

$$A^k_{;r} = A^k_{,r} + \Gamma^k_{ra} A^a$$

$$A^k_{;rs} = (A^k_{;r})_{,s} + \Gamma^k_{sa} (A^a_{;r}) - \Gamma^b_{rs} (A^k_{;b})$$

$$= \boxed{A^k_{,rs}} + \Gamma^k_{ra,s} A^a + \Gamma^k_{ra} A^a_{,s}$$

$$+ \Gamma^k_{sa} (A^a_{,r} + \Gamma^a_{rb} A^b)$$

$$- \boxed{\Gamma^b_{rs}} (A^k_{,b} + \Gamma^k_{ab} A^b)$$

$$A^k_{;rs} - A^k_{;sr} = (\Gamma^k_{ra,s} - \Gamma^k_{sa,r}) A^a + (\Gamma^k_{sa} \Gamma^a_{rb} - \Gamma^k_{ra} \Gamma^a_{sb}) A^b$$

$$= (\Gamma^k_{rb,s} - \Gamma^k_{sb,r} + \Gamma^k_{sa} \Gamma^a_{rb} - \Gamma^k_{ra} \Gamma^a_{sb}) A^b$$

$$= R^k_{b\,rs} A^b$$