The Electron Cyclotron Drift Instability

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1 Acronyms

1. Electron Cyclotron Drift Instability $\equiv$ (ECDI)
2. Electron Cyclotron Harmonic $\equiv$ (ECH)
3. Ion-Acoustic Waves $\equiv$ (IAWs)
4. Electron-Acoustic Waves $\equiv$ (EAWs)
5. Lower Hybrid Waves $\equiv$ (LHWs)
6. Electrostatic $\equiv$ (ES)
7. Electromagnetic $\equiv$ (EM)
8. Modified Two-Stream Instability (MTSI)
9. Let ECHWs include: Bernstein, totem-pole, and (n + 1/2) waves

2 Other Names

1. Electron Cyclotron Drift Instability (ECDI)
2. Beam Cyclotron Instability [*Lampe et al.*, 1971a,b, 1972]
3. Electrostatic (ES) Electron-Ion Streaming Instability [*Wong*, 1970]

3 Ashour-Abdalla and Kennel *et. al.*, [1978a]

Ashour-Abdalla and Kennel [1978] examined nonconvective and convective ECHIs ($f \simeq (n + 1/2)f_{ce}$) finding:

1. n_{ce} controls which harmonic band can be excited through the upper hybrid frequency
2. T_{ce} controls the spatial amplification $\rightarrow$ when $0 < T_{ce}/T_{he} \lesssim 10^{-2} \Rightarrow$ instability is nonconvective while larger values will eventually cause the instability to become convective
3. if $T_{ce}/T_{he} \lesssim 5 \times 10^{-2}$, quasi-linear diffusion increases T_{ce} faster than resonant diffusion can heat/scatter hot electrons into the loss-cone
4. if $n_{ce}/n_{he} \simeq$ 3-5, the instability does not occur

4 Ashour-Abdalla and Kennel *et. al.*, [1980]

Ashour-Abdalla et al. [1980] examined ECHIs finding that the waves heated the cold electrons perpendicular to the field faster than parallel.

5 Forslund *et. al.*, [1970]

Forslund et al. [1970] examined the electron cyclotron drift instability (ECDI), which is an instability that occurs when ions drift relative to electrons, V_d, across a magnetic field. The dispersion relation he used was:

$$(K\lambda_{De})^2 = -1 + e^{-\lambda} I_o(\lambda) + 2\omega^2 \sum_{n=1}^{\infty} \frac{e^{-\lambda} I_n(\lambda)}{\omega^2 - (n\Omega_{ce})^2} + \frac{T_e}{2T_i} Z' \left(\frac{\omega - kV_d}{kV_{Ti}} \right) \tag{1}$$

where Ω_{ce} = e B/(m_e c), λ + (k r_e)^2/2, λ = 1/2 $\sqrt{k_\perp V_{Te}/\Omega_{ce}}$, $r_e = V_{Te}/\Omega_{ce}$, $V_{Te}{}^2 = 2\ T_e/m_e$, and Z'(x) is the derivative of the plasma dispersion function given by Z'(x) = -2 [1 + x Z(x)]. The plasma dispersion function can be written as:

$$Z(x) = i\frac{k}{|\,k\,|}\sqrt{\pi}e^{-x^2} - \left[\frac{1}{x} + \frac{1}{2x^3} + \frac{3}{4x^5} + \cdots\right] \text{ (for x} \gg 1) \tag{2a}$$

$$Z(x) = i\frac{k}{|\,k\,|}\sqrt{\pi}e^{-x^2} - \left[2x - \frac{4}{3}x^3 + \frac{8}{15}x^5 + \cdots\right] \text{ (for x} \ll 1) \tag{2b}$$

$$Z'(x) = -2i\frac{k}{|\,k\,|}\sqrt{\pi}xe^{-x^2} + \left[\frac{1}{x^2} + \frac{3}{2x^4} + \frac{15}{4x^6} + \cdots\right] \text{ (for x} \gg 1) \tag{2c}$$

$$Z'(x) = -2i\frac{k}{|\,k\,|}\sqrt{\pi}xe^{-x^2} - \left[2 - 4x^2 + \frac{8}{3}x^4 + \cdots\right] \text{ (for x} \ll 1) \tag{2d}$$

The instability results from a resonance between the otherwise purely electron cyclotron waves (ECWs) and ions where $\partial F_i/\partial v > 0$, or where $v < V_d$. Due to the resonance with the ions, γ_{max} for each harmonic occurs near MAX($\partial F_i/\partial v$), which is about at $\omega/k \sim V_d - V_{Ti}$. Note that for all $\omega_R < k\ V_d$, $\gamma/\Omega_{ce} > 0 \Rightarrow$ unstable growth. From this, one can see:

