Flight Model of a Golf Ball

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March of 2007

Project for Fortran and C++ for Scientists

"What other people may find in poetry or art museums, I find in the flight of a good drive." ~Arnold Palmer

<u>Purpose:</u>
To model the flight path of a golf ball.

<u>Introduction</u>:
Golf is a very simple yet complicating game. All you have to do is put the ball in the hole, but for most of us this is harder than it sounds. For someone to know where their shot is going to end up, they must know how to control what the golf ball does after it leaves the clubface. In this project we will take a close look into what makes the golf ball do what it does what it is in the air and why a golf shot is slightly more complicated than simple projectile motion.

In order to model the flight path of a golf ball we must first understand what cause the golf ball to fly off into the distance, or just a few feet in front of us. The basics of the golf shot come from the swinging the golf club, which has a set loft, and hitting the golf ball up into the air. How far the ball goes depends on how fast you swing the club and how high you hit the ball. Before we look at how the golf ball flies in the air, let's first examine the point of impact between the golf club and the golf ball.

In order to know how far the ball will travel, we first need to know how fast the ball is going. The initial speed of the golf ball depends on how fast the golf club is being swung, as well as the masses of the club head and the golf ball.
The relationship between the speeds and masses of the club head to the golf ball can be equated by the conservation of mass equation.
$m1*v1 = m2*v2$
Using this equation we would be able to figure out how fast the golf ball would be sent off at based on the speed of the club head, the mass of the club head, and the mass of the golf ball. Here is where I will use the first of many assumptions throughout the rest of the problem. The USGA, which is the governing body for the game of golf in the United States, has numerous legal limits on the equipment that can be used in the game of golf. One of these limits is that the mass of a golf ball can be no more than 45.93 grams, so I will use this value for the mass of a golf ball. Another value that I am going to use is for the mass of the club head. Throughout this problem we will be looking at only the shots off of a driver.
The average mass of a driver club head is reported to be approximately 200 grams [15]. Now that we know the masses, we can figure out the velocity of the golf ball, well not quite yet. The conservation of momentum equation is for the case of a perfect elastic collision, whereas with the case of a golf ball the collision between the club head and the golf ball is not quite perfect. The transfer of energy between the club head and the golf ball is governed by a number call the coefficient of restitution which is a measure of the efficiency of the kinetic energy transfer between club face and ball. The range of this number is between 0 and 1, with 0 corresponding to no energy transfer at all where the ball would just stick to the club head, and a value of 1 corresponding to perfect elastic collisions where 100% of the kinetic energy of the club head is transferred to the golf ball. The formula for the coefficient of restitution is given by:
$Cr = (V2f - V1f) / (V1 - V2)$
V2f is the final velocity of the first object, V1f is the final velocity of the second object after impact, V1 is the velocity of the first object before impact, and V2 is the velocity of the second object before impact.
When applied to our case of conservation of momentum, our equation for the initial velocity of the golf ball becomes
$Vball = Vclub * (1 + Cr) / (1 + (m\_ball/M\_club))$ [15]

Now that we know the initial velocity of the golf ball, we can begin to examine the behavior of the ball during the course of its flight.

In order to model the flight of the golf ball, we need to know all of the forces that are acting on the golf ball. In the case of simple projectile motion, the only thing that is affecting the flight of the golf ball is the force of gravity. As we all know the force on an object is governed by the famous law of $F=ma$, and for the case of gravity we know that the rate of acceleration is approximately constant with a value of 9.8 m/s^2. Through the use of derivatives we can come up with some equations of motion for a simple projectile.
$a = dv/dt$ along with
$v = a*t + v\_y(o)$ $x = v\_x*t$
$y = (1/2)a*t^2 + v\_y*t + y(o)$

Where v_y is the y component of the velocity and v_x is the component of the velocity in the x direction.

So from knowing the rate of acceleration caused by gravity we are able to find the velocity in the y direction at any time. Then from the velocity we are able to find the height of the projectile at any point in time. From the velocity we are also able to find the distance of the projectile at any time t during the its flight.

