon from Enoch? If New Years was 1 Kopow 0:00, when was the next New Years?

8. Since hydrogen is the most common element seen in the universe, its absorption lines are quite important. Lyman  $\alpha$  ( $L\alpha$ ) is the transition:  $n = 1 \rightarrow n = 2$ . Calculate its wavelength and energy in eV. What type (X-ray, UV, visible, IR, ...) of light is it? What wavelength of light would just ionize an H-atom in its ground state? Calculate the wavelength and energy in eV of  $H\alpha$  (the lowest energy Balmer series line:  $n = 2 \rightarrow n = 3$ ). Highly excited H-atoms can be seen with radio telescopes. Calculate the frequency and wavelength of  $n = 200 \rightarrow n = 201$  (called  $200\alpha$ ).
9. While hydrogen is the most common element in the atmospheres of stars, most stars lack prominent visible-light (e.g.,  $H\alpha$ ) absorption lines. Explain why these lines are not prominent in stars like the Sun or cooler and in very hot stars. ( $H\alpha$ ,  $H\beta$ , etc., are very prominent in A stars.)
10. Derive a formula giving the temperature of a fast-spinning planet in terms of the distance the planet is from the Sun,  $r$ , the radius of the Sun,  $R_\odot$ , the temperature of the surface of the Sun,  $T_\odot$ , and the albedo of the planet,  $a$ . What factors limit the accuracy of your formula? Plug in numbers that you think are appropriate for the Earth. Explain your result.
11. Consider a binary star system: star A (with apparent magnitude  $m_A = 1$ ) and star B (with  $m_B = 2$ ) in orbit about each other. What magnitude do they have together? If A's absolute magnitude is 4.77, what is the distance to A? If you learn that the temperature of A is twice the temperature of the Sun, how does A's radius compare to the Sun?

12. When we talk about the spectrum of a star we are talking about a plot of  $F_\nu$  vs  $\nu$ . What exactly is  $F_\nu$ ? Give a *careful* definition of  $F_\nu$  including its units and how it might be experimentally determined (*determined*: give me the details of what you would measure, what would you divide by what, etc., to get  $F_\nu$ ).
13. We often use the ratio of  $F_\nu$  at a “high” frequency (e.g., blue =  $\nu_1$ ),  $F_\nu(\nu_1) = F_1$  and at a “low” frequency (e.g., “visible” =  $\nu_2$ )  $F_\nu(\nu_2) = F_2$  to determine the temperature,  $T$ , of a star. Write down a formula for  $F_1/F_2$  if the star radiates as a blackbody at temperature  $T$  and use your favorite graphing program to graph  $F_1/F_2$  vs  $T$ . Carefully show/derive the behavior of the graph as  $T \rightarrow \infty$  and  $T \rightarrow 0$ .
14. Sketch an HR diagram and approximately place the stars in the table on that diagram. Answer the questions using the data in the table.

Star Name	Absolute Magnitude $M_V$	Apparent Magnitude $m_V$	Spectral Type	Luminosity Class
1. Spica	-3.5	+1.0	B1	III
2. Antares	-3.8	+1.0	M2	I
3. Sirius	+1.4	-1.5	A1	V
4. Rigel Kent	+4.4	+0.0	G2	V
5. Fomalhaut	+2.0	+1.2	A3	V
6. Deneb	-7.2	+1.3	A2	I
7. Canopus	-3.5	-0.7	F0	II
8. Regulus	-0.7	+1.4	B7	V
9. Aldebaran	-0.5	+0.9	K5	III

- (a) Which star has the greatest apparent brightness?
- (b) Which star has the same spectral type as the Sun?
- (c) Which star is intrinsically the brightest?
- (d) Which star has the highest surface temperature?
- (e) Which star has the lowest surface temperature?
- (f) Which star is furthest away?
- (g) Which star has the largest radius?