1. when (k λ_{De}) > 1, there is an attenuation of $\gamma \propto (k\ \lambda_{De})^{-4}$, which acts as a cutoff for large (k λ_{De})
2. for (k r_e) large, but (k λ_{De}) < 1, all harmonics have roughly equal γ's because the resonance condition is (k r_e) $\simeq n_o\ V_{Te}/(V_d - V_{Ti})$
3. when $V_d > V_{Ti}$ and (k λ_{De}) < 1, (noting that $\Im$[Z'] $\sim$ 1) $\gamma \propto \Omega_{ce}\ V_d/V_{Te}$
4. $\gamma \neq \gamma(m_e/M_i)$
5. for larger values of ω_{pe}/Ω_{ce}, the Debye length cutoff occurs at smaller values of $V_d/V_{Te} \Rightarrow$ more harmonics grow for larger ω_{pe}/Ω_{ce}

Due to γ's dependence on $\Re$[Z'], there is a strong interaction between the ECWs and ion modes. Note also that T_e/T_i has little effect on γ. Since the Debye length cutoff implies the instability is most effective when $\omega_{pe} \sim \Omega_{ce}$ for $V_d > V_{Te}$, we assume that $\omega/k > V_{Te}$ and $\gg V_{Ti}$. Using these assumptions, we can reduce Equation 1 to the following:

$$1 = \frac{\omega_{pe}{}^2}{\omega^2 - \Omega_{ce}} + \frac{\omega_{pe}{}^2 m_e/M_i}{(\omega - kV_d)^2} \tag{3}$$

which nicely reduces to the usual two-stream instability for $\omega_{pe}/\Omega_{ce} \gg 1$. Equation 3 represents an interaction between the upper hybrid and a Doppler-shifted lower hybrid mode.

There are three conditions which can squelch the instability:

1. when $V_d \rightarrow 0$ (which could result from field diffusion due to the instability itself)
2. when the instability heats the electrons to the Debye-length cutoff
3. when the ions are resonantly heated until the MAX($\Im$[Z']) for the fundamental is very small and/or lies beyond the Debye-length cutoff

When $V_{Te} \gg V_d$ and $\omega_{pe}/\Omega_{ce} \gg 1$, the instability will preferentially heat the ions instead of the electrons. Since the resonance occurs near the MAX($\partial F_i/\partial v$), the ECDI can be an effective ion heating mechanism. In addition to heating the plasma, the ECDI produces an anomalous resistivity which causes a drift and diffusion across the magnetic field.

The ECDI is mostly a longitudinal instability until the particles become relativistic and it does not appear to be affected by finite β_e.

6 Forslund *et. al.*, [1971]

Forslund et al. [1971] examined the nonlinear ECDI, finding significant electron heating due to adiabatic and non-adiabatic trapping. The ions were heated as well due to trapping. So the electrons are dragged across the magnetic field by the drifting ions, which produces an effective drag on the ions causing them to break the *frozen-in* condition and gain some thermal energy while losing bulk kinetic. Also, ion trapping does not reduce the heating rate.

7 Forslund *et. al.*, [1972]

Forslund et al. [1972] examined the ECDI, examining the perpendicular anomalous resistivity it causes at collisionless shocks and in lab plasmas. They consider a relative drift, V_d, between electrons and ions that is perpendicular to both the magnetic field and the shock normal. They treat the ion trajectories as straight because the $\gamma_{ECDI} \gg \Omega_{ci}$. If one assumes that $\lambda \gg 1$, then one can approximate:

$$e^{-\lambda} I_n(\lambda) \simeq \sqrt{\frac{1}{2\pi\lambda}} \tag{4}$$

They also assume that the plasma dispersionn functions are of the form:

$$Z'_i = Z'\left[\frac{\omega_R - \mathbf{k}\cdot\mathbf{V}_d}{kV_{Ti}}\right] \tag{5a}$$

$$Z'_e = Z'\left[\frac{\omega_R - n\Omega_{ce}}{k_{\parallel} V_{Te}}\right] \tag{5b}$$

As one might expect, the largest values of γ occur when $k_{\parallel} \rightarrow 0$, which results in $\Re[Z'] = \Im[Z'_e] = 0$. This simplifies the growth rate calculation and real frequency result to:

$$\frac{\gamma}{\Omega_{ce}} \simeq \frac{n_o}{\sqrt{\pi} k r_e}\left[\frac{T_e/(2T_i)\Im[Z'_i]}{[1+(k\lambda_{De})^2 - (T_e/2T_i)\Re[Z'_i]]^2 + [(T_e/2T_i)\Im[Z'_i]]^2}\right] \tag{6a}$$

$$\frac{\omega_R - n\Omega_{ce}}{\Omega_{ce}} \simeq \frac{\gamma}{\Omega_{ce}}\left[\frac{1+(k\lambda_{De})^2 - (T_e/2T_i)\Re[Z'_i]}{(T_e/2T_i)\Im[Z'_i]}\right] \tag{6b}$$