With the case of the golf ball we are always dealing with the effects of gravity, however what sets the flight of a golf ball apart from simple projectile motion is due to the spin put on the golf ball from the golf club as well as the air resistance of the golf ball as it flies through the air. The force caused by the spin of the ball is known as the Magnus effect and the force cause by the air resistance of the golf ball is called the drag force. Below are the equations that represent these forces,

Magnus Force
 Flift = (1/2)*rho*A*V^2*Cl rho is the density of air, A is the cross sectional area of the golf ball, V is the velocity of
the golf ball, and Cl is called the lift coefficient.

Drag Force
 Fdrag = (1/2)*rho*A*V^2*Cd rho is the density of air, A is the cross sectional area of the golf ball, V is the velocity of
the golf ball, and Cd is called the drag coefficient.

Here we must introduce a few more assumptions for use throughout this problem. First being the cross-sectional area of the golf ball. The USGA has a set limit for the diameter of the golf ball being that the diameter can be no less than 42.67 mm. Using this value as the diameter of the golf ball we then know the cross-sectional area of the golf ball according to A=pi*r^2.

The other assumption that is made deals with the density of the air the ball is traveling through. The density of the air has a large effect on the flight of the golf ball because there can be very large differences between two different location based on the current temperature, elevation, and other weather conditions. For this program it will be assumed that the golf shot is being made at standard ambient temperature and pressure of 25 degrees C (70 F) and 100KPa.

The next assumptions that we must make deal with the Cl and Cd, the lift coefficient and drag coefficient of the golf ball.

The lift coefficient, according to the US patent for a golf ball, usually falls within the range of 0.14 to 0.19. The drag coefficient of the golf ball, also according the US patent for a golf ball, usually falls within the range of 0.21 and 0.255. Each of these coefficients depends greatly on the spin ratio of the individual shot as well as the dimple characteristics of the individual golf ball. Since we are dealing with the flight characteristics of a shot from a driver, we will use the value on the lower end of the ranges. (With a shot with a driver produces a much lower spin rate than most of the other clubs in the bag.)

The lift force is perpendicular to the direction of the velocity of the golf ball and the drag force is in the opposite direction of the velocity of the golf ball. The tricky part is that the direction of the golf ball's velocity in relation to the ground changes as the ball flies through the air. The way that we can break down the flight characteristics is through the use of components when looking at the accelerations and velocities.

Looking at components allows us to examine at the behavior of the golf ball in the y (height) direction and in the x (distance) direction separately.

It's best to consider the force equation we have if we want to come up with equations of motion.
Ftotal = Flift + Fdrag + Fgravity
Looking at each of the forces separately
Flift = (1/2)*rho*A*V^2*Cl
Fdrag = (1/2)*rho*A*V^2*Cd
Allows us to come up with accelerations caused from each of the forces using F=ma,
a_lift = ((1/2)*rho*A*V^2*Cl) / m_ball
a_drag = ((1/2)*rho*A*V^2*Cd) / m_ball
We can now use sines and cosines to come up with the x and y components of each of these accelerations,
a_x = a_drag*-cos(theta) + a_lift*-sin(theta)
a_y = a_drag*-sin(theta) + a_lift*cos(theta) - g

Where theta is the current angle between the direction of the velocity of the golf ball and the ground. Since the angle theta changes throughout the flight of the ball, the angle theta can be defined as a function of time

Theta(t) = ArcTangent(V_y(t) / V_x(t))

Now that we have equations for the acceleration of the golf ball in the x and y directions we can apply them to our equations of motion to produce an equation for the velocity in the x and y directions, as well as distance in the x and y directions.

a_x(t) = a_drag*-cos(theta(t)) + a_lift*-sin(theta(t))

v_x(t) = a_x*t + v_x_initial

x(t) = v_x(t)

The same can be applied for the y direction,

a_y(t) = a_drag*-sin(theta(t)) + a_lift*cos(theta(t)) - g

v_y(t) = a_x*t + v_y_initial

y(t) = v_y(t)

Procedure:

With the aid of Fortran 90, I was then able to create a program based on all of the above to create a graph that shows the flight of a golf ball. The program asks for the swing speed of the golf club in mph and the loft of their driver in degrees. The program then converts the swing speed from mph to m/s. Also, in order to be able to use the separate x and y components of the initial velocity of the golf ball, the launch angle of the golf ball must also be known. Here I was required to make another assumption about the moment of impact between the club and the ball. For a tee shot with the driver, most golfers catch the golf ball on the upswing of the club. Knowing this I assumed that the launch angle of the golf ball was 3 degrees more than the loft of the driver. This relationship between the launch angle of the golf ball and the loft of the club is not always true, but for the average drive this will be a generally good approximation.

