Evaluating the Wheatstone Bridge

To find the Thévenin equivalent circuit for the Wheatstone bridge pictured to the right requires finding $V_{TH}$ when terminals $a$ and $b$ are unconnected and $I_{TH}$ when terminals $a$ and $b$ are shorted together. For $V_{TH}$ we have voltage dividers on the left and right sides that give us $V_a$ and $V_b$.

\[
V_a = \frac{(12 \text{ V})}{\frac{3.3 \text{ k}}{1.0 \text{ k} + 3.3 \text{ k}}} = 9.21 \text{ V} \quad V_b = \frac{(12 \text{ V})}{\frac{2.2 \text{ k}}{1.5 \text{ k} + 2.2 \text{ k}}} = 7.14 \text{ V}
\]

\[
V_{TH} = V_a - V_b = 2.07 \text{ V}
\]

For $I_{TH}$ the majority of the current follows the “path of least resistance” indicated by the bold arrows in the picture to the right. However, there is still a non-zero current through the 1.5 k and 3.3 k resistors, because there is a potential difference across these resistors.

To find $I_{TH}$ redraw the resistors as in the figure to the right. Since the terminals are shorted together we have a parallel combination of resistors in series with another parallel combination of resistors. The top parallel combination gives a 600 Ω effective resistance and the bottom parallel combination gives a 1.32 kΩ effective resistance for a total effective resistance of 1.92 kΩ.

\[
I_{TH} = \frac{12 \text{ V}}{1.92 \text{ kΩ}} = 6.25 \text{ mA} \quad \Rightarrow \quad R_{TH} = \frac{V_{TH}}{I_{TH}} = \frac{2.07 \text{ V}}{6.25 \text{ mA}} = 331 \text{ Ω}
\]

This means that the voltage drop across the upper parallel combination of resistors is:

\[
V_u = I_{TH}(600 \text{ Ω}) = 3.75 \text{ V}
\]

and the voltage drop across the lower parallel combination of resistors is:

\[
V_l = I_{TH}(1.32 \text{ kΩ}) = 8.25 \text{ V}
\]

The currents (labeled $I_R$) are then:

\[
I_{1.0k} = \frac{3.75 \text{ V}}{1.0 \text{ kΩ}} = 3.75 \text{ mA} \quad I_{1.5k} = \frac{3.75 \text{ V}}{1.5 \text{ kΩ}} = 2.5 \text{ mA}
\]

\[
I_{3.3k} = \frac{8.25 \text{ V}}{3.3 \text{ kΩ}} = 2.5 \text{ mA} \quad I_{2.2k} = \frac{8.25 \text{ V}}{2.2 \text{ kΩ}} = 3.75 \text{ mA}
\]