If we drop the term associated with $\Re[Z'_i]$ in the denominator of Equation 6a (typically okay when $T_e \simeq T_i$), and noting that $\mathrm{MAX}(\Im[Z'_i]) \simeq 1.5$ (when its argument is $\simeq$ -0.7), then Equations 6a and 6b reduce to:

$$\frac{\gamma}{\Omega_{ce}} \simeq \frac{n_o}{\sqrt{\pi} k r_e}\left(\frac{T_e}{2T_i}\right)\left[\frac{3/2}{[1+(k\lambda_{De})^2]^2 + [(3T_e/4T_i)]^2}\right] \tag{7a}$$

$$\frac{\omega_R - \mathbf{k}\cdot\mathbf{V}_d}{kV_{Ti}} \simeq -0.7 \tag{7b}$$

assuming $\omega_R \simeq n\Omega_{ce}$ and $\mathrm{Cos}\theta = \mathbf{k}\cdot\mathbf{V}_d/(k\, V_d)$, then we can reduce Equations 7a and 7b down to:

$$k \simeq \frac{n\Omega_{ce}}{V_d - 0.7V_{Ti}} \tag{8a}$$

$$\frac{\gamma}{\Omega_{ce}} \simeq \frac{Cos\theta}{\sqrt{\pi}}\left(\frac{V_d}{V_{Te}}\right)\left(\frac{T_e}{2T_i}\right)\left[\frac{3/2}{[1+(k\lambda_{De})^2]^2}\right] \tag{8b}$$

$$\simeq \frac{Cos\theta}{\sqrt{\pi}}\left(\frac{V_d}{V_{Te}}\right)\left(\frac{T_e}{2T_i}\right)\left[\frac{3/2}{1+\left[(n/Cos\theta)(V_d/V_{Te})(\Omega_{ce}/\sqrt{2}\omega_{pe})\right]^2}\right] \tag{8c}$$

where I have ignored any electron-electron or electron-ion collisions.

Not that for all harmonics with $(k\,\lambda_{De}) < 1$, $\gamma_{max} \neq \gamma_{max}(n, m_e/M_i)$ and occurs at the same $k_{\perp}$ ($= k\,\mathrm{Cos}\theta$) for all $k_{\perp}$ in the plane perpendicular to $\mathbf{B}_o$. In this case, the last term in the brackets of Equation 8c reduces to unity leaving the growth rate to be:

$$\frac{\gamma}{\Omega_{ce}} \simeq \frac{Cos\theta}{\sqrt{\pi}}\left(\frac{V_d}{V_{Te}}\right)\left(\frac{T_e}{2T_i}\right) \tag{9}$$

The expected turbulence fills a relatively wide range of angles in the plane perpendicular to $\mathbf{B}_o$ but parallel to $\mathbf{V}_d$. Anisotropic heating due to cyclotron interactions can change this fan-like resonance into a cone that extends into the plane containing $\mathbf{B}_o$. From Equation 8c when (k λ_{De}) > 1, there is a strong Debye-length cutoff $\propto$ (k $\lambda_{De})^{-4}$ which allows us to estimate an approximate instability criterion:

$$\frac{V_d}{V_{Te}} \gtrsim \frac{n}{Cos\theta}\frac{\Omega_{ce}}{\sqrt{2}\omega_{pe}} \tag{10}$$

From the full resonance condition (Equation 8a) one can see that if $V_{Ti} \rightarrow > V_d$, then k $\rightarrow >$ Debye-length cutoff for all harmonics. Thus one also must demand that $V_d \gtrsim V_{Ti}$ for an instability to occur.

There are four possible ways to stabilize this instability:

1. V_d is reduced by resistive broadening of $\mathbf{B}_o$
2. V_{Te} is increased by resistive heating which results in successively lower harmonics being stabilized until the n = 1 (fundamental) is stabilized at $V_d/V_{Te} \simeq \Omega_{ce}/\omega_{pe}$. The primary electron heating will be perpendicular to $\mathbf{B}_o$, various longitudinal and transverse instabilities driven by the resulting anisotropy could rapidly convert some electron thermal energy into parallel momentum, thus producing effectively an increase in electron thermal conductivity.
3. V_{Ti} can be increased by heating until $V_d \simeq V_{Ti}$, however this is less likely than (1) or (2) because electron heating is more effective
4. plasma compression $\perp$-$\mathbf{B}_o$ can reduce Ω_{ce}/ω_{pe} by $\sqrt{N_2/N_1} \Rightarrow$ increases the instability threshold.

When considering the full dispersion relation (not shown), there are effects to consider when $k_\parallel \neq 0$ that enter through Z'_e (= $Z'[(\omega_R - n\,\Omega_{ce})/(k_\parallel V_{Te})]$) in the combinations 1/2 $\Im[Z'_e]$ and (1 + $\Re[Z'_e]$/2). Both of these terms go to zero (from above and below, respectively) for argument $\rightarrow \infty$, and through their quotient ($\Im[Z'_e]$/2)/(1 + $\Re[Z'_e]$/2) [argument = ∞(0) $\rightarrow$ 0(-∞, from the $k_\parallel > 0$ side)]. However, when all terms are considered, γ/Ω_{ce} decreases monotonically $\propto$ increasing $k_\parallel$. The cutoff occurs when the argument of $Z'_e \rightarrow 1$, thus $(\omega_R - n\Omega_{ce})/\Omega_{ce} \sim n/(\sqrt{\pi}\ k\ r_e)$ and the resonance condition simplifies to $\omega_R/k \simeq V_d\ Cos\theta$. If we assume (k λ_{De}) < 1, then the spread in $k_\parallel$ can be shown to be:

$$\frac{k_\parallel}{k} \lesssim \frac{n}{\sqrt{\pi}}(kr_e)^2 \simeq \left(\frac{1}{\sqrt{\pi}n}\right)\left(\frac{V_d Cos\theta}{V_{Te}}\right)^2 \tag{11}$$

