FLIGHT OF A BATTED BASEBALL

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ABSTRACT

The Fortran 90 computer language was used to create an accurate simulation of the flight of a baseball after making contact with the bat. A two-dimensional analysis of the forces on the baseball during the flight was produced using Newtonian projectile motion equations as a function of time. Using input variables of bat velocity, ball velocity, and trajectory of the ball off the bat, the program "long_ball" plots the flight of the batted baseball and prints the total distance of the ball. Holding chosen variables to be constant to simulate the flight in windless stadium at sea level, the program gives most notable insight on the necessary parameters for a ball to travel home run distance.

INTRODUCTION

Projectile motion is one of the first concepts physics major will encounter. Students use Newton's laws to determine the height, distance, time, and components of velocity and acceleration. To expand on these concepts "long_ball" takes into account drag and lift forces in addition to gravity. The force of drag and lift are not constant and depend on experimentally measured coefficients. The coefficients used in the program are found in the research of Robert K. Adair. Velocity of the baseball off the bat is also an experimentally measured relationship. The model used was built on the research done by Alan M. Nathan. The program assumes a collision at the node of the bat which is the ideal placement for maximum resulting velocity. This is what is known in the baseball world as the "sweet spot." Because of this assumption, the program is the best applied as a simulation of home run hitting.

As the number of home runs continue to rise in Major League Baseball, the demand for explanation increases. Current research has developed more accurate models of pitching, batting, and post-impact flight of a base ball. Research done by Sawicki, Hubbard, and Stronge has even disproved the age-old assumption that fast-balls can be hit further than curve-balls. Lift has been found to be a large contributor to distance. The lift coefficient is found to be function of spin. Since curve-balls have more spin, an optimally hit curve-ball will have more lift and, therefore, more distance, than an optimally hit fast-ball.

To apply current research of the flight of a baseball and the physics of projectile motion to computer programming, Fortran 90 was an ideal choice because of the built-in mathematical and array functions. These functions, as well as loop and "if" statement techniques, were utilized to calculate the range of velocities and positions of the baseball after collision with the bat. The ready-to-use libraries also made Fortran 90 a convenient choice. The "pgplot" library was employed to plot the array calculations of position of the baseball.

DETERMINING LAUNCH SPEED

There are many factors to take into account when calculating the exit speed of the ball after the collision with the bat. The primary factors are the speed of the ball and the speed of the bat. However, the vibration energy transferred as a result of the collision $\mathbf{R_0}$ plays an important role. Nathan expresses this relationship as

$$R_0 = \frac{m_{ball}}{M}\left[1 + \left(\frac{z_k - z_{cm}^2}{r_\gamma}\right)\right] \qquad (1)$$

Where r_γ is the radius of gyration. $\mathbf{z_k}$ is the location of the kth slice (the impact point) and $\mathbf{z_{cm}}$ is the centers of mass, which were assumed to be equal for the program. This means the baseball contacts the node, or the "sweet spot" of the bat. The equation then reduces to

$$R_0 = \frac{m_{ball}}{M} \quad (2)$$

The mass of a baseball $\mathbf{m_{ball}}$ is .145 kg and the mass of the bat **M** is .885 kg. This means $\mathbf{R_o}$ is a constant. $\mathbf{R_o}$ is then related to ball velocity and bat velocity to produce the following equation that leads to launch velocity.

$$v_f = \left[\frac{e_{eff} - R_0}{1 + R_0}\right] v_{ball} + \left[\frac{e_{eff} + 1}{1 + R_0}\right] v_{bat} \quad (3)$$

Where $\mathbf{e_{eff}}$ is an effective coefficient of restitution approximated to be .69 when used in the program. After the velocity of the ball $\mathbf{v_{ball}}$ and bat $\mathbf{v_{bat}}$ are inputted by the user of the program, equation **(3)** determines the launch velocity of the baseball after collision.

POSITION OF A BASEBALL IN FLIGHT

Calculating the position of a baseball in flight begins by breaking down the forces acting on it.

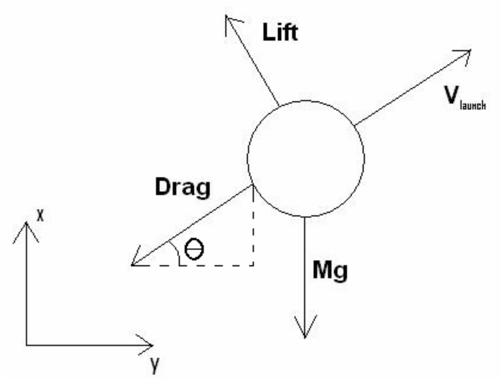

Figure 1
Diagram of the forces acting on a baseball in flight. The Drag force is always opposite the launch velocity as the angle θ of velocity changes. The force of lift is always perpendicular to the Drag force and the velocity. Gravity will always be a downward force on the baseball.