The main component of the program deals with the accelerations, velocities, and distances in the x and y directions. Here is the code that was used to create values for the y heights and the x distances.

```
DO i=2,100000
  a_drag(i) = ( (.5*air_density*area_cross*dC)/ball_mass) * (V(i-1)**2.)
  a_lift(i) = ( (.5*air_density*area_cross*lC)/ball_mass) * (V(i-1)**2.)

  a_x(i) = a_drag(i)*(-COS(theta(i))) + a_lift(i)*(-SIN(theta(i-1)))
  v_x(i) = v_x(i-1) + a_x(i)*delta_t
  x(i) = x(i-1) + v_x(I)*delta_t

  a_y(i) = -g + a_drag(i)*(-SIN(theta(i))) + a_lift(i)*(COS(theta(i-1)))
  v_y(i) = v_y(i-1) + a_y(i)*delta_t
  y(i) = y(i-1) + v_y(I)*delta_t

  V(i) = SQRT( v_x(i)**2. + v_y(i)**2. )
  theta(i) = ATAN( v_y(i) / v_x(i) )

  last_point = 0
   IF ( (y(i)<0) .AND. (last_point == 0)) THEN
    max_distance_m = x(i)
     EXIT
   END IF
 END DO
```

Using this format for creating values for the x and y values of a certain golf shot I was then able to plot a number of these values on a graph that shows the flight path of the golf ball.

Here is an example of a run-through of the program:

```
Welcome to the driver distance figure-outer.
  The driver distance figure outer will ask you for your swing speed along with
the loft of your driver,
  and in return figure out how far your shot will travel.
 So let's get started.

 What is your swing speed in mph?
125

 What is the loft of you driver (in degrees)?
8.5

 The length of you shot was          333 yards
 Wow, great shot!

 Have a great day!
```
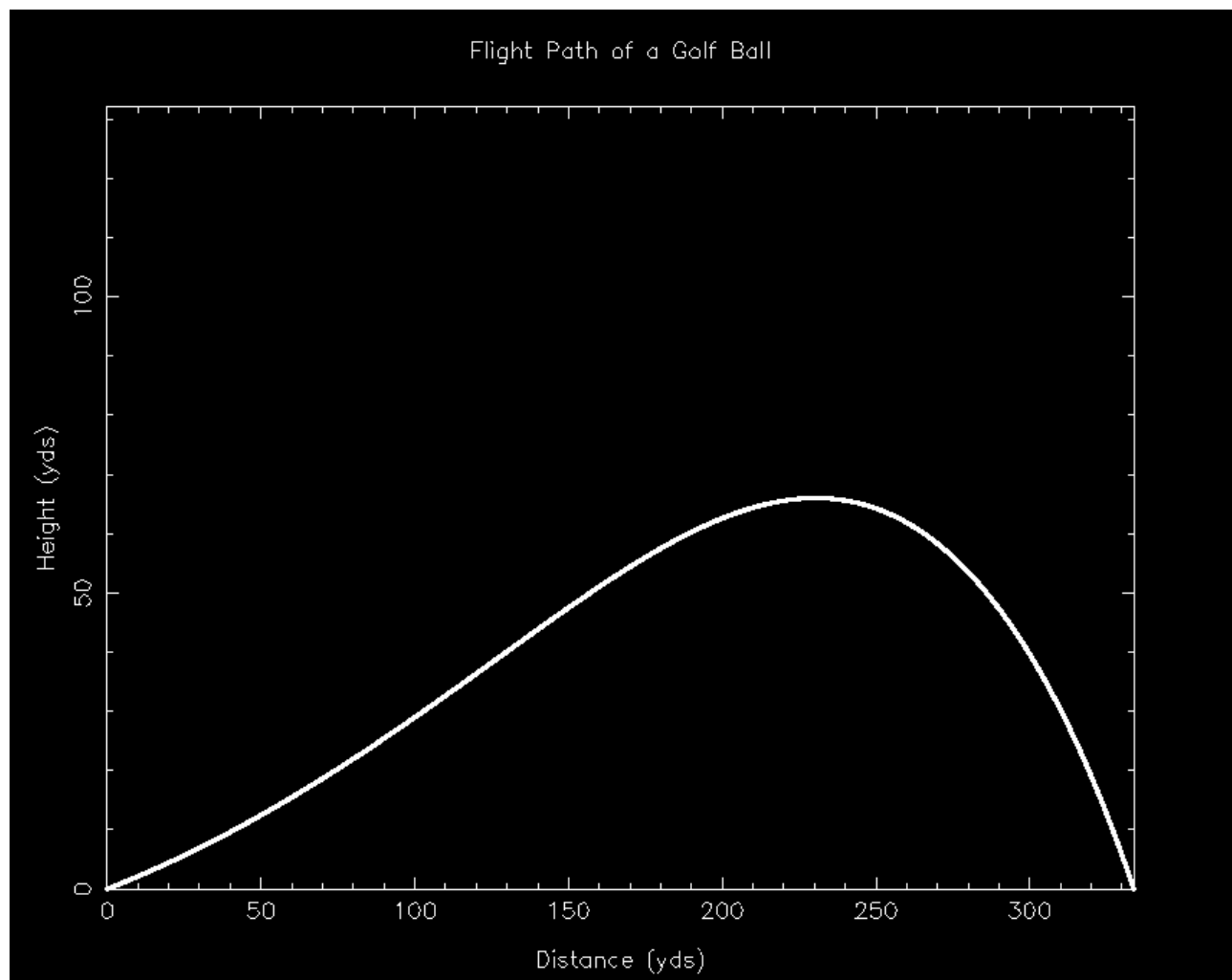
And here is the graph that was created from this trial.