The spread in $k_\parallel$ is less for higher harmonics and in general, much narrower than for $k_\perp$. The damping which limits $k_\parallel$ is cyclotron, NOT Landau, damping! Thus, as the Debye-length (or collisional) cutoff of a harmonic is approached and exceeded, its $k_\parallel$ spread $\rightarrow 0$. The important conclusions are:

1. an instability still exists if a weak magnetic field and $T_e \sim T_i$ but does not exist if the magnetic field $\rightarrow 0$
2. the instability is difficult to stabilize (linearly) without significant magnetic field diffusion or electron heating
3. electron collision frequency $\propto \gamma$ is required to stabilize
4. instability occupies a broad cone of angles in the plane $\perp$-$\mathbf{B}_o$ but a very narrow cone of angles in the plane $\parallel$-$\mathbf{B}_o$

7.1 Analytical Linear Theory

The IAW mode still exists in the presence of a magnetic field and couples strongly to the ECDI when $T_e > T_i$. However, they find that the IAW is never unstable, but the Bernstein roots are. However, when the full dispersion relation is solved, the dependence of γ/Ω_{ce} on T_e/T_i is even weaker than suggested by Equation 6a. In fact, *Forslund et al.* [1972] claims that there appears to be no real distinction between the cold and warm plasma solutions since the dependence of γ/Ω_{ce} on T_e/T_i is so weak. Even more, when $T_i \gtrsim T_e$, the largest γ's occur at the lower harmonics.

Using $\Omega_{ce}/\omega_{pe} \sim 1/50$, $m_e/M_i \sim 1/1836$, and $T_e/T_i \sim 1$, they find that γ/Ω_{ce} increases rather dramatically for higher harmonics as a function of V_d/V_{Te}.

7.2 Numerical Nonlinear Theory

Consider the case where the electric field parallel to the shock normal is zero, thus the canonical particle momentum in that direction will be a constant for every particle. If we also consider a class of electrons whose undisturbed gyromotion guiding centers result in a velocity, $v_n = (\mathbf{x} \cdot \mathbf{V}_d/V_d)\, \Omega_{ce}$. Thus the total force along the $\mathbf{V}_d$ direction (define as x) is given by:

$$F_x = -e\left[-\phi(x) + x\Omega_{ce}\frac{B_o}{c}\right] \tag{12}$$

which gives an effective combined potential of:

$$\Phi = -e\phi + \frac{m_e}{2}\left(\Omega_{ce}x\right)^2 \tag{13}$$

where the second term simply describes the gyromotion, the electrostatic part, ϕ(x), remains tied to the ions and moves with about V_d. The effect could be seen as a superposed ripple on the parabolic potential defined by the second term.

For the situation where (k λ_{De}) < 1, and noting that (k λ_{De}) $\simeq n_e\ (V_{Te}/V_d)$, if $V_d \gtrsim V_{Te}$, few electrons are resonant with the wave. Note that the growth saturates as the perturbed electron velocities reach V_d, which means the electrons break their *frozen-in* trajectories and start to become trapped in the potential wells of the waves. If no magnetic field is present, this occurs at $e\phi_o \simeq m_e V_d{}^2/2$, where ϕ_o is the magnitude of oscillations of ϕ(x). However, the addition of a magnetic field reduces the saturation level of eϕ due to the Lorentz force term $\mathbf{v} \times \mathbf{B}$. The equation of motion for an electron in an oscillating electric field is given by:

$$v_x = -\left(\frac{ieE_x}{m_e\omega}\right)\left[1 + \left(\frac{\Omega_{ce}}{\omega}\right)^2\right]^{-1} \tag{14}$$

where we can replace E_x with -ikϕ_o (= -iϕ_o ω/V_d) for a wave traveling at velocity V_d with respect to the magnetic field. If we also let v_x = -V_d, then the trapping saturation estimate goes to:

$$e\phi_o = m_e V_d{}^2\left[1 - \left(\frac{\Omega_{ce}}{\omega}\right)^2\right] \tag{15}$$

Note that in the low density regime one needs $V_d \gtrsim V_{Te}$ to overcome the Debye-length cutoff and when $\omega \simeq \Omega_{ce}$ the saturation potential is greatly reduce. When $\omega \simeq \Omega_{ce}$, the electrons respond by coiling up into ordered spirals in phase space (*i.e.* gyrophase restricted) while the ions suffer considerable heating because of resonant breaking of their *frozen-in* trajectories.

When the dominant modes satisfy $1 <$ (k λ_{De}) $< 2\pi$, a transistion behavior is observed. There is considerable electron heating in this regime.