Adair presents the relation for the force of drag on a sphere as

$$F_{drag} = \frac{C_d A \rho v^2}{2} \quad (4)$$

Here $\mathbf{A = \pi r^2}$, with the **r = .0366 m** as the radius of the baseball, $\boldsymbol{\rho}$ **= 1.23 kg/m³** is the density of the air, **v** is the velocity, and $\mathbf{C_d}$ is the drag coefficient. For $\mathbf{C_d}$ **=2,** this is just the force required to move a column of air the size of the ball to match the velocity of the ball. A $\mathbf{C_d}$ **=.5** was chosen for the program because it has been found to be the most accurate for a baseball.

The force of lift is similar to the drag relation and is expressed

$$F_{lift} = \frac{C_l A \rho v^2}{2} \quad (5)$$

A, $\boldsymbol{\rho}$**,** and **v** are the same values as in the drag expression, but $\mathbf{C_l}$ is not found to be a constant coefficient as in the drag force. It strongly depends on the spin parameter which is

$$S = \frac{r\omega}{v} \quad (6)$$

Where **r, ω,** and **v** are the radius of the ball, the spin in rpm, and velocity, respectively. The program assumes an initial spin of 500 rpm which is typical of a home run distance hit. According to Adair, the spin decreased at a rate of 1/5th of the spin per second. Sawicki, Hubbard, and Stronge suggest the spin parameter relates to the $\mathbf{C_l}$ by

$$C_l = 1.5S \qquad S \le 0.1$$
$$C_l = 0.09 + 0.6S \qquad S > 0.1 \quad (7)$$

As the spin and velocity decrease and change the spin parameter, the coefficient of lift will reach a critical point and change accordingly. This is calculated in the program by placing an "if" statement inside of a "do" loop to simulate the change in spin parameter. The force of lift, drag, and gravity can be combined in classical mechanics using Newton's second law.

$$F_{lift} + F_{drag} + F_{gravity} = m_{ball}\, a \quad \textbf{(8)}$$

Where **a** is the acceleration of the baseball. Solving equation (8) for **a** and breaking into x and y components we get

$$a_x = \frac{-F_{lift}\,\mathrm{Sin}\theta - F_{drag}\,\mathrm{Cos}\theta}{m_{ball}} \quad (9)$$

And

$$a_y = \frac{F_{lift}\,\mathrm{Cos}\theta - F_{drag}\,\mathrm{Sin}\theta - F_{gravity}}{m_{ball}} \quad (10)$$

By substituting equations (9) and (10) into a classical projectile motion equation, we are able to find the velocity components as a function of time of the baseball in flight

$$v_x = v_o \cos\theta + t \left(\frac{-F_{lift} \sin\theta - F_{drag} \cos\theta}{m_{ball}} \right) \quad (11)$$

And

$$v_y = v_o \sin\theta + t \left(\frac{F_{lift} \cos\theta - F_{drag} \sin\theta - F_{gravity}}{m_{ball}} \right) \quad (12)$$

With the components of velocity, expressions for the height and distance can now be found using the following recurrence relation:

$$x_i = v_x t + x_{i-1} \quad \textbf{(13)}$$

And

$$y_i = v_y t + y_{i-1} \quad \textbf{(14)}$$

In equations 11 –14, t is a differential unit of time. The program uses a t of .01, which means the position of baseball is calculated 100 times for every second the baseball is in flight. Since the average flight of a baseball is about 5 seconds, the number of iterations i of the "do" loop is 10,000 to ensure it plots the entire flight of the baseball. When the baseball reaches a height of zero, an "if" statement stops the loop.

CONCLUSIONS

Current research on the physics of baseball has been to create used to create an accurate model of a batted baseball in flight. There are factors in the model that are neglected to simplify the program. Most notably, an ideal collision between the bat and baseball are assumed. This is not reasonable in an average game of baseball, but it is more applicable to an event such as a home run derby. The spin of the pitch was assumed to have no effect on the resulting spin of the ball after impact with the bat. Air pressure and wind could be neglected by assuming the ball is hit in a dome at sea level elevation. Under these assumptions, "long_ball" presents an accurate simulation of a baseball traveling home run distance.

REFERENCES

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