Observations:
Looking at the graph we can see the initial rise of the golf ball created from the lift force, and then at the end of the shot we can see how the drag force causes the ball to seem to fall down after the golf ball reaches it's apex.

Conclusions:
Through the consideration of the lift force and the drag force along with the force of gravity, I was able to model the expected flight of a golf ball. The lift force causes the ball to rise up into the air while the drag force causes the ball to slow down throughout the shot.

Here is a list of the assumptions that were made throughout the creation of the Fortran 90 program.
  *The golf shot is being made at standard ambient temperature and pressure of 25 degrees C (70 F) and 100KPa.
  That the average driver club head mass is 200 grams.
  The drag coefficient of a golf ball is 0.21
  The lift coefficient of a golf ball is 0.14
  The radius of a golf ball is 21.335 mm
  The mass of a golf ball is 45.93 grams
  The coefficient of restitution of the club being used is at the USGA limit of 0.83
  The average mass of a driver club head is 200 grams
  *The golf ball maintains a constant spin rate throughout the flight of the shot.
  *The launch angle of the shot is 3 degrees more than the loft of the driver.
  *The rotation of the golf ball is only in the x-direction and does not have any (what would be) z-components
  That the force of gravity is a constant, would actually depend upon elevation => distance from the center of the earth.

Here is the code that was used in the Fortran 90 program:

```
! Final Project for Fortran - Andrew Werner - March 13, 2007 - Flight of a Golf
Ball
! Design a project that allows you to investigate a physics problem in depth
throw the use of a Fortran program.
! The problem I chose to investigate is the flight path of a golf ball.


! Assumptions being used(made)


!  *The golf shot is being made at standard ambient temperature and pressure of
25 degrees C (70 F) and 100KPa.
!  That the average driver club head mass is 200 grams.
!  The drag coefficient of a golf ball is 0.21
!  The lift coefficient of a golf ball is 0.14
!  The radius of a golf ball is 21.335 mm
!  The mass of a golf ball is 45.93 grams
!  The coefficient of restitution of the club being used is at the USGA limit of
0.83
!  The average mass of a driver club head is 200 grams
!  *The golf ball maintains a constant spin rate throughout the flight of the
shot.
!  *The launch angle of the shot is 3 degrees more than the loft of the driver.
!  *The rotation of the golf ball is only in the x-direction and does not have
any (what would be) z-components
!  That the force of gravity is a constant, would actually depend upon elevation
=> distance from the center of the earth.

PROGRAM golf_ball
 IMPLICIT NONE

REAL(KIND=8)::launch_angle,swing_speed,loft_club,ball_speed,delta_t=.001,lC=0.14
,dC=0.21,air_density
REAL(KIND=8)::mass_clubhead, drag_force, lift_force, g=9.8,radius,ball_mass
REAL(KIND=8)::area_cross, swing_speed_mph, launch_angle_deg, last_point
REAL,DIMENSION(100000)::a_drag,a_lift,a_x,v_x,x_m,a_y,v_y,y_m,theta,V
,y_yards,x_yards
INTEGER::i
INTEGER:: PGBEG, IER, max_distance_m, max_distance_yds
!LOGICAL:: y_y
REAL(KIND=10)::one=1, zero=0, n_one=(-1), pi !used to determine the value of pi
pi = (ACOS(n_one))


!Launch angle = launch angle of the golf ball in relation to the ground, figure
from loft of club, in degrees
!swing_speed = the speed of the particular golfer, prompt for mph then convert
to m/s
!loft_club = the loft of the club being used in degrees
!ball_speed = the speed of the golf ball in m/s
!t = time in seconds