When the dominant modes satisfy (k λ_{De}) $\gg 2\pi$, and if $V_d < V_{Te}$ and $\omega_{pe}/\Omega_{ce} \gg 1$, as in the solar wind, the wave is resonant with the bulk of the electron distribution. To modify the linear growth by a nonlinear distortion of the electron velocity distribution, the electrons must have time execute a trapping oscillation in the potential wells. Unlike the field-free case, the resonant interaction of the electrons with the wave is limited by the smaller of the following two: 1) the time a gyrating electron remains in resonance with a wave ($\sim \Omega_{ce}{}^{-1}$), or 2) the time that a well, ϕ(x), remains in existence [$\sim$ (eϕ_o/m_e) (k/$\Omega_{ce}V_d$)]. From this, we can estimate the threshold for which electron trapping modifies the linear growth rate as:

$$e\phi_o \simeq m_e V_d{}^2\left[2\pi\left(\frac{\Omega_{ce}}{kV_d}\right)^2\right]^{2/3} \tag{16}$$

Note, however, that the potential estimated by Equation 16 tends to be a very small number. However, if a sufficiently large number of electrons become trapped in the wells, the potentials will enhance an possibly grow to larger than the thermal energy of the electrons. If this happens, the trapped electrons will be carried along by the potentials to be eventually released at a higher energy which increases the energy associated with gyration (since they released into the larger magnetic potential well). In other words, the electrons are first energized along the shock normal and their increased energy perpendicular to the magnetic field, thus they gain energy in the x-direction too! Also, if $V_d \ll V_{Te}$ and ϕ is large enough to trap electrons above their thermal energy, then the electrons remain trapped for much longer than a gyroperiod. Their perturbed charge density is also shifted relative to the potentials.

The ion perturbed charge density, on the other hand, is almost entirely defined by ϕ(x) alone. Thus a phase shift is produced between the perturbed charge density of each species which drives the magnitude of ϕ well beyond the value in

Equation 16. This makes the instability become very efficient at heating electrons. However, the nonlinear instability is no longer resonant with the bulk of the ions but only their high energy tails.

7.3 Discussion

The anomalous resistance is ultimately caused by clumps of trapped electrons being pulled across the magnetic field by the drifting density maxima of the ions. The electrons cause an effective drag on the ion drift causing the ions to lose drift energy and gain thermal energy by breaking. This also causes the electrons to get heated by increasing their velocity along the shock normal until they are pulled out of their potential wells (*i.e.* perpendicular heating). The requirement for no net current along the shock normal direction causes the convective electric field to adjust itself to give an $\mathbf{E} \times \mathbf{B}$ drift along the shock normal which cancels out the net drift of the electrons in that direction.

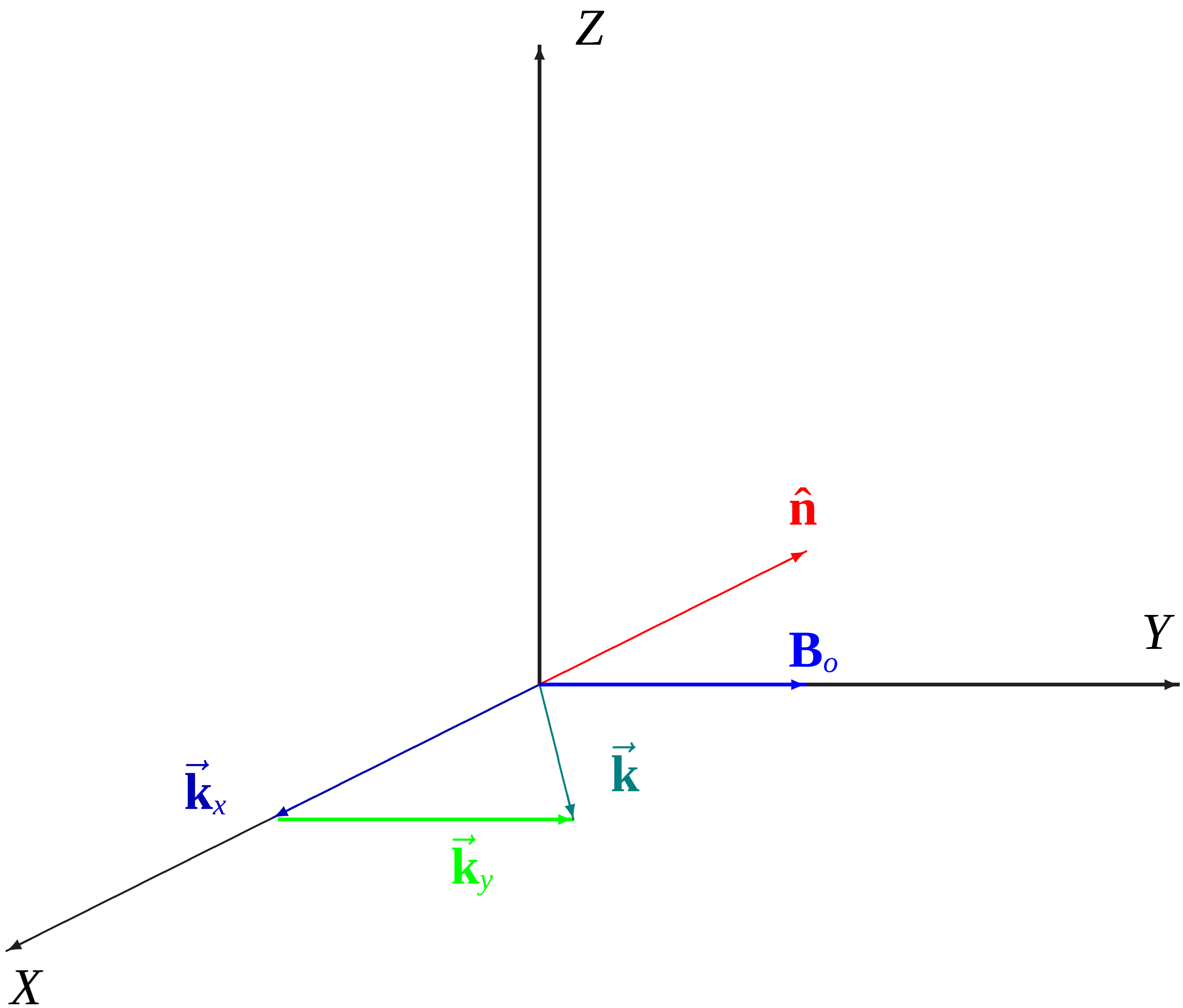
8 Lampe *et. al.*, [1971]

