Equations for PHYS 320 Final Exam

Equations:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\nu = \nu_0\sqrt{1 - v^2/c^2} \qquad \nu = \nu_0\sqrt{\frac{1 + v/c}{1 - v/c}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x' = \gamma(x - vt) \quad y' = y \quad z' = z \quad t' = \gamma(t - vx/c^2)$$

$$v_{x'} = \frac{v_x - v}{1 - \frac{v_x v}{c^2}} \quad v_{y'} = \frac{v_y}{\gamma(1 - \frac{v_x v}{c^2})} \quad v_{z'} = \frac{v_z}{\gamma(1 - \frac{v_x v}{c^2})}$$

Inverses:

$x = \gamma(x' + vt')$, etc.

Also:

$$\vec{p} = \gamma_v m\vec{v} \qquad E = \gamma mc^2 \qquad E^2 = (mc^2)^2 + (pc)^2$$
$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$\epsilon_n = nh\nu \quad \text{and} \quad E_\gamma = h\nu = \frac{hc}{\lambda}$$

$$u(\nu)d\nu = \frac{8\pi h\nu^3\, d\nu}{c^3 e^{h\nu/kT} - 1}$$

$$h\nu = (KE)_{max} + \phi$$

$$2d\sin(\theta) = n\lambda$$

$$\lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos\phi)$$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} \quad \text{and} \quad h\nu = \gamma mc^2$$

$$W = A\cos(\omega t - kx) \quad \text{where} \quad \omega = 2\pi\nu, \quad k = 2\pi/\lambda$$

$$v_p = \nu\lambda = \omega/k = c^2/v \quad \text{and} \quad v_g = d\omega/dk = v$$

$$\Delta x\Delta p \geq \hbar/2 \quad \text{and} \quad \Delta E\Delta t \geq \hbar/2$$

$$N(\theta) = \frac{N_i ntZ^2e^4}{(8\pi\epsilon_o)^2 r^2 (KE)^2 \sin^4(\theta/2)}, \quad \cot(\theta/2) = \frac{4\pi\epsilon_o KE}{Ze^2}b, \quad f = nt\pi(\frac{\cot(\theta/2)Ze^2}{4\pi\epsilon_o KE})^2$$

$$\frac{1}{\lambda} = R\,(\frac{1}{n_f^2} - \frac{1}{n_i^2})$$

$$PE = \frac{Qq}{4\pi\epsilon_0 r}$$

$$n\lambda = 2\pi r_n, \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi e^2 m}\,, \quad E_n = \frac{-Z^2 e^4 m}{8\epsilon_0^2 h^2}\frac{m'}{m}(\frac{1}{n^2}) = \frac{Z^2 m'}{mn^2}E_1$$

$$(m' = (mM)/(m+M), \quad E_1 = -13.6\,\mathrm{eV})$$

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x,t)\Psi(x,t)$$

IF $U(x,t) = U(x)$, $\Psi(x,t) = \psi(x)\phi(t)$ POSSIBLE:

$$\phi(t) = e^{-(iE/\hbar)t}, \quad -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

$$\int_{-\infty}^{+\infty}\psi_n\psi_m dx = \delta_{mn}$$

OPERATORS: $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$, $\hat{E} = i\hbar\frac{\partial}{\partial t}$, $\hat{KE} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} = -\frac{\hbar^2}{2m}\nabla^2$

HARMONIC OSCILLATOR: $U(x) = \frac{kx^2}{2} = \frac{m\omega^2}{2}$

$$E_n = (n+\tfrac{1}{2})h\nu = (n+\tfrac{1}{2})\hbar\omega$$

$$\psi_n(x) = (\tfrac{2m\nu}{\hbar})^{1/4}(2^n n!)^{-1/2}H_n(y)e^{-y^2/2} \quad = \quad (\tfrac{\sqrt{mk}}{\pi\hbar})^{1/4}(2^n n!)^{-1/2}H_n(y)e^{-y^2/2}$$

y	$H_n(y)$	y	$H_n(y)$	$(y = (\sqrt{mk}/\hbar)^{1/2}x)$
0	1	3	$8y^3 - 12y$	
1	$2y$	4	$16y^4 - 48y^2 + 12$	
2	$4y^2 - 2$			

$$(r,\theta,\phi):\ \nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}, \quad dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\Psi = a\,\psi_n(\vec{r})e^{-iE_n/\hbar} + b\,\psi_m(\vec{r})e^{-iE_m/\hbar} \qquad \int_{space} u\,\psi_n\psi_m^* dV$$

$$\vec{\mu}_l = -\frac{e}{2m}\vec{L} \qquad U_m = -\vec{\mu}\cdot\vec{B} = \frac{e}{2m}L_z B \qquad \vec{\mu}_s = -\frac{e}{m}\vec{S} \qquad U_m = \frac{e}{m}S_z B$$

$$\psi_S = \tfrac{1}{\sqrt{2}}[\psi_{a1}(1)\psi_{a2}(2) + \psi_{a1}(2)\psi_{a2}(1)] \qquad \psi_S = \tfrac{1}{\sqrt{2}}[\psi_{a1}(1)\psi_{a2}(2) - \psi_{a1}(2)\psi_{a2}(1)]$$

$$E_n = Z_{eff}^2 E_1/n^2$$

$K\ :\ (Z-1) \qquad L\ :\ (Z-7.4)$

Integrals: definite

$$\int_0^{+\infty} u^n e^{-u/a} du = n!\, a^{n+1}$$

$$\int_0^{+\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi}\frac{(2n)!}{n!}(\frac{b}{2})^{2n+1}$$

$$\int_0^{+\infty} x^{2n+1} e^{-x^2/b^2} dx = \frac{n!}{2} b^{2n+2}$$

indefinite

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + C$$

$$\int x\sin(ax)dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} + C$$

$$\int x\sin^2(ax)dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2} + C$$

$$\int x^2\sin(ax)dx = \frac{2x\ \sin(ax)}{a^2} - \frac{(a^2x^2 - 2\cos(ax)}{a^3} + C$$

$$\int x^2\sin^2(ax)dx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin(2ax) - \frac{x\cos(2ax)}{4a^2} + C$$

Constants: $c = 2.998\times10^8$ m/s $\quad e = 1.602\times10^{-19}C \quad R = 0.01097\ \text{nm}^{-1}$

$m_e = 9.11\times10^{-31}\text{kg} = 0.5110\, MeV/c^2 \quad m_p = 1.67\times10^{-27}\text{kg} = 938.28\, MeV/c^2$

$h = 6.626\times10^{-34}Js = 4.136\times10^{-15}\text{eV s} \quad \hbar = 1.054\times10^{-34}Js$

$\epsilon_o = 8.854^{-12}\, C^2/Nm^2$ and $1/(4\pi\epsilon_0) = 8.99\times10^9\, Nm^2/C^2$

n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}}e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\, a_0^{3/2}}e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\, a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\cos\theta$
2	1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\, a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\, a_0^{3/2}}\left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right)e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\, a_0^{3/2}}\left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right)e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\cos\theta$
3	1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}}\left(6 - \frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta - 1)$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}(3\cos^2\theta - 1)$
3	2	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{15}}{2}\sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	±2	$\frac{1}{\sqrt{2\pi}}e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\, a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$