!delta_t = the increment of time, being used to create a number of heights at
certain t's
!lC = Lift coefficient
!dC = drag coefficient
!air_density = density of the air being traveled through, in g/m^3
!radius = golf ball radius in mm
!ball_mass = mass of the golf ball in grams
```

```
!mass_clubhead = mass of the club head of the golf club
!area_cross = the cross sectional area of a golf ball, pi*r^2
!drag_force = the drag force felt by the golf ball
!lift_force = the lift force felt by the golf ball


PRINT *, 'Welcome to the driver distance figure-outer'
PRINt *, ' The driver distance figure outer will ask you for your swing speed
along with the loft of your driver,'
PRINT *, ' and in return figure out how far your shot will travel.'
PRINT *, "So let's get started."
PRINT *,' '
PRINT *, 'What is your swing speed in mph?'
READ *, swing_speed_mph
PRINT *, ' '
PRINT *, 'What is the loft of you driver (in degrees)?'
READ *, loft_club

!To decrease complications in this project, I will use the case of standard
temperature and pressure (25 C and 100KPA)
!At standard ambient temperature and pressure the density of air is 1.168
kg/(m^3 => 1168 g/(m^3)
air_density = 1.168 !kg/m^3

!The properties of the golf ball of radius and mass will be those as defined by
the USGA in the rules of golf.
!The minimum diameter of a golf ball is specified as 42.67 mm, radius =>21.335
mm.
radius = .021335 !m

!The maximum mass of the golf ball is specified as 45.93 grams by the USGA in
the rules of golf.
ball_mass = .04593 !kg

!The average mass of the driver club head is said to be 200 grams.
mass_clubhead = .200 !kg

!Here I am going to convert the ball_speed_mph from mph into meters/second
!5280 ft = 1 mile, 3600 secs = 1 hour, .304800610 m = 1 ft
swing_speed = swing_speed_mph * (5280./3600.) * .30480061

!Here is the relationship between the swing speed and the resulting ball speed.
!By the conservation of momentum, we know m1v1 = m2v2
!ball_speed = (swing_speed * ball_mass) / clubhead_mass   This equation would
seem to be of the correct form,
!However one of the hot topics in golf is the coefficient restitution,
!which is a measure of the efficiency of the kinetic energy transfer between
club and ball
! The USGA has also put a limit on this attribute of the golf club, which is
0.83
! Applying this attribute to our conservation of momentum equation we get
ball_speed = swing_speed * ( (1.+0.83)/(1.0+ (ball_mass/mass_clubhead) ) )

!For the launch angle, I'm going to take into consideration that most golfers
catch the ball on the slight
! upswing with their driver, adding a few degrees on top of their club head loft
for their launch angle
```

```
launch_angle_deg = loft_club + 3.0
launch_angle = (launch_angle_deg / 180) * pi   !Converting the launch angle into
radians in order to use intrinsic functions.

!In order to be congruent with the forms of equations I am using from my
derivations I will define
! the launch angle to = theta, and the angle of (90 - theta) to = phi
!theta = launch_angle
!phi = 90. - theta

!For the cross_sectional area of the golf ball, we get the form of
area_cross = pi * (radius**2.)
!For the value of pi, above I used a method from one of my previous homeworks




!Now that we have the initial velocity of the ball and the launch angle, we can
go to town on the
! equations that determine the flight of the ball

! The forces on the golf ball are the drag force, the lift force, and the force
of gravity.
! While the force of gravity is always downward (-y), the drag force is always
parallel to the direction of motion,
! while the lift force is always perpendicular to the direction of motion, and
in the case of a golf ball,