Lampe et al. [1971a] examined the nonlinear development of the ECDI. If $V_d > C_s$, then the electron Bernstein modes can couple to the IAWs. They also found that the ECDI saturations after a sufficient amplitude and mode converts to the IAW. For warm plasmas, the IAWs are stabilized by Landau damping (*i.e.* parallel heating).

9 Matsukiyo and Scholer, [2006]

Matsukiyo and Scholer [2006] investigated microinstabilities at a perpendicular supercritical collisionless shock. They used a 2D PIC simulation with realistic mass ratio ($\sim$1836), $\beta_{inc} = 0.04$, $\beta_{ref} = 0.01$, $\beta_e = 0.05$, $n_{ref}/n_{inc} = 0.25$, $(\omega_{pe}/\Omega_{ce})^2 = 4$, $\Delta t \sim 0.02\ \omega_{pe}{}^{-1}$, $\Delta x = \Delta y = 0.5\ \lambda_{De} \sim 0.04\ c/\omega_{pe}$, $U_{inc}/V_A = +2.14$, and $U_{ref}/V_A = -8.57$. There are three instabilities of interest, though they observed 6, ECDI, MTSI-1, MTSI-2.

The simulation geometry is shown as:


Z
$\hat{\mathbf{n}}$
Y
$\mathbf{B}_o$
$\vec{\mathbf{k}}$
$\vec{\mathbf{k}}_x$
$\vec{\mathbf{k}}_y$
X

The properties of the ECDI observed in this simulation can be seen as:

1. **Free Energy Source:** [reflected ion] - [incident electron] relative drift
2. $k_x \sim n\Omega_{ce}/U_r$, where U_r is the reflected ion speed
3. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 10$, $-2 \gtrsim k_y c/\omega_{pe} \gtrsim 2$
4. 1st two harmonics are seen
5. mostly in δE_x and δB_y, but **VERY** diffuse in k_x-k_y space
6. X-mode polarization $\Rightarrow$ compressional δB
7. nonlinearly couples to MTSI-1
8. heats electrons strongly perpendicular to magnetic field and slightly heats reflected ions

The properties of the MTSI-1 observed in this simulation can be seen as:

1. **Free Energy Source:** [incident ion] - [locally decelerated electron] relative drift
2. **k** mostly $\perp$-$\mathbf{B}_o$
3. $k_x > 0 \Rightarrow$ anti-$\|$-**n**
4. k_y is both positive and negative
5. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 3$, $-0.5 \gtrsim k_y c/\omega_{pe} \gtrsim 0.5$ (seen in δB_z)
6. nonlinearly couples to ECDI
7. heats incident ions strongly and maintains their density profile
8. drives ES perpendicular whistler waves

The properties of the MTSI-2 observed in this simulation can be seen as:

1. **Free Energy Source:** [reflected ion] - [incident electron] relative drift
2. **k** oblique-$\mathbf{B}_o$
3. $k_x < 0 \Rightarrow \|$-**n**
4. k_y is both positive and negative
5. $0 \gtrsim k_x c/\omega_{pe} \gtrsim 2$, $-1 \gtrsim k_y c/\omega_{pe} \gtrsim 1$ (seen in δB_z)
6. little to no heating of reflected ions
7. drives oblique EM whistler waves, electron holes
8. through a two-step heating process the effects of the MTSI-2 cause tremendous electron heating

The two-step heating process occurs in the following manner:

1. ECDI drives a perpendicular anisotropy in the electrons which is unstable to whistlers (seen in δB_x and δB_z)
2. the MTSI-2 drives oblique EM whistler waves and electron holes
3. the electron holes produce double-peaked electron velocity distribution functions which are unstable to EAWs (seen in δE_y) which strongly heat electrons $\|$-$\mathbf{B}_o$
4. the electron temperature is observed to increase by a factor of $\sim$5 while the ions only increase by $\sim$5/4 for reflected and $\sim$2 for incident

10 Tsutsui *et. al.*, [1975]

Tsutsui et al. [1975] examined, in a lab plasma, the nonlinear decay of electron Bernstein modes into IAWs and ES electron cyclotron harmonic waves (ECHWs).

11 Kumar and Tripathi, [2006]

Kumar and Tripathi [2006] examined electron Bernstein modes in the presence of IAWs finding that they convert into ECHWs. Electron beams can excite EM waves in two stages: 1) ES wave excitation through Cerenkov or slow cyclotron interaction, and 2) ES wave undergoes resonance mode conversion into an EM wave under a density gradient or in the presence of a low frequency mode. The electron Bernstein waves cause the electrons to oscillate with a velocity, $\mathbf{v}_o$, that couples to the density perturbation, $\delta n(\omega,\mathbf{k})$, due to the IAW. This coupling produces a nonlinear current density at the sum and difference frequency which generates ECHs.

If we assume that a Bernstein wave exists in a plasma with an ES potential given by:

$$\phi_o = a_o e^{-i(\omega_o t - \mathbf{k}_o \cdot \mathbf{r})} \tag{17}$$

where $\mathbf{k}_o = k_{ox}\,\hat{x} + k_{oz}\,\hat{z}$, $k_{ox} \gg k_{oz}$, and an electron velocity distribution, $f_e = f_{oe} + \delta f_e$, where f_{oe} is a Maxwellian and δf_e is governed by the linearized Vlasov equation:

$$\frac{\partial \delta f_e}{\partial t} + \mathbf{v} \cdot \nabla (\delta f_e) = -\frac{e}{m_e} \nabla \phi_o \cdot \frac{\partial f_{oe}}{\partial \mathbf{v}} \tag{18}$$

which results in a solution for δf_e going as:

$$\delta f_e = \frac{2ie}{m_e V_{Te}{}^2} (f_{oe} \phi_o) I \tag{19}$$

where:

$$I = \sum_{p=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \frac{e J_l(\alpha)}{(\omega_o - p\Omega_{ce} - k_{o\|} v_z)} \left[\frac{i k_{o\perp} v_\perp}{2} (J_{p+1}(\alpha) + J_{p-1}(\alpha)) + i k_{o\|} v_z J_p(\alpha) \right] \tag{20}$$

where $\alpha = k_{o\perp}\, v_\perp/\Omega_{ce}$, $\theta = \mathrm{Tan}^{-1}(v_y/v_x)$ is the gyrophase angle, $v_\perp = (v_x{}^2 + v_y{}^2)^{1/2}$, and they used the identity:

$$e^{i\alpha Sin\theta} = \sum_{l=-\infty}^{\infty} J_l(\alpha) e^{il\theta} \tag{21}$$

From this, we can find the electron drift velocity due to the Bernstein mode as:

$$\mathbf{v}_o = \int_0^\infty \int_{-\infty}^\infty \int_0^{2\pi} dv_\perp dv_z d\theta\; \mathbf{v} v_\perp\; \delta f_e = \mathbf{u}_o \phi_o \tag{22}$$

where $\mathbf{u}_o$ can be represented as:

$$u_{ox} = \left(\frac{2\omega_{pe}{}^2 \Omega_{ce}{}^2}{e\pi V_{Te}{}^3 k_{o\perp} k_{o\|}} \right) \left[l(l+1) I_l(b) e^{-b} + I' + \frac{V_{Te} k_{o\|}}{4\Omega_{ce}} \left(1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) l\, I_l(b) e^{-b} \right] \tag{23a}$$

$$u_{oy} = \left(\frac{\omega_{pe}{}^2}{2\pi e k_{o\|} V_{Te}} \right) \left[\frac{k_{o\perp}}{k_{o\|}} (I_c - I_D) + (I_E - I_A) \left(1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) \right] \tag{23b}$$

$$u_{oz} = \left(\frac{\omega_{pe}{}^2 \Omega_{ce}}{2\pi e k_{o\|} V_{Te}{}^2} \right) \left(1 - \frac{\omega_o}{\omega_o - l\Omega_{ce}} \right) l\, I_l(b) e^{-b} \tag{23c}$$

where the terms I', I_c, I_D, I_E, and I_A are given as:

$$I' = -\left(\frac{2k_{o\perp}}{\Omega_{ce}{V_{Te}}^2}\right) I_A - \left(\frac{3{k_{o\perp}}^2}{4{\Omega_{ce}}^2{V_{Te}}^2}\right)(I_B + I_c) \tag{24a}$$

$$I_A = \frac{1}{{V_{Te}}^3}\int_0^\infty dv_\perp\ J_l(\alpha)J_{l-1}(\alpha)\ {v_\perp}^2 e^{-(v_\perp/V_{Te})^2} \tag{24b}$$

$$I_B = \frac{1}{{V_{Te}}^4}\int_0^\infty dv_\perp\ {J_l}^2(\alpha){v_\perp}^3 e^{-(v_\perp/V_{Te})^2} \tag{24c}$$