! since there is always backspin imparted on the golf ball the lift force will
always be upward (+y)
!From these thoughts we can determine that the both the lift force and the drag
force will have horizontal and
!vertical components.
!Using the form of F=ma, and Ftotal = Fdrag + Flift + mg = ma, we can come up
with the forms of
! a = (Flift + Fdrag + mg) / ball_mass
!Separating this form of the acceleration of the golf ball into x and y
components yields
! x direction: a_x = ( lift_force*COS(phi) + drag_force*SIN(theta) ) / ball_mass
! y direction: a_y = ( lift_force*SIN(phi) + drag_force*COS(theta) + ball_mass*g
) / ball_mass
! After taking two derivatives of y''=a, y'=at + v, y=(1/2)at^2 + vt + y, and
the same for x, I come up with
! x = .5*a_x*(t**2) + t*(ball_speed*SIN(theta))
! y = .5*a_y*(t**2) + t*(ball_speed*COS(theta))
!As equations of the y and x ball positions as a function of time (and currently
theta)

!Now for the drag and lift forces.
!lift_force = .5*air_density*(ball_speed**2)*cross_area*lC
!drag_force = .5*air_density*(ball_speed**2)*cross_area*dC




!lift_force = .5*air_density*(ball_speed**2)*area_cross*lC
!drag_force = .5*air_density*(ball_speed**2)*area_cross*dC

!a_x = ( -lift_force*COS(phi) - drag_force*SIN(theta) ) / ball_mass
!a_y = ( lift_force*SIN(phi) - drag_force*COS(theta) - ball_mass*g ) / ball_mass
```

```fortran
  theta(1) = launch_angle
!With V(i) being the magnitude of the velocity of the golf ball, we can us V =
v_x^2 + v_y^2 for the magnitude
 v_x(1) = ball_speed * COS(launch_angle)
 v_y(1) = ball_speed * SIN(launch_angle)
 V(1) = SQRT( v_x(1)**2. + v_y(1)**2. )
 a_x(1) = 0
 a_y(1) = 0

 x_m(1) = 0
 y_m(1) = 0
 a_lift(1) = 0
 a_drag(1) = 0

  !PRINT *, swing_speed, ball_speed
 !PRINT *, a_y(1:5)
 !PRINT *, v_y(1:5)
 !PRINT *, y(1:5)
 !PRINT *,' '
 !PRINT *, air_density, area_cross, dC, ball_mass, lC, V(1)
 DO i=2,100000
  a_drag(i) = ( (.5*air_density*area_cross*dC)/ball_mass) * (V(i-1)**2.)
  a_lift(i) = ( (.5*air_density*area_cross*lC)/ball_mass) * (V(i-1)**2.)

  a_x(i) = a_drag(i)*(-COS(theta(i))) + a_lift(i)*(-SIN(theta(i-1)))
  v_x(i) = v_x(i-1) + a_x(i)*delta_t
  x_m(i) = x_m(i-1) + v_x(I)*delta_t
  x_yards(i) = x_m(i)*(1./0.9144)

  a_y(i) = -g + a_drag(i)*(-SIN(theta(i))) + a_lift(i)*(COS(theta(i-1)))
  v_y(i) = v_y(i-1) + a_y(i)*delta_t
  y_m(i) = y_m(i-1) + v_y(I)*delta_t
  y_yards(i) = y_m(i)*(1./0.9144)                              !There are 0.9144
meters in a yard.

  V(i) = SQRT( v_x(i)**2. + v_y(i)**2. )
  theta(i) = ATAN( v_y(i) / v_x(i) )

  last_point = 0
   IF ( (y_m(i)<0) .AND. (last_point == 0)) THEN
    max_distance_m = x_m(i)
     EXIT
    END IF
 END DO

 !PRINT *, a_drag(1:5)              Used for debugging the program.
 !PRINT *, a_lift(1:5)
 !PRINT *, a_x(1:5)
 !PRINT *, v_x(1:5)
 !PRINT *, x(1:5)

     !IER = PGBEG(0,'ballflight.ps/PS',1,1)
     IER = PGBEG(0,'/xserve',1,1)                             !Naming the graph,
selecting the graphics device.
     IF (IER.NE.1) STOP
```

```fortran
  CALL
PGENV(0,MAXVAL(REAL(x_yards(1:100000))),0,2*MAXVAL(REAL(y_yards(1:10000))),0,1)
  !Defining plot scales according to the min and max array values.
  CALL PGLAB('Distance (yds)','Height (yds)','Flight Path of a Golf Ball')
!Creating labels for the graph
  CALL PGPT(20000, REAL(x_yards(1:20000)), REAL(y_yards(1:20000)),20)
!Creating the point markers in the graph.
  CALL PGEND


  max_distance_yds = max_distance_m * (1./ 0.9144)
!There are 0.9144 meters in a yard.

 PRINT *,' '
 PRINT *, 'The length of you shot was', max_distance_yds,'yards'
 PRINT *, 'Wow, great shot!'
 PRINT *, ' '
 PRINT *, 'Have a great day!'

END PROGRAM golf_ball
```