$$I_c = \frac{1}{{V_{Te}}^4}\int_0^\infty dv_\perp\ J_l(\alpha)J_{l+2}(\alpha){v_\perp}^3 e^{-(v_\perp/V_{Te})^2} \tag{24d}$$

$$I_D = \frac{1}{{V_{Te}}^4}\int_0^\infty dv_\perp\ J_l(\alpha)J_{l-2}(\alpha){v_\perp}^3 e^{-(v_\perp/V_{Te})^2} \tag{24e}$$

$$I_E = \frac{1}{{V_{Te}}^3}\int_0^\infty dv_\perp\ J_l(\alpha)J_{l+1}(\alpha){v_\perp}^2 e^{-(v_\perp/V_{Te})^2} \tag{24f}$$

(24g)

where they assumed that $\omega_o \approx l\ \Omega_{ce}$ and they retained only one term in Equation 20.

In addition to the Bernstein mode, there also exists a low frequency IAW or lower hybrid mode with potential, ϕ, of the same form as Equation 17 and an electron density perturbation of the form:

$$\delta n_e = \frac{k^2}{4\pi e}\chi\phi \tag{25}$$

where χ is the electron susceptibility and can be of the form:

$$\chi_{IAW} = 2\left(\frac{\omega_{pe}}{kV_{Te}}\right)^2 \quad \text{(for IAWs)} \tag{26a}$$

$$\chi_{LHW} = \left(\frac{\omega_{pe}k_\perp}{\Omega_{ce}k}\right)^2 - \left(\frac{\omega_{pi}}{\omega}\right)^2 - \left(\frac{\omega_{pe}k_z}{k}\right)^2 \quad \text{(for Lower Hybrid Mode.)} \tag{26b}$$

The density perturbation couples with the $\mathbf{v}_o$ to produce a current, ${\mathbf{j}_1}^{NL}$ (= -1/2 n_e e $\mathbf{v}_o$), with frequency, $\omega_1 = \omega + \omega_o$, and wave vector, $\mathbf{k}_1 = \mathbf{k} + \mathbf{k}_o$. The frequency of the IAW needed for resonant coupling can be shown as:

$$\omega_{IAW} = kC_s = C_s\sqrt{({k_1}^2 + {k_o}^2 - 2k_ok_1 Sin\psi)} \tag{27}$$

where C_s is the ion-acoustic speed ($\simeq \sqrt{k_BT_e/M_i}$) and $\psi = \text{Cos}^{-1}(\mathbf{k}_1 \cdot \mathbf{B}_o/k_1B_o)$. The electric field of the EM wave produced by the coupling can be written as (after considerable approximations) as:

$$\mathbf{E}_1 = 2\pi i\frac{\omega_1 n_e e}{{\omega_1}^2 - (k_1c)^2}\left(\mathbf{v}_o - \frac{c^2\mathbf{k}_1(\mathbf{k}_1\cdot\mathbf{v}_o)}{{\omega_1}^2}\right) \tag{28}$$

11.1 Parametric Conversion of EM Wave to Bernstein Wave

Consider a circularly polarized EM wave near a cyclotron harmonic in a plasma with an electric field given by:

$$\mathbf{E}_1 = \mathbf{A}_1 e^{i(\omega_1 t - k_1 z)} \tag{29}$$

where $\mathbf{A}_1 = A_{1x}\ \hat{x} + A_{1y}\ \hat{y}$, A_{1x} = -i A_{1y}, $k_1 = \omega_1/c\ [1 - {\omega_{pe}}^2/(\omega_1\ \{\omega_1 + \Omega_{ce}\})]$. This wave oscillates the electrons at a velocity given by:

$$\mathbf{v}_1 = \frac{eA_{1x}(\hat{x} - i\hat{y})}{m_e(\omega_1 + \Omega_{ce})} \tag{30}$$

The pump wave decays into an IAW and Bernstein wave or LHW and Bernstein wave, each with potentials of the form seen in Equation 17 and $\omega_o = \omega - \omega_1$, $\mathbf{k}_o = \mathbf{k} - \mathbf{k}_1$. The Bernstein wave produces an oscillatory electron velocity, $\mathbf{v}_o = \mathbf{u}_o\ \phi_o$, which was derived earlier. The density perturbation, δn_e, due to the IAW couples to the oscillatory velocity $\mathbf{v}_1$ produced by the pump EM wave. This coupling between the density perturbation and oscillatory velocity produce a nonlinear current density which drives the Bernstein wave at (ω_o, $\mathbf{k}_o$).

11.2 Discussion

Electron Bernstein waves, in the presence of low frequency high(low) wave number(length) IAWs, will efficiently mode convert into EM radiation at the cyclotron harmonics. This radiation can far exceed background if $\mathbf{v}_o \sim C_s$. The reverse process, parametric excitation of electron Bernstein waves, is also efficient for high temperature plasmas with optimal growth for $k_{o\perp} r_e \sim 2$ and $\gamma/\Omega_{ci} \sim 1$.

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