References:

1 http://math.ucr.edu/home/baez/physics/General/golf.html
 Magnus effect - effect of the upward force from the spin of a rotating ball

2 http://ffden-2.phys.uaf.edu/211_fall2002.web.dir/josh_fritts/flight.html

3 http://probablegolfinstruction.com/science_golf_ball_flight.htm

4 http://en.wikipedia.org/wiki/Magnus_effect

5 http://en.wikipedia.org/wiki/Golf_ball
 good list of external links

6 http://en.wikipedia.org/wiki/Lift_%28force%29 -- Lift Force

7 http://www.answers.com/topic/magnus-effect
The following equations demonstrate the manipulation of characteristics needed to determine the lift force generated by inducing a mechanical rotation on a ball.

   {F}={1\2} {rho} {V^2 * A * Cl}

   F = lift force
   Ï   = density of the fluid
   V = velocity of the ball
   A = cross sectional area of ball
   Cl = lift coefficient

The lift coefficient is the ratio between the weight of the ball and the air density, velocity squared, and the cross sectional area of the ball. The lift coefficient is very dependent on the spin ratio ((angular velocity*diameter)/(2* linear velocity) of the object (a ball in this case). Lift coefficient may be determined from graphs of experimental data using Reynolds number and spin ratio. Typical lift coefficients of a smooth ball range from 0.2 to 0.6 for spin ratios ranging from 0.5 to 4.5.

8 http://www.grc.nasa.gov/WWW/K-12/airplane/cyl.html
 Lift of a Rotating Cylinder

9 http://www.grc.nasa.gov/WWW/K-12/airplane/beach.html
 Approximate Force on a spinning ball

10
http://www.acoustoscan.com.au/ISSS/Free%20Flight%20Aerodynamics%20of%20Sports%20Balls%20II.pdf
 Flight Aerodynamics of Sports balls - Experimentally determined drag coefficient of a falling golf ball

11 http://pdf.aiaa.org/preview/CDReadyMAAC06_1186/PV2006_2996.pdf
 Flow Analysis around golf balls without and with spinning

12 http://www.patentstorm.us/patents/5935023-description.html
 Golf Ball Patent

13 http://pdf.aiaa.org/jaPreview/AIAAJ/2001/PVJAIMP1278.pdf
 Influence of Aerodynamic Loads of Flight Trajectory of Spinning Spherical Projectile

14 http://en.wikipedia.org/wiki/Air_density
 At standard ambient temperature and pressure (25 Â°C and 100 kPa), dry air has a density of Ï SATP = 1.168 g/L =>kg/m^3

15 http://probablegolfinstruction.com/PGI%20Newsletter/news02-12-04.htm
Equation for ball speed taking into account the COR
 also, mass of the average driver club head is around